WORKED EXAMPLES
IN ELECTRICAL ENGINEERING

BY THE SAME AUTHOR

Worked Examples in Electrotechnology

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IN
ELECTRICAL ENGINEERING

by

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Section I.—THE THEORY, PERFORMANCE AND DESIGN OF ELECTRICAL MACHINERY

Section II.—THE GENERATION AND TRANSMISSION OF ELECTRICAL ENERGY

PREFACE

In an earlier volume (*Worked Examples in Electrotechnology*) the author presented a set of problems taken chiefly from examination papers set for the Ordinary National Certificate in Electrical Engineering, the Preliminary and Intermediate Examinations of the City and Guilds of London Institute in Electrical Engineering Practice and the former Part I Examination for Associate Membership of the Institution of Electrical Engineers. The reception given to *Worked Examples in Electrotechnology* has prompted the compilation of a further set of examples the aim of which is to provide a sequel to it.

It is hoped that the present book will meet the needs of students preparing for the Higher National Certificate in Electrical Engineering, the Final Certificate of the City and Guilds in Electrical Engineering Practice and Part B of the I.E.E. Examination. Owing to the fact that the new Regulations and Syllabuses of the I.E.E. have been in force for a comparatively short time, the examples from I.E.E. papers are drawn in the main from the papers set at the former Part II examinations, but these examples should be found useful by students now taking the present Part B examination. Once again the author's thanks are accorded to the City and Guilds of London Institute, the Institution of Electrical Engineers and the Senate of the University of London for permission to make extracts from their examination papers.

The questions are reproduced exactly as set, but to avoid the repetition of much descriptive matter which can be found in well-known textbooks on the subject, in general only the solutions to the numerical parts of the questions are given. In a few cases the descriptive part is answered where it is deemed necessary to reinforce the numerical solution which follows.

The choice of examples has had to be made with a view to keeping the size and price of the book within reasonable limits. To do this the book has been given a bias to the heavy-current side of electrical engineering, which has meant excluding questions on electronics and electrical measurements.

The author is indebted to those friends and correspondents who have offered suggestions resulting from the publication of the earlier volume, and also to those of his students who by their conscientious attention to their studies have confirmed many of the solutions given here. Each of the solutions has been carefully checked but notification of any errors which may be found will be gratefully received.

W.T.P.

November, 1948.

DEFINITIONS OF TERMS

The following is an explanation of the more important terms used, given under the chapter headings in which they first occur. The definitions are in general conformity with those recommended by the British Standards Institution where such definitions exist.

CHAPTER I

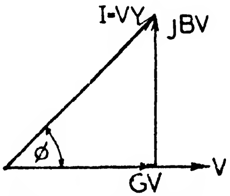
Admittance. The reciprocal of impedance. The ratio of the R.M.S. current to the R.M.S. electromotive force which produces it. The unit in which it is measured is the *mho*, which is the admittance of a circuit whose impedance is one ohm.

Conductance. The component of the current in phase with the applied voltage divided by the applied voltage. The resistance divided by the square of the impedance. The unit is the *mho.*

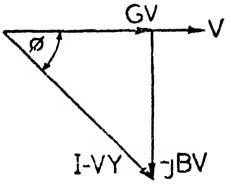
Susceptance. The component of the current in quadrature with the applied voltage divided by the voltage. The reactance divided by the square of the impedance. The unit is the *mho.*

The susceptance of an inductive circuit is conventionally regarded as a negative quantity while that of a capacitive circuit is a positive quantity. Using the j-notation

$Y = G + jB$ for a capacitive circuit.
$Y = G - jB$ for an inductive circuit.
$I = VY$ in each case.



Capacitive circuit



Inductive circuit

It follows from the admittance triangles in the above diagram that $G = Y \cos \phi$ and $B = Y \sin \phi$, where ϕ is the phase angle between the applied voltage and the current.

Phase sequence. The order in which the phases of a polyphase system reach a maximum voltage in the same direction.

CHAPTER II

Distribution factor. The ratio of the vector sum of all the coil e.m.f.s in a distributed winding to their arithmetic sum. It is given by the expression

$$k_m = \frac{\sin \frac{1}{2} g'\psi}{g' \sin \frac{1}{2}\psi} \text{ for the fundamental frequency}$$

and

$$k_{mn} = \frac{\sin \frac{1}{2} g'n\psi}{g' \sin \frac{1}{2}\cdot n\psi} \text{ for the n th. harmonic of the}$$

fundamental frequency.

Coil span factor. The ratio in which the e.m.f. of a distributed winding is reduced due to the coils having a span less than a pole-pitch. It is equal to the cosine of half the angle by which the coil span is less than 180°, 180° corresponding to a full-pitched coil.

$k_e = \cos \tfrac{1}{2}\epsilon$ for the fundamental frequency

and $k_{en} = \cos \tfrac{1}{2} n\epsilon$ for the n th. harmonic of the fundamental frequency.

Synchronous impedance. The ratio of the e.m.f. developed on open circuit by a synchronous generator running at normal speed and with a given excitation to the current flowing on short circuit, the speed and excitation remaining unchanged. Its ohmic value varies according to the excitation at which it is measured, due to the curvature of the magnetization curve of the magnetic circuit and other reasons.

Synchronous reactance. The component of the synchronous impedance which is in quadrature with the short circuit current.

Percentage regulation (of an alternator). The percentage change in voltage which occurs when the load is reduced from the rated output (at the rated power factor and rated voltage) to no-load, the speed and excitation current being maintained constant. The change is expressed as a percentage of the rated voltage.

Leakage inductance. The inductance of a winding due to those magnetic leakage fluxes which are set up in non-useful paths outside the magnetic circuit of a machine. Its value in henries is given by the number of leakage flux-linkages per ampere $\times 10^{-8}$. The *leakage reactance* due to this inductance is the leakage inductance $\times 2\pi \times$ frequency and affects the inductive reactance of the windings and the percentage regulation of the machine.

Armature reaction. A magnetic effect produced by the magneto-motive forces set up by the currents in the stator windings of an alternator.

Output coefficient. A measure of the output of the generator per unit volume of the stator bore per revolution per second. It is given by

$$G = \frac{\text{kVA output}}{D^2L\,N}$$

Specific magnetic loading. The average flux density in which the stator conductors of an alternator lie. Its value depends on the size, type and speed of the machine. For slow speed alternators its value may be between 5500—6500 gauss, while for turbo-alternators a somewhat lower value of 4500—5500 gauss is usual.

Specific electric loading. The number of ampere-conductors per centimetre of stator periphery. For slow speed alternators its value is usually about 300 and for high-speed turbo-alternators about 500. The value adopted affects the copper loss in the stator and the armature reaction. Since the copper loss must be dissipated by adequate ventilation this accounts for the higher value allowed in turbo-alternators where the higher speed assists ventilation.

The output coefficient, specific magnetic loading and specific electric loading are related by the expression

$$G = \pi^2 k_f k_m \bar{B}.ac.\ 10^{-11}$$

Assuming sinusoidal waveform for which $k_f = 1{\cdot}11$ and a distributed winding for which $k_m = 0{\cdot}955$ this relation becomes

$$G = 10{\cdot}45\ \bar{B}.ac.\ 10^{-11}$$

CHAPTER III

Equivalent resistance. The total resistance which, if assumed to be concentrated in one of the windings of a transformer, either primary or secondary, and carrying the full-load current of that winding, causes the same resistive voltage drop at the secondary terminals on full-load as that which actually occurs in the transformer.

Equivalent reactance. The total leakage reactance which, if assumed to be concentrated in one of the windings of a transformer, either primary or secondary, and carrying the full-load current of that winding, causes the same inductive reactive voltage drop at the secondary terminals of the transformer on full-load as that which actually occurs in the transformer.

The winding other than that for which the equivalent resistance and reactance are calculated is regarded as having zero resistance and reactance respectively.

The equivalent resistance and reactance depend upon the actual resistances and reactances of the windings and the turns ratio.

$$R_o = R_1 + k^2 R_2 \quad \text{referred to the primary,}$$

$$X_o = X_1 + k^2 X_2 \quad \text{,,} \quad \text{,,} \quad \text{,,}$$

$$R'_o = R_2 + \frac{1}{k^2} R_1 \quad \text{referred to the secondary,}$$

$$X'_o = X_2 + \frac{1}{k^2} X_1 \quad \text{,,} \quad \text{,,} \quad \text{,,}$$

Equivalent impedance. If the equivalent resistance and equivalent reactance are combined a value for the equivalent impedance referred to one winding or the other is obtained, thus

$$Z_o = \sqrt{R_o^2 + X_o^2} \quad \text{referred to the primary,}$$

$$Z'_o = \sqrt{R_o'^2 + X_o'^2} \quad \text{referred to the secondary.}$$

Percentage resistance. The voltage drop across the equivalent resistance of a transformer on full-load, expressed as a percentage of the no-load voltage of the winding for which the equivalent resistance is calculated.

Percentage reactance. The voltage drop across the equivalent reactance of a transformer on full-load, expressed as a percentage of the no-load voltage of the winding for which the equivalent reactance is calculated.

Percentage impedance. The voltage drop across the equivalent impedance of a transformer on full-load, expressed as a percentage of the no-load voltage of the winding for which the equivalent impedance is calculated.

$$e_r = \frac{I_1 R_o}{V_1} \times 100 = \frac{I_2 R'_o}{V_2} \times 100$$

$$e_x = \frac{I_1 X_o}{V_1} \times 100 = \frac{I_2 X'_o}{V_2} \times 100$$

$$e_z = \frac{I_1 Z_o}{V_1} \times 100 = \frac{I_2 Z'_o}{V_2} \times 100$$

Percentage regulation. The change in secondary voltage which occurs when the load is reduced from the rated output (at rated power factor and rated voltage) to no-load expressed as a percentage of the no-load secondary voltage, the primary applied voltage being maintained constant.

Load factor of losses. The ratio of the average power loss of a transformer calculated over a stated period to the power loss in the transformer when working on the maximum load demanded from it at any time during that period.

Heating time constant. The time from switching on in which a transformer attains 0·632 of its maximum temperature rise, assuming that heat continues to be generated at a constant rate. It is also the time in which the maximum temperature would be attained if the initial rate of temperature rise at the instant of switching on were uniformly maintained.

Chapter IV

Synchronous speed. The speed of an a.c. machine which corresponds to the speed of rotation of the magnetic flux. It is related to the frequency, in cycles per second, and the number of poles on the machine by the expression

$$N_s = \frac{2f}{p} \text{ revolutions per second.}$$

Slip. The ratio of the difference between the synchronous speed of an induction motor and the actual speed of the rotor to the synchronous speed, the difference being generally expressed as a percentage of the latter.

$$s = \frac{N_s - N_r}{N_s} \times 100 \text{ per cent.}$$

Standstill reactance. The leakage reactance of the rotor winding when the rotor is locked and unable to rotate, with the stator supplied at normal voltage and frequency. When the rotor is rotating the leakage reactance is equal to the product of the standstill reactance and the slip (expressed as a fraction not as a percentage).

Synchronous watt. A unit for the measurement of the torque of an a.c. machine. It is that torque which, at the synchronous speed of the machine, would develop a power of one watt. To convert a torque from synchronous watts to the more conventional lb.-ft. units:

$$\text{Torque in lb.-ft.} = \frac{\text{Synchronous watts} \times 33000}{2\pi N_s \times 746}$$

N_s in this case being in revolutions per minute.

Chapter V

Pull-out torque. The torque applied to a synchronous motor which, if exceeded, will result in the speed of the motor falling below synchronous speed.

Synchronous overload capacity. The difference between the load on a synchronous motor at which it falls out of synchronism and the normal full-load of the motor, expressed as a percentage of the full-load.

Chapter VII

Diametral connection. The system of connections between a supply transformer and a synchronous convertor whereby the terminals of each secondary phase of the transformer are connected via the convertor slip-rings to points on the armature 180 electrical degrees apart.

Copper loss ratio. The ratio of the copper loss in a conductor or armature when the machine is running as a synchronous convertor on a given output to the copper loss in the conductor or armature when the machine is being driven as a d.c. generator on the same output.

Chapter VIII

Interphase transformer. A device for equalizing the potentials of two anodes of a rectifier at a time, so that at any instant the load is always being shared by two anodes working effectively in parallel. A six-anode rectifier therefore functions as two three-phase rectifiers connected in parallel.

Transition load. There is a certain load for a six-anode rectifier working with an interphase transformer below which the interphase transformer does not function and the rectifier becomes a simple six-phase one. This minimum load is called the transition load.

Overlap angle. The interval, in electrical degrees, during which two anodes of a mercury arc rectifier conduct at the same time due to their potentials being equalized by the presence of reactance in the anode circuits.

Ignition angle. The interval, in electrical degrees, by which the normal firing instants of the rectifier anodes are delayed by control of the potentials of grids placed between the anodes and the cathode.

Percentage regulation. The change in the output voltage of a rectifier which occurs when the load is reduced from the rated output (at rated voltage) to no-load with rated input a.c. voltage and frequency, expressed as a percentage of the voltage at the rated output.

Chapter IX

Output coefficient. A measure of the output of a d.c. machine per unit volume of the armature core per revolution per second. It is given by

$$G = \frac{\text{Watts output}}{D^2LN}$$

The output coefficient, specific magnetic loading and specific electric loading and the ratio of pole arc to pole pitch are related by the expression

$$G = \pi^2.\delta.\bar{B}.ac.10^{-8}$$

where $\bar{B}$ and ac are defined as in Chapter II.

Slot span. The distance around the circumference of an armature separating the two sides of a coil, given in slot pitches.

Commutator span. The distance around the circumference of a commutator between the points of connection of the start and finish of an armature coil, measured in terms of commutator segments.

Chapter X

Thermal efficiency. The percentage of the heat energy developed by fuel consumption in a generating station which is converted into electrical energy output.

Maximum demand. The maximum current, power or volt-amperes supplied to a consumer during a prescribed period. The power maximum demand usually employed is determined by integrating the power during successive equal intervals of time and recording the highest. The duration of the interval usually prescribed is half an hour.

Load factor. The ratio of the total number of kilowatt-hours supplied to a consumer during a given period to the total number of kilowatt-hours which would have been supplied if the maximum demand had been maintained throughout the period.

Chapter XI

Sectional busbar. When a number of generators operate in parallel each generator is usually connected to its own busbar, known as a sectional busbar, in order to limit the fault current flowing in the event of a fault occurring on any one generator. The sectional busbars are linked together through current-limiting reactors.

Interconnector. A feeder connecting two generating stations, or a generating station and an important sub-station, having no intermediate connections and in which the energy may normally flow in either direction.

Current-limiting reactor. A reactor inserted in a circuit, e.g. between a generator and a busbar, for the purpose of limiting the current to a predetermined value.

Percentage rating of a reactor. The voltage drop across a reactor, when the rated current at rated frequency is flowing through it, expressed as a percentage of the voltage between lines on single-phase circuits or of the voltage between line and neutral on three-phase circuits.

$$\text{Percentage rating} = \frac{\text{Ohmic reactance} \times \text{rated current}}{\text{Rated voltage}} \times 100$$
$$= \frac{\text{Ohmic reactance} \times \text{rated kVA}}{(\text{Rated kV})^2 \times 10}$$

Chapter XII

Kelvin's law. The most economical size of conductor for a transmission line is that for which the annual cost of the energy dissipated in the line is equal to the cost of interest and depreciation on that part of the conductor whose capital cost is proportional to its cross sectional area.

Chapter XIII

Corona. A discharge of electricity which appears round a conductor when the potential gradient at the surface of the conductor exceeds a certain value.

Disruptive critical voltage. The minimum voltage required between the conductors of a transmission line (or between line and neutral of a three-phase line) to produce sufficient potential gradient at the surface of the conductor to cause breakdown of the surrounding air and the passage of a current.

Air density factor. The ratio in which the disruptive critical voltage of a transmission line is reduced due to the density of the surrounding air being other than its normal value.

Irregularity factor. The ratio in which the disruptive critical voltage of a transmission line is reduced due to the conductors not being completely smooth and round.

String efficiency. The ratio, given as a percentage, of the potential difference between the line and earth over a string of suspension insulators to

the product of the number of insulators in the string and the potential difference across the insulator connected to the line.

Surge impedance. The ratio of the potential difference to the current flowing at any point in a transmission line due to a surge causing a travelling wave to exist upon it.

Chapter XV

Diversity factor. The ratio of the arithmetical sum of the individual maximum demands of the components of a group of power supplies to the simultaneous maximum demand of the group.

SYMBOLS

A list of the symbols used is given below. Some of these symbols are frequently used with subscripts and where this occurs the meaning of the symbol is again stated in the text.

Symbol	*Meaning*
a, A	Cross sectional area
a	Number of armature paths in parallel
ac	Specific electric loading
B	Susceptance
	Flux density
$\bar{B}$	Specific magnetic loading
C	Capacitance
	Number of commutator segments or coils.
	Weight of fuel consumed
d	Diameter of circumscribing circle (of a transformer core)
	Diameter of a conductor
	Spacing of conductor centres
D	Diameter of armature core or stator bore
	Electric flux density
	Sag of a transmission line
	Spacing of core centres (transformers)
	Winding depth
e	Base of Napierian logarithms
	Percentage regulation
e_r	Percentage resistance
e_x	Percentage reactance
e_z	Percentage impedance
E	Electromotive force
f	Frequency
F	Magnetomotive force
g	Potential gradient
	Slots per pole
g_0	Breakdown strength
g'	Number of slots per pole per phase
G	Conductance
	Output coefficient
h	Window height
H	Magnetic force
	Field excitation
	Overall height of a transformer core
i, I	Current
j	Vector operator signifying a 90° phase advance
k	Permittivity
	Turns ratio
k_f	Form factor

Symbol	*Meaning*
k_m	Distribution factor
k_e	Coil span factor
k_w	Winding space factor
l	Half span of a transmission line
L	Armature core length or length of stator bore
	Inductance
	Length of winding or turn
m_o	Irregularity factor
N	Number of slip-rings, anodes, brush sets, turns, etc.
	Speed of rotation
p	Number of poles
	Pressure
P	Power
	Volt-amperes or Kilovolt-amperes
q, Q	Electric charge
r, R	Resistance
r	Radius of conductor
	Radial thickness of ice, insulation, etc.
	Rate of interest per cent
$\left.\begin{matrix} R_o \\ R_o' \end{matrix}\right\}$	Equivalent resistance of a transformer
R	Overall radius
s	Slip
	Specific gravity
S	Number of slots
t	Thickness of laminations
	Time interval
T	Line tension
	Number of turns
	Torque
u	Number of conductors per slot
v, V	Potential difference; voltage
w	Density of water
	Weight per foot run of transmission line
	Window width
W	Overall width of transformer core
	Weight of water evaporated
X	Reactance
$\left.\begin{matrix} X_o \\ X_o' \end{matrix}\right\}$	Equivalent reactance of a transformer
y_c	Commutator span
Y	Admittance
	Length of pole pitch
Z	Impedance
	Number of armature or stator conducto
$\left.\begin{matrix} Z_o \\ Z_o' \end{matrix}\right\}$	Equivalent impedance of a transformer
α, β, θ, ϕ	Phase angles

Symbol	*Meaning*
α	Ratio of rotor resistance to standstill reactance
	Heating time constant
	Overlap angle
β	Ignition angle
	Ratio of rotor circuit resistance on successive starter studs
δ	Air density factor
	Current density
	Ratio of pole arc to pole pitch
ϵ	Angle of chording
	Electric force
θ	Temperature rise
Φ	Flux per pole
η	Efficiency
ρ	Resistivity
ω	$2\pi \times$ frequency

Vector quantities are indicated by a dot printed below the symbol. The magnitude of a quantity (vector or scalar) is indicated by the plain symbol.

SECTION I
THE THEORY, PERFORMANCE AND DESIGN OF ELECTRICAL MACHINERY

CHAPTER I

SYMBOLIC NOTATION AND ITS APPLICATION TO COMPLEX CIRCUITS

(i) The operator 'j'.

1. *Explain the symbolic method of vector representation and express the vector* $4 + j5$ *in its various forms.*

Draw the following vectors to a scale of inches: (1) $(1 + j2) - (j1{\cdot}5 - 1)$, (2) $(j - 1) \times (j0{\cdot}5 - 2)$, (3) $(1{\cdot}5\angle\pi/3)^2$ (*C. and G. Final, Pt. I,* 1924)

The vector $4 + j5$ may be expressed in the following forms:

(1) In rectangular co-ordinates,

$$4 + j5 = (4, 5)$$

(2) In polar co-ordinates,

$$4 + j5 = A\angle\theta \text{ or } Ae^{j\theta}$$

$$\text{where}\quad A = \sqrt{4^2 + 5^2} = \sqrt{41} = 6{\cdot}4$$

$$\text{and}\ \tan\theta = \frac{5}{4} = 1{\cdot}25$$

$$\theta = 51°20' = 0{\cdot}285\pi \text{ radians.}$$

$$\text{Hence,}\quad \mathbf{4 + j5 = 6{\cdot}4\angle 51°20'} \text{ or } \mathbf{6{\cdot}4e^{j0{\cdot}285\pi}}$$

$(1 + j2) - (j1{\cdot}5 - 1) = (2 + j0{\cdot}5)$ *see* Fig. 1(a)

$(j - 1) \times (j0{\cdot}5 - 2) = (j^2 0{\cdot}5 - j2{\cdot}5 + 2)$

$= (1{\cdot}5 - j2{\cdot}5)$ *see* Fig. 1(b)

$(1{\cdot}5\angle\pi/3)^2 = 2{\cdot}25\angle 2\pi/3$ *see* Fig. 1(c)

These three vectors are drawn in Fig. 1 to a scale of inches half full size:

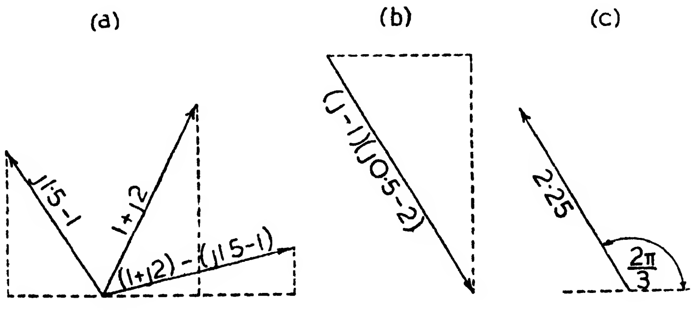


Fig. 1

2. *Explain the method of representing a vector quantity by the 'j' notation. The current in a circuit is given by* $(4{\cdot}5 + j12)$ *when the applied voltage is* $(100 + j150)$. *Determine (a) the complex expression for the impedance, stating whether it is inductive or capacitive, (b) the power, (c) the phase angle between the current and the voltage.* (*C. and G. Final, Pt. I,* 1936)

(a)

$$\begin{aligned} \underset{\cdot}{V} &= 100 + j150 \text{ volts,} \\ \underset{\cdot}{I} &= 4{\cdot}5 + j12 \text{ amperes,} \\ \text{Impedance } \underset{\cdot}{Z} &= \underset{\cdot}{V} \div \underset{\cdot}{I} \\ &= \frac{100 + j150}{4{\cdot}5 + j12} \text{ ohms.} \end{aligned}$$

Rationalize the denominator by multiplying both numerator and denominator by $4{\cdot}5 - j12$.

$$\begin{aligned} \underset{\cdot}{Z} &= \frac{100 + j150}{4{\cdot}5 + j12} \times \frac{4{\cdot}5 - j12}{4{\cdot}5 - j12} \\ &= \frac{2250 - j525}{4{\cdot}5^2 + 12^2} \\ &= \mathbf{13{\cdot}7 - j3{\cdot}196 \text{ ohms.}} \end{aligned}$$

Since the quadrature term in this expression is negative, **the impedance is capacitive.**

(b) Power

$$\begin{aligned} P &= V.I. \cos\,(\alpha\text{-}\beta) \\ V &= \sqrt{100^2 + 150^2} = 180{\cdot}3 \text{ volts.} \\ I &= \sqrt{4{\cdot}5^2 + 12^2} = 12{\cdot}82 \text{ amperes.} \\ \alpha &= \text{arc tan } \frac{150}{100} = 56^\circ\ 19' \\ \beta &= \text{arc tan } \frac{12}{4{\cdot}5} = 69^\circ\ 27' \end{aligned}$$

Hence,

$$\begin{aligned} P &= 180{\cdot}3 \times 12{\cdot}82 \times \cos\,(56^\circ\ 19' - 69^\circ\ 27') \\ &= \mathbf{2250 \text{ watts.}} \end{aligned}$$

Alternatively,

$$\begin{aligned} P &= (\text{Product of quadrature terms of } \underset{\cdot}{V} \text{ and } \underset{\cdot}{I}) \\ &\quad + (\text{Product of in-phase terms of } \underset{\cdot}{V} \text{ and } \underset{\cdot}{I}) \\ &= (150 \times 12) + (100 \times 4{\cdot}5) \\ &= (1800 + 450) \\ &= \mathbf{2250 \text{ watts.}} \end{aligned}$$

(c) Phase angle between voltage and current

$$\begin{aligned} &= (\alpha\text{-}\beta) \\ &= (56^\circ 19' - 69^\circ 27') \\ &= \mathbf{10^\circ 8'}, \textbf{ the current leading.} \end{aligned}$$

(ii) Parallel and series-parallel circuits.

3. *A voltage of* $200\angle\underline{30^\circ}$ *volts is applied to two circuits connected in parallel. The currents in the respective branches are* $20\angle\underline{60^\circ}$ *amperes and* $40\angle\underline{-30^\circ}$ *amperes. Find the kVA and kW in each branch circuit and in the main circuit. Express the current in the main circuit in the form* $A + jB$.

(*C. and G. Final, Pt. I,* 1938)

$$\begin{aligned} \underset{\cdot}{V} = 200\angle\underline{30^\circ} &= 200(\cos 30^\circ + j\sin 30^\circ) \\ &= 200(0{\cdot}866 + j0{\cdot}5) \text{ volts.} \\ \underset{\cdot}{I}_1 = 20\angle\underline{60^\circ} &= 20(\cos 60^\circ + j\sin 60^\circ) \end{aligned}$$

$= 20(0{\cdot}5 + j0{\cdot}866)$ amperes.

$I_2 = 40\angle\underline{-30^\circ} = 40(\cos\text{-}30^\circ + j\sin\text{-}30^\circ)$

$= 40(0{\cdot}866 - j0{\cdot}5)$ amperes.

Volt-amperes in first circuit $= 200 \times 20$

$= 4000$ VA $=$ **4 kVA.**

Power in first circuit $= V \times I_1 \times$ (cosine of angle between V and I_1)

$= 200 \times 20 \times (\cos 30^\circ)$

$=$ 3464 watts

$=$ **3·464 kW.**

Power in second circuit $= V \times I_2 \times$ (cosine of angle between V and I_2)

$= 200 \times 40 \times \cos(30^\circ\text{-}(\text{-}30^\circ))$

$= 200 \times 40 \times \cos 60^\circ$

$= 4000$ watts

$=$ **4 kW.**

Volt-amperes in second circuit $= 200 \times 40$

$= 8000$ VA

$=$ **8 kVA.**

Current in main circuit $I = I_1 + I_2$

$= 20(0{\cdot}5 + j0{\cdot}866) + 40(0{\cdot}866 - j0{\cdot}5)$

$= (44{\cdot}64 - j2{\cdot}68)$ **amperes.**

$I = \sqrt{44{\cdot}64^2 + 2{\cdot}68^2}$ amperes

$=$ 44·72 amperes.

kVA in main circuit $= 200 \times 44{\cdot}72 \div 1000$

$=$ **8·944 kVA.**

Power in main circuit $=$ Sum of powers in branch circuits

$= 3{\cdot}464$ kW $+ 4$ kW

$=$ **7·464 kW.**

4. *A coil having an impedance of* 8 + *j*6 *ohms is connected across a* 200-*volt supply. Express the current in the coil in polar and in rectangular co-ordinate forms.*

If a condenser having a susceptance of 0·1 *mho is placed in parallel with the coil, find the magnitude of the current taken from the supply.*

(*C. and G. Final, Pt. I,* 1940)

Coil admittance

$$Y = \frac{1}{Z}$$

$$= \frac{1}{8 + j6}$$

$$= \frac{1}{8 + j6} \times \frac{8 - j6}{8 - j6}$$

$$= \frac{8 - j6}{8^2 + 6^2}$$

$$= 0{\cdot}08 - j0{\cdot}06 \text{ mho}$$

Current in the coil $I = VY$

$= 200\,(0{\cdot}08 - j0{\cdot}06)$ amperes

$=$ **(16 − j12) amperes.**

In polar form, $\underset{\cdot}{I} = A\angle\theta$

where $A = \sqrt{16^2 + (-12)^2} = 20$

$\theta = \text{arc tan } (-12/16) = -36°52'$

Hence, $\underset{\cdot}{\mathbf{I}} = \mathbf{20\angle\underline{-36°52'}}$ **amperes.**

$$\text{Admittance of the condenser} = \frac{1}{-j\frac{1}{\omega C}} = -\frac{\omega C}{j} = j\omega C = j0{\cdot}1 \text{ mho.}$$

Total admittance of parallel circuit $= 0{\cdot}08 - j0{\cdot}06 + j0{\cdot}1$

$= 0{\cdot}08 + j0{\cdot}04$ mho.

Therefore, total current $\underset{\cdot}{I}$ = Applied p.d. × total admittance,

$= 200\ (0{\cdot}08 + j0{\cdot}04)$ amperes

$= \mathbf{16 + j8{\cdot}0}$ **amperes**

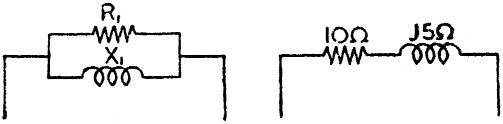
$I = \sqrt{16^2 + 8{\cdot}0^2}$

$= \mathbf{17{\cdot}89}$ **amperes.**

5. *Define conductance, susceptance, and admittance with reference to an alternating-current circuit.*

Find the values of the conductance and susceptance which, when placed in parallel with each other, will be equivalent to a circuit consisting of a resistance of 10 *ohms in series with a reactance of* 5 *ohms.*

(*C. and G., Final Pt. I*, 1943)



(a) (b)

Fig. 2

In the parallel circuit, Fig. 2(a),

$$\text{Conductance } G = \frac{1}{R_1}$$

$$\text{Susceptance } B = \frac{1}{X_1}$$

In the equivalent series circuit of Fig. 2(b),

$\underset{\cdot}{Z} = 10 + j5$ ohms,

$$\text{Admittance } \underset{\cdot}{Y} = \frac{1}{10 + j5} = \frac{1}{10 + j5} \times \frac{10 - j5}{10 - j5} = \frac{10 - j5}{10^2 + 5^2} = 0{\cdot}08 - j0{\cdot}04 \text{ mho.}$$

For the circuits to be equivalent,

$$G = \frac{1}{R_1} = \mathbf{0{\cdot}08\ mho}, \text{ and } B = -\frac{1}{X_1} = \mathbf{-0{\cdot}04\ mho.}$$

6. *Two coils of resistance* 10 *ohms and* 20 *ohms and inductance* 0·02 *henry and* 0·03 *henry respectively are connected in parallel across a* 200-*volt*, 50-*frequency supply. Calculate* (*a*) *the conductance, susceptance, and admittance of each coil,* (*b*) *the total current taken by the two coils connected in parallel,* (*c*) *the resistance and inductance of a single coil which will take the same current and power as the original circuit.* (*C. and G., Final Pt. I,* 1941)

For coil A, $X_a = 2\pi \times 50 \times 0{\cdot}02 \text{ ohms} = 6{\cdot}28 \text{ ohms.}$

For coil B, $X_b = 2\pi \times 50 \times 0{\cdot}03 \text{ ohms} = 9{\cdot}42 \text{ ohms.}$

For coil A, $Z_a = R_a + jX_a = 10 + j6{\cdot}28 \text{ ohms.}$

$$\begin{aligned} Y_a &= \frac{1}{Z_a} \\ &= \frac{1}{10 + j6{\cdot}28} \\ &= \frac{1}{10 + j6{\cdot}28} \times \frac{10 - j6{\cdot}28}{10 - j6{\cdot}28} \\ &= 0{\cdot}0717 - j0{\cdot}045 \text{ mho.} \end{aligned}$$

$$Y_a = \sqrt{0{\cdot}0717^2 + (-0{\cdot}045)^2} = 0{\cdot}0845 \text{ mho.}$$

Hence, for coil A, **Conductance = 0·0717 mho.**

Susceptance = – 0·045 mho.

Admittance = 0·0845 mho.

For coil B, $Z_b = R_b + jX_b = 20 + j9{\cdot}42 \text{ ohms.}$

$$\begin{aligned} Y_b &= \frac{1}{Z_b} \\ &= \frac{1}{20 + j9{\cdot}42} \\ &= \frac{1}{20 + j9{\cdot}42} \times \frac{20 - j9{\cdot}42}{20 - j9{\cdot}42} \\ &= 0{\cdot}0408 - j0{\cdot}01925 \text{ mho.} \end{aligned}$$

$$Y_b = \sqrt{0{\cdot}0408^2 + 0{\cdot}01925^2} = 0{\cdot}0451 \text{ mho.}$$

Hence, for coil B, **Conductance = 0·0408 mho.**

Susceptance = – 0·01925 mho.

Admittance = 0·0451 mho.

For the parallel circuit,

$$\begin{aligned} \text{total admittance} \quad Y &= (0{\cdot}0717 - j0{\cdot}045) + (0{\cdot}0408 - j0{\cdot}01925) \\ &= 0{\cdot}1125 - j0{\cdot}06425 \text{ mho.} \\ Y &= \sqrt{0{\cdot}1125^2 + (-0{\cdot}06425)^2} \\ &= 0{\cdot}1294 \text{ mho.} \end{aligned}$$

$$\begin{aligned} \text{Total current flowing} \quad I &= VY \\ &= 200 \times 0{\cdot}1294 = \mathbf{25{\cdot}88\ amperes} \end{aligned}$$

Impedance of parallel

$$\text{circuit } Z = \frac{1}{Y}$$

$$=\frac{1}{0{\cdot}1125-j0{\cdot}06425}\times\frac{0{\cdot}1125+j0{\cdot}06425}{0{\cdot}1125+j0{\cdot}06425}$$
$$=6{\cdot}7+j3{\cdot}83 \text{ ohms.}$$

Hence, **equivalent resistance** of single coil = **6·7 ohms,**
equivalent reactance of single coil = 3·83 ohms, which is the reactance offered at 50 c./s. by **0·0122 henry** inductance.

7. *Define "conductance," "susceptance," and "admittance."*

Two circuits are connected in parallel across a 200-*volt,* 50-*frequency supply. One of the circuits takes a current of* 10 *amperes lagging by* 30° *and the other a current of* 12 *amperes leading by* 45°. *Find the conductance and susceptance of each circuit. If a condenser of* 100 *microfarads capacitance is placed in parallel with the above arrangement, find the resultant current, expressing it in the form $A + jB$.* (*C. and G. Final, Pt. I,* 1939)

Current in coil A, $\underset{\cdot}{I}_a = 10\angle\underline{-30^\circ} = 10\,(0{\cdot}866 - j0{\cdot}5)$ amperes.

Current in coil B, $\underset{\cdot}{I}_b = 12\angle\underline{45^\circ} = 12\,(0{\cdot}707 + j0{\cdot}707)$ amperes.

Impedance of coil A, $\underset{\cdot}{Z}_a = \dfrac{\underset{\cdot}{V}}{\underset{\cdot}{I}_a}$

$$= \frac{200}{10(0{\cdot}866 - j0{\cdot}5)}$$
$$= \frac{200(0{\cdot}866 + j0{\cdot}5)}{10(0{\cdot}866^2 + 0{\cdot}5^2)}$$
$$= 20(0{\cdot}866 + j0{\cdot}5) \text{ ohms.}$$

Admittance of coil A, $\underset{\cdot}{Y}_a = \dfrac{1}{\underset{\cdot}{Z}_a}$

$$= \frac{\underset{\cdot}{I}_a}{\underset{\cdot}{V}}$$
$$= \frac{10(0{\cdot}866 - j0{\cdot}5)}{200}$$
$$= 0{\cdot}0433 - j0{\cdot}025 \text{ mho.}$$

i.e., **Conductance of A = 0·0433 mho: susceptance of A = − 0·025 mho.**

Admittance of coil B, $\underset{\cdot}{Y}_b = \dfrac{1}{\underset{\cdot}{Z}_b}$

$$= \frac{\underset{\cdot}{I}_b}{\underset{\cdot}{V}}$$
$$= \frac{12(0{\cdot}707 + j0{\cdot}707)}{200}$$
$$= 0{\cdot}0424 + j0{\cdot}0424 \text{ mho.}$$

i.e., **Conductance of B = 0·0424 mho; susceptance of B = 0·0424 mho.**

Susceptance of 100 microfarad condenser $= j\omega C = j2\pi \times 50 \times 100 \times 10^{-6}$
$$= j0{\cdot}0314 \text{ mho.}$$

Total admittance of parallel circuit $= (0{\cdot}0433 - j0{\cdot}025) + (0{\cdot}0424 + j0{\cdot}0424) + j0{\cdot}0314$
$$= (0{\cdot}0857 + j0{\cdot}0488) \text{ mho} = \underset{\cdot}{Y}$$

Total current taken

$$\begin{aligned} &= \underset{\cdot}{V}\underset{\cdot}{Y} \\ &= 200(0{\cdot}0857 + j0{\cdot}0488) \\ &= \mathbf{17{\cdot}14 + j9{\cdot}76\ amperes.} \end{aligned}$$

8. *Three impedances,* $6 + j5$, $8 - j6$, *and* $8 + j10$ *ohms, are connected in parallel. Calculate the current in each branch when the current in the main circuit is* 20 *amperes.* (*C. and G. Final, Pt. I,* 1944)

Admittance of first branch

$$\begin{aligned} \underset{\cdot}{Y}_1 &= \frac{1}{6 + j5} \\ &= \frac{1}{6 + j5} \times \frac{6 - j5}{6 - j5} \\ &= \frac{6 - j5}{61} = 0{\cdot}0983 - j0{\cdot}08195 \text{ mho.} \end{aligned}$$

Admittance of second branch

$$\begin{aligned} \underset{\cdot}{Y}_2 &= \frac{1}{8 - j6} \\ &= \frac{1}{8 - j6} \times \frac{8 + j6}{8 + j6} \\ &= \frac{8 + j6}{100} \\ &= 0{\cdot}08 + j0{\cdot}06 \text{ mho.} \end{aligned}$$

Admittance of third branch

$$\begin{aligned} \underset{\cdot}{Y}_3 &= \frac{1}{8 + j10} \\ &= \frac{1}{8 + j10} \times \frac{8 - j10}{8 - j10} \\ &= \frac{8 - j10}{164} \\ &= 0{\cdot}04878 - j0{\cdot}06098 \text{ mho.} \end{aligned}$$

Total admittance of circuit

$$\begin{aligned} \underset{\cdot}{Y} &= \underset{\cdot}{Y}_1 + \underset{\cdot}{Y}_2 + \underset{\cdot}{Y}_3 \\ &= 0{\cdot}2271 - j0{\cdot}08293 \text{ mho.} \end{aligned}$$

Current in first branch

$$\begin{aligned} \underset{\cdot}{I}_1 &= \frac{\underset{\cdot}{Y}_1}{\underset{\cdot}{Y}} \times \underset{\cdot}{I} \\ &= \frac{0{\cdot}0983 - j0{\cdot}08195}{0{\cdot}2271 - j0{\cdot}08293} \times 20 \\ &= \frac{20(0{\cdot}0983 - j0{\cdot}08195) \times (0{\cdot}2271 + j0{\cdot}08293}{(0{\cdot}2271)^2 + (0{\cdot}08293)^2} \\ &= 9{\cdot}968 - j3{\cdot}575 \text{ amperes.} \\ I_1 &= \sqrt{9{\cdot}968^2 + 3{\cdot}575^2} \\ &= \mathbf{10{\cdot}59\ amperes.} \end{aligned}$$

Current in second branch

$$\begin{aligned} \underset{\cdot}{I}_2 &= \frac{\underset{\cdot}{Y}_2}{\underset{\cdot}{Y}} \times \underset{\cdot}{I} \\ &= \frac{20(0{\cdot}08 + j0{\cdot}06)\,(0{\cdot}2271 + j0{\cdot}08293}{0{\cdot}2271^2 + 0{\cdot}08293^2} \\ &= 4{\cdot}513 + j6{\cdot}933 \text{ amperes} \\ I_2 &= \sqrt{4{\cdot}513^2 + 6{\cdot}933^2} \\ &= \mathbf{8{\cdot}27\ amperes.} \end{aligned}$$

Current in third branch

$$I_3 = \frac{Y_3}{Y} \cdot I$$

$$= \frac{20(0{\cdot}04878 - j0{\cdot}06098) \times (0{\cdot}2271 + j0{\cdot}08293)}{0{\cdot}2271^2 + 0{\cdot}08293^2}$$

$$= 5{\cdot}524 - j3{\cdot}355 \text{ amperes.}$$

$$I_3 = \sqrt{5{\cdot}524^2 + 3{\cdot}355^2}$$

$$= \mathbf{6{\cdot}46\ amperes.}$$

9. *A resistance of* 5 *ohms is in parallel with an inductance* (*of negligible resistance*) *having a reactance of* 8 *ohms. This circuit is joined in series with another consisting of a resistance of* 2 *ohms in series with a capacitive reactance of* 4 *ohms. The combination is connected to* 100-*volt,* 50-*c./s. mains. Find* (*a*) *the current taken from the supply and its power factor,* (*b*) *the p.d. across each circuit.* (*H.N.C.* 1938)

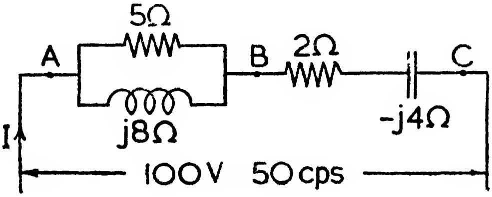


Fig. 3

Referring to the circuit diagram in Fig. 3.

Admittance of circuit AB,

$$Y_{AB} = \frac{1}{5} + \frac{1}{j8}$$

$$= 0{\cdot}2 - j0{\cdot}125 \text{ mho.}$$

Impedance of circuit AB,

$$Z_{AB} = \frac{1}{Y_{AB}}$$

$$= \frac{0{\cdot}2 + j0{\cdot}125}{0{\cdot}2^2 + 0{\cdot}125^2}$$

$$= 3{\cdot}595 + j2{\cdot}25 \text{ ohms.}$$

Hence, impedance of whole circuit from A to C $= 3{\cdot}595 + j2{\cdot}25 + 2 - j4$

$$Z = 5{\cdot}595 - j1{\cdot}75 \text{ ohms.}$$

Current flowing

$$I = \frac{V}{Z}$$

$$= \frac{100}{5{\cdot}595 - j1{\cdot}75}$$

$$= \frac{100(5{\cdot}595 + j1{\cdot}75)}{5{\cdot}595^2 + 1{\cdot}75^2}$$

$$= 16{\cdot}27 + j5{\cdot}09 \text{ amperes.}$$

$$I = \sqrt{16{\cdot}27^2 + 5{\cdot}09^2}$$

$$= \mathbf{17{\cdot}06\ amperes.}$$

Power factor

$$= \cos\,(\text{arc tan } 1{\cdot}75/5{\cdot}595)$$

$$= \cos 17^\circ 24' = \mathbf{0{\cdot}954}$$

Since the quadrature term in the above expression for the current is positive, the **power factor is leading.**

p.d. between A and B $\quad \underset{\cdot}{V}_{AB} = (16{\cdot}27 + j5{\cdot}09)\ (3{\cdot}595 + j2{\cdot}25)$
$= 47{\cdot}08 + j54{\cdot}91 =$ **72·33 volts.**

p.d. between B and C $\quad \underset{\cdot}{V}_{BC} = (16{\cdot}27 + j5{\cdot}09)\ (2 - j4)$
$= 52{\cdot}9 - j54{\cdot}9 =$ **76·26 volts.**

(iii) Unbalanced 3-phase circuits.

10. *A* 440/254-*volt,* 3-*phase,* 4-*core cable supplies an unbalanced load represented by the following impedances in ohms connected between the R, Y, and B phases respectively and the neutral:* 16 + *j*12, 14 − *j*21, *and* 25. *The phase sequence is RYB. Calculate the current in each conductor of the cable and the readings on each of three wattmeters connected in each line to neutral.*
(*C. and G. Final, Pt. II,* 1938)

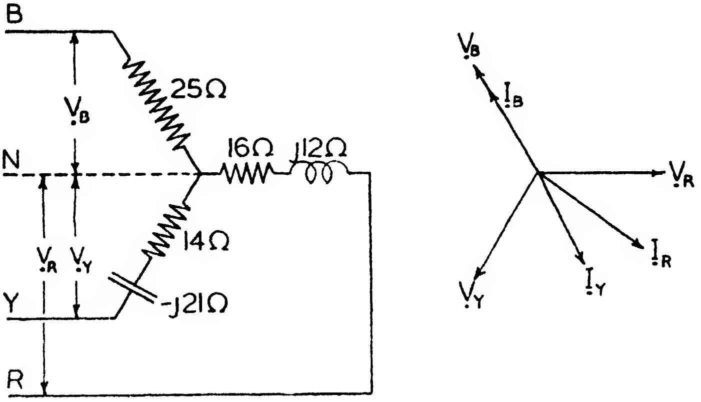


Fig. 4

The p.d.s between the three lines and neutral are given by:

$$\underset{\cdot}{V}_R = 254\ (1 + j0)\ :\ \underset{\cdot}{V}_Y = 254(-0{\cdot}5 - j0{\cdot}866)$$

and $\underset{\cdot}{V}_B = 254\ (-0{\cdot}5 + j0{\cdot}866)$, taking $\underset{\cdot}{V}_R$ as the reference vector.

Then
$$\underset{\cdot}{I}_R = \frac{254\ (1 + j0)}{16 + j12}$$
$$= \frac{254\ (16 - j12)}{16^2 + 12^2}$$
$$= 254\ (0{\cdot}04 - j0{\cdot}03) \text{ amperes.} \qquad (1)$$
$$\underset{\cdot}{I}_Y = \frac{254\ (-0{\cdot}5 - j0{\cdot}866)}{14 - j21}$$
$$= \frac{254\ (-0{\cdot}5 - j0{\cdot}866)\ (14 + j21)}{14^2 + 21^2}$$
$$= 254\ (0{\cdot}01756 - j0{\cdot}03551) \text{ amperes.} \qquad (2)$$
$$\underset{\cdot}{I}_B = \frac{254\ (-0{\cdot}5 + j0{\cdot}866)}{25}$$
$$= 254\ (-0{\cdot}02 + j0{\cdot}03464) \text{ amperes.} \qquad (3)$$

Now $$\begin{aligned} I_N &= -(I_R + I_Y + I_B) \\ &= 254\,[(-0·04 - 0·01756 + 0·02) \\ &\quad + j(0·03 + 0·03551 - 0·03464)] \\ &= 254\,(-0·03756 + j0·03087) \text{ amperes.} \qquad (4) \end{aligned}$$

Evaluating these currents,

from (1), $I_R = 254\sqrt{(0·04)^2 + (0·03)^2}$
$= \mathbf{12·7}$ **amperes**

„ (2), $I_Y = 254\sqrt{(0·01756)^2 + (0·03551)^2}$
$= \mathbf{10·06}$ **amperes**

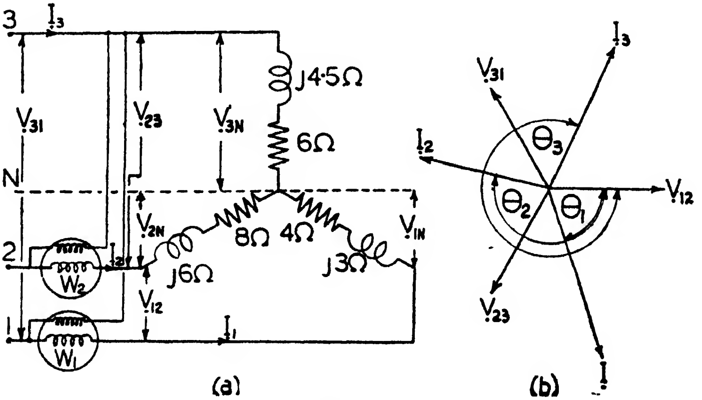
„ (3), $I_B = 254\sqrt{(0·02)^2 + (0·03464)^2}$
$= \mathbf{10·16}$ **amperes**

„ (4), $I_N = 254\sqrt{(0·03756)^2 + (0·03087)^2}$
$= \mathbf{12·35}$ **amperes**

Power in each phase $= (\text{Current})^2 \times \text{Resistance}$
Power in phase R $= (12·7)^2 \times 16$ watts
$= \mathbf{2580}$ **watts.**
Power in phase Y $= (10·06)^2 \times 14$ watts
$= \mathbf{1416}$ **watts.**
Power in phase B $= (10·16)^2 \times 25$ watts
$= \mathbf{2580}$ **watts.**

11. *Three impedances,* $Z_1 = 4 + j3$ *ohms,* $Z_2 = 8 + j6$ *ohms, and* $Z_3 = 6 + j4·5$ *ohms are star-connected across symmetrical* 3-*phase,* 400-*volt mains. Determine the current in each line, the total power supplied, and the reading on each of two wattmeters connected to read the total power. The current coils are in lines* 1 *and* 2 *and the phase sequence is* 1 2 3.

(*C. and G. Final, Pt. I,* 1943)



(a) (b)

Fig. 5

Let the line and phase p.d.s be designated as in Fig. 5(a). Then, expressing the line p.d.s in symbolic notation with V_{12} as the reference vector, $V_{12} = 400(1 + j0)$: $V_{23} = 400(-0·5 - j0·866)$: and $V_{31} = 400(-0·5 + j0·866)$.

The p.d. across each load is given by:

$\underset{\cdot}{V}_{1N} = \underset{\cdot}{I}_1\,(4+j3) : \underset{\cdot}{V}_{2N} = \underset{\cdot}{I}_2\,(8+j6) : \underset{\cdot}{V}_{3N} = \underset{\cdot}{I}_3\,(6+j4{\cdot}5).$

Also, $\underset{\cdot}{V}_{12} = \underset{\cdot}{V}_{1N} - \underset{\cdot}{V}_{2N},\ \underset{\cdot}{V}_{23} = \underset{\cdot}{V}_{2N} - \underset{\cdot}{V}_{3N},\ \underset{\cdot}{V}_{31} = \underset{\cdot}{V}_{3N} - \underset{\cdot}{V}_{1N}$

Hence,

$$\underset{\cdot}{I}_1\,(4+j3) - \underset{\cdot}{I}_2(8+j6) = 400(1+j0) \tag{1}$$

$$+\,\underset{\cdot}{I}_2(8+j6) - \underset{\cdot}{I}_3(6+j4{\cdot}5) = 400(-0{\cdot}5 - j0{\cdot}866) \tag{2}$$

$$-\underset{\cdot}{I}_1\,(4+j3) + \underset{\cdot}{I}_3(6+j4{\cdot}5) = 400(-0{\cdot}5 + j0{\cdot}866) \tag{3}$$

Also, $\underset{\cdot}{I}_1 + \underset{\cdot}{I}_2 + \underset{\cdot}{I}_3 = 0$, i.e. $\underset{\cdot}{I}_1 + \underset{\cdot}{I}_2 = -\,\underset{\cdot}{I}_3$ (4)

Substitute (4) in (2), $\underset{\cdot}{I}_2\,(8+j6) + (\underset{\cdot}{I}_1 + \underset{\cdot}{I}_2)\,(6+j4{\cdot}5) = 400(-0{\cdot}5 - j0{\cdot}866)$

$$\text{i.e. } \underset{\cdot}{I}_1\,(6+j4{\cdot}5) + \underset{\cdot}{I}_2\,(14+j10{\cdot}5) = 400\,(-0{\cdot}5 - j0{\cdot}866) \tag{5}$$

Multiply (1) by $(14+j10{\cdot}5)$,

$$\underset{\cdot}{I}_1\,(4+j3)\,(14+j10{\cdot}5) - \underset{\cdot}{I}_2\,(8+j6)\,(14+j10{\cdot}5) = 400\,(14+j10{\cdot}5)$$

$$\text{i.e. } \underset{\cdot}{I}_1\,(24{\cdot}5+j84) - \underset{\cdot}{I}_2\,(8+j6)\,(14+j10{\cdot}5) = 400\,(14+j10{\cdot}5) \tag{6}$$

Multiply (5) by $(8+j6)$,

$$\underset{\cdot}{I}_1\,(6+j4{\cdot}5)\,(8+j6) + \underset{\cdot}{I}_2\,(8+j6)\,(14+j10{\cdot}5) = 400\,(-0{\cdot}5 - j0{\cdot}866)\,(8+j6)$$

$$\text{i.e. } \underset{\cdot}{I}_1\,(21+j72) + \underset{\cdot}{I}_2\,(8+j6)\,(14+j10{\cdot}5) = 400\,(1{\cdot}196 - j9{\cdot}928) \tag{7}$$

Adding (6) and (7), $\underset{\cdot}{I}_1\,(45{\cdot}5+j156) = 400\,(15{\cdot}196+j0{\cdot}572)$

$$\begin{aligned}
\underset{\cdot}{I}_1 &= \frac{400\,(15{\cdot}196+j0{\cdot}572)}{45{\cdot}5+j156}\\
&= \frac{400\,(15{\cdot}196+j0{\cdot}572)\,(45{\cdot}5-j156)}{45{\cdot}5^2+156^2}\\
&= \frac{400\,(780{\cdot}7-j2345)}{45{\cdot}5^2+156^2}\\
&= 400\,(0{\cdot}0296 - j0{\cdot}0888)
\end{aligned}$$

$$I_1 = 400\sqrt{0{\cdot}0296^2 + 0{\cdot}0888^2} = \mathbf{37{\cdot}44\ amperes.}$$

Substitute for $\underset{\cdot}{I}_1$ in (1),

$$\begin{aligned}
\underset{\cdot}{I}_2\,(8+j6) &= 400\,(0{\cdot}0296 - j0{\cdot}0888)\,(4+j3) - 400\,(1+j0)\\
&= 400\,(0{\cdot}3847 - j0{\cdot}2665) - 400\,(1+j0)\\
&= 400\,(-0{\cdot}6153 - j0{\cdot}2665)\\
\underset{\cdot}{I}_2 &= \frac{400\,(-0{\cdot}6153 - j0{\cdot}2665)}{8+j6}\\
&= \frac{400\,(-0{\cdot}6153 - j0{\cdot}2665)\,(8-j6)}{8^2+6^2}\\
&= \frac{400\,(-6{\cdot}521 + j1{\cdot}56)}{100}\\
&= 400\,(-0{\cdot}06521 + j0{\cdot}0156)
\end{aligned}$$

$$I_2 = 400\sqrt{0{\cdot}0652^2 + 0{\cdot}0156^2} = \mathbf{26{\cdot}81\ amperes.}$$

Substitute for $\underset{\cdot}{I}_1$ in (3),

$$\begin{aligned}
\underset{\cdot}{I}_3\,(6+j4{\cdot}5) &= 400\,(-0{\cdot}5 + j0{\cdot}866) + 400\,(0{\cdot}0296 - j0{\cdot}0888)\,(4+j3)\\
&= 400\,(-0{\cdot}5 + j0{\cdot}866) + 400\,(0{\cdot}3847 - j0{\cdot}2665)\\
&= 400\,(-0{\cdot}1153 + j0{\cdot}5995)
\end{aligned}$$

$$I_3 = \frac{400\,(-0{\cdot}1153 + j0{\cdot}5995)}{6 + j4{\cdot}5}$$

$$= \frac{400\,(-0{\cdot}1153 + j0{\cdot}5995)\,(6 - j4{\cdot}5)}{6^2 + 4{\cdot}5^2}$$

$$= \frac{400\,(2{\cdot}006 + j4{\cdot}116)}{6^2 + 4{\cdot}5^2}$$

$$= 400\,(0{\cdot}0357 + j0{\cdot}0732)$$

$$I_3 = 400\sqrt{0{\cdot}0357^2 + 0{\cdot}0732^2} \qquad = \mathbf{32{\cdot}56\ amperes.}$$

Hence the line currents are:—37·44 amperes: 26·81 amperes: 32·56 amperes.

Wattmeter reading $W_1 = V_{13} \times I_1 \times \cos$ (angle between I_1 and V_{13})

$= 400 \times 37{\cdot}44 \times \cos(\theta_1 - 60°)$

Note: V_{13} is V_{31} **reversed**

$$\tan\theta_1 = \frac{0{\cdot}0888}{0{\cdot}0296} \text{ hence } \theta_1 = 71°36'$$

Therefore, **wattmeter W_1 reading** $= 400 \times 37{\cdot}44 \times \cos 11°36'$

$= \mathbf{14670\ watts}$

Wattmeter W_2 reading $= V_{23} \times I_2 \times \cos$ (angle between I_2 V_{23})

$= 400 \times 26{\cdot}81 \times \cos(\theta_2 - 120°)$

$$\tan\theta_2 = \frac{0{\cdot}0156}{-0{\cdot}0652} \text{ hence } \theta_2 = 193°27'$$

Therefore, **wattmeter reading W_2** $= 400 \times 26{\cdot}81 \times \cos 73°27'$

$= \mathbf{3055\ watts.}$

Total power supplied $= W_1 + W_2$

$= \mathbf{17725\ watts.}$

12. *A 3-phase, mesh-connected load is joined to a 400-volt, 3-phase, 3-wire system. The load impedances are, respectively,* $Z_1 = 20 + j0$; $Z_2 = 10 - j5$; *and* $Z_3 = 15 + j10$ *ohms.*

Determine the line currents.

The transformer feeding the system has its primary star and its secondary mesh connected, the line impedance being negligible. Determine also the current in each of the secondary windings of the transformer.

(London B.Sc. Eng., July, 1945)

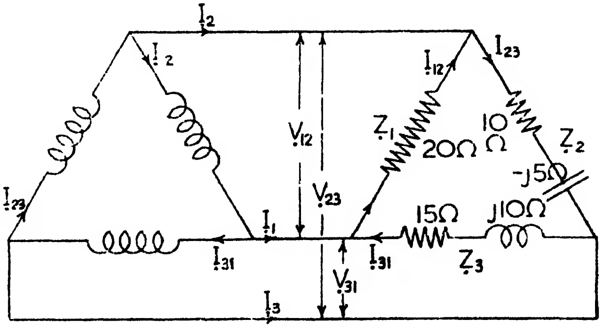


Fig. 6

Let $V_{12} = 400\ (1 + j0)$ volts $= 400\ \angle 0^\circ$ volts.

$V_{23} = 400\ (-0{\cdot}5 - j0{\cdot}866) = 400\ \angle -120^\circ$ volts.

$V_{31} = 400\ (-0{\cdot}5 + j0{\cdot}866) = 400\ \angle -240^\circ$ volts.

Then
$$I_{12} = \frac{V_{12}}{Z_1} = \frac{400\ (1 + j0)}{20 + j0} = 20 + j0 \text{ amperes.}$$

$$I_{23} = \frac{V_{23}}{Z_2} = \frac{400\ (-0{\cdot}5 - j0{\cdot}866)}{10 - j5} = \frac{400\ (-0{\cdot}5 - j0{\cdot}866)\ (10 + j5)}{125} = -2{\cdot}14 - j35{\cdot}71 \text{ amperes.}$$

$$I_{31} = \frac{V_{31}}{Z_3} = \frac{400\ (-0{\cdot}5 + j0{\cdot}866)}{15 + j10} = \frac{400\ (-0{\cdot}5 + j0{\cdot}866)\ (15 - j10)}{325} = 1{\cdot}43 + j22{\cdot}10 \text{ amperes.}$$

The line currents are given by:

$$\begin{aligned} I_1 &= I_{12} - I_{31} \\ &= (20 + j0) - (1{\cdot}43 + j22{\cdot}10) \\ &= 18{\cdot}57 - j22{\cdot}10 = \mathbf{28{\cdot}87\ \angle -50^\circ \text{ amperes.}} \end{aligned}$$

$$\begin{aligned} I_2 &= I_{23} - I_{12} \\ &= (-2{\cdot}14 - j35{\cdot}71) - (20 + j0) \\ &= -22{\cdot}14 - j35{\cdot}71 = \mathbf{42{\cdot}01\ \angle -121^\circ 48' \text{ amperes.}} \end{aligned}$$

$$\begin{aligned} I_3 &= I_{31} - I_{23} \\ &= (1{\cdot}43 + j22{\cdot}10) - (-2{\cdot}14 - j35{\cdot}71) \\ &= 3{\cdot}57 + j57{\cdot}81 = \mathbf{57{\cdot}93\ \angle 86^\circ 30' \text{ amperes.}} \end{aligned}$$

The transformer secondary currents will be the same as those in the corresponding phases of the load, i.e.,

$$\begin{aligned} I_{12} &= 20 + j0 &&= \mathbf{20\ \angle 0^\circ \text{ amperes.}} \\ I_{23} &= -2{\cdot}14 - j35{\cdot}71 &&= \mathbf{35{\cdot}78\ \angle -93^\circ 24' \text{ amperes.}} \\ I_{31} &= 1{\cdot}43 + j22{\cdot}10 &&= \mathbf{22{\cdot}14\ \angle 86^\circ 18' \text{ amperes.}} \end{aligned}$$

CHAPTER II

ALTERNATORS

(i) Generated e.m.f.

13. *Explain the effect on the R.M.S. value of the e.m.f. of an alternator of distributing the armature coils and of skewing the armature slots.*

Define distribution factor (or breadth coefficient) and calculate its value for a 3-phase, single-layer winding with 3 slots per pole per phase. Assume a sinusoidal flux distribution. (*C. and G. Final, Pt. I,* 1938)

The distribution factor of a winding is defined as the ratio $k_m = \frac{\text{vector sum}}{\text{arithmetic sum}}$ of the e.m.f.s in all the coils forming the winding. The vector and arithmetic sums of the coil e.m.f.s are different because of the phase displacement between the e.m.f.s of conductors in adjacent slots. For a 3-phase, single-layer winding with 3 slots per pole per phase the slot arrangement and vector diagram for the e.m.f.s would be as shown in Fig. 7.

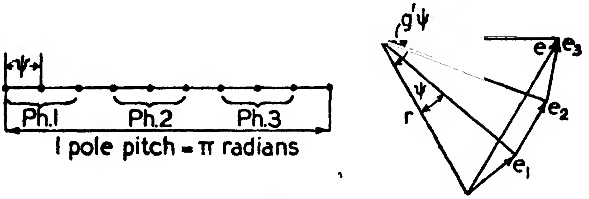


Fig. 7

Then, from the vector diagram,

$$k_m = \frac{2r \sin \frac{g'\psi}{2}}{2rg' \sin \frac{\psi}{2}} = \frac{\sin \frac{g'\psi}{2}}{g' \sin \frac{\psi}{2}}$$

where g' = number of wound slots per pole per phase,
ψ = angular displacement in electrical degrees of e.m.f.s in successive coils in series.

In this problem $g' = 3$, and $\psi = \frac{\pi}{9}$

$$\text{Therefore,}\quad k_m = \frac{\sin\left(3 \times \frac{\pi}{18}\right)}{3 \times \sin \frac{\pi}{18}}$$

$$= \frac{0{\cdot}5}{3 \times 0{\cdot}1736} = 0{\cdot}96$$

i.e. **Distribution factor for this winding is 0·96.**

14. *What are the effects of the distribution and coil span factors on the output and waveform of an alternator? Determine their values for a 3-phase winding with* 4 *slots per pole per phase, the coil span being* 10 *slot pitches. Calculate the percentage increase in the R.M.S. value of the phase voltage due to a* 25 *per cent third harmonic.*

(*C. and G. Final, Pt. II, Sect. A,* 1942)

Distribution factor. The effect of the distribution factor on the output of an alternator is to reduce it by an amount depending on the spread of the winding. The vector sum of the coil e.m.f.s is less than their arithmetic sum which would be given if all the coils were located in the same slot.

The distribution factor is not always the same for harmonics as for the fundamental. For a phase spread of 120° $k_m = 0$ for the third harmonic and multiples of three, which are thus eliminated from the waveform.

Coil span factor. At the fundamental frequency this factor is $k_{e_1} = \cos \frac{1}{2}\epsilon$ where ϵ is the angle in electrical degrees by which the span of the coil is less than a pole-pitch.

The output at the fundamental frequency is reduced in the same ratio as this factor. The nth harmonic is reduced in the ratio

$$k_{en} = \cos \frac{1}{2} n\epsilon$$

Short-chording can thus be used to reduce or eliminate troublesome harmonics.

In the problem given, we see from Fig. 8 that

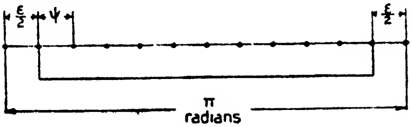


Fig. 8

$$\psi = \pi/12, \quad \epsilon = \pi/6, \text{ i.e. coil span} = 5\pi/6$$

Distribution factor $= \dfrac{\sin \frac{g'\psi}{2}}{g' \sin \frac{\psi}{2}}$

$$= \frac{\sin \frac{4 \times \pi/12}{2}}{4 \sin \frac{\pi/12}{2}}$$

$$= \frac{0{\cdot}5}{4 \times 0{\cdot}1305} = \mathbf{0{\cdot}958}$$

Coil span factor $= \cos \dfrac{\epsilon}{2}$

$$= \cos \frac{\pi}{12} \quad = \mathbf{0{\cdot}966}$$

R.M.S. value of a complex wave $= E = \sqrt{E_1^2 + E_h^2}$

where E_1 = R.M.S. value of the fundamental,

E_h = R.M.S. value of the harmonic.

Hence,
$$E = E_1\sqrt{1 + (E_h/E_1)^2}$$
$$= E_1\sqrt{1 + (0\cdot25)^2} = 1.03\ E_1$$

i.e. **a 25 per cent harmonic increases the R.M.S. value of the phase e.m.f. by 3 per cent.**

15. *Derive an expression for the voltage induced in an alternator armature, assuming full-pitched coils connected in series and sinusoidal flux distribution.*

A star-connected, 3*-phase,* 4*-pole,* 50*-frequency alternator has a single layer winding in* 24 *stator slots. There are* 50 *turns in each coil and the flux per pole is* 5 *megalines. Find the open-circuit line voltage.*

(*C. and G. Final, Pt. I,* 1940)

R.M.S. value of e.m.f. per phase $= 4k_f\ k_m\ f\ \Phi\ T_s \times 10^{-8}$ volts.

where k_f = form factor of the wave = 1·11 for sine wave

k_m = distribution factor

f = frequency = 50 cycles/sec.

Φ = flux per pole = 5×10^6 lines.

T_s = turns in series per phase.

Number of slots per pole $= g = 24/4 = 6$

Number of slots per pole per phase $= g' = g/3 = 2$

Phase difference between adjacent coil sides connected in series $= \pi/g = \pi/6 = \psi$

$$\text{Distribution factor} = \frac{\sin\ (g'\psi/2)}{g'\ \sin\psi/2}$$
$$= \frac{\sin\dfrac{(2 \times \pi/6)}{2}}{2 \times \sin\ \pi/12}$$
$$= \frac{0\cdot5}{2 \times 0\cdot2588} = 0\cdot966$$

$$T_s = \frac{\text{number of coils on stator} \times \text{turns per coil}}{\text{number of phases}}$$
$$= \frac{12 \times 50}{3} = 200$$

Therefore, e.m.f. per phase $= 4 \times 1\cdot11 \times 0\cdot966 \times 50 \times 5 \times 10^6 \times 200 \times 10^{-8}$

$= 2145$ volts.

R.M.S. line e.m.f. $= 2145\sqrt{3} =$ **3715 volts.**

16. *Draw to scale a diagram showing the m.m.f. distribution over a pole-pitch of a synchronous machine with* 3 *slots per pole per phase and* 10 *conductors per slot each carrying a current of* 70·7 *amperes R.M.S.,* (*a*) *for an instant when the current in one phase is a maximum, and* (*b*) *one-twelfth of a cycle later. Indicate the peak value of the sine wave fundamental common to the two m.m.f. wave-shapes.* (*I.E.E., Pt. II, Nov.,* 1942)

The m.m.f. waveform is built up from a consideration of the current distribution over the pole-pitch. Fig. 9 (a) and (c) shows this for the two instants specified.

In the m.m.f. waveforms

$$F' = \frac{g'}{\sqrt{2}}\cdot u.I$$

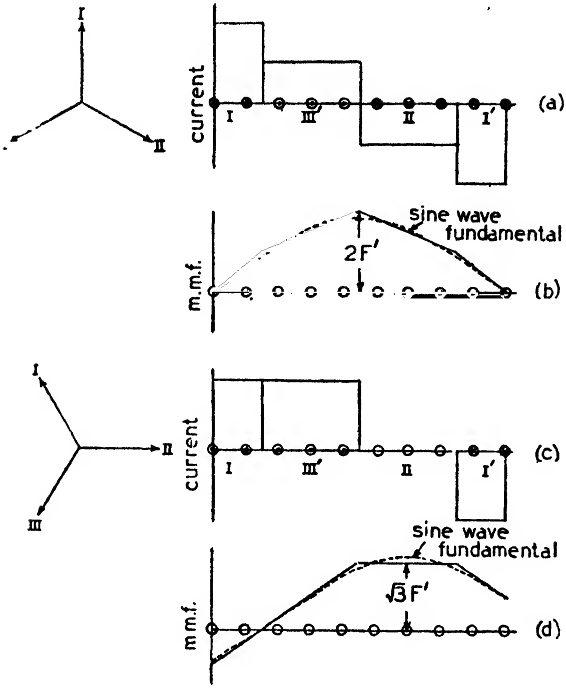


Fig. 9

where $g' =$ number of slots per pole per phase,
$u =$ number of conductors per slot,
$I =$ R.M.S. value of current in conductors in amperes.

Hence, $$F' = \frac{3}{\sqrt{2}} \times 10 \times \frac{100}{\sqrt{2}}$$
$$= 1500 \text{ ampere-turns.}$$

If the m.m.f. waveform is reduced to its corresponding Fourier series it will be found that it has a sinusoidal fundamental component whose peak value is

$$F = \frac{18}{\pi^2} \times F'$$
$$= \frac{18 \times 1500}{\pi^2} = \textbf{2735 ampere-turns.}$$

This fundamental component is common to both waveshapes shown.

17. *Why may an alternator winding be chorded? Find the no-load terminal e.m.f. of a* 4*-pole a.c. generator from the following data: Flux per pole* (*assumed sinusoidally distributed*), 12·0 *mega-maxwells: slots per pole per phase,* 4*: conductors per slot,* 4*: two-layer winding: coil-span,* 150°*: connection, star.*

(*I.E.E., Pt. II, Nov.,* 1938)

An alternator winding may be chorded in order to reduce or eliminate certain harmonics in the e.m.f. waveform. Thus if the chording is 60° the

third harmonic disappears. Similarly a chording of 30° will eliminate the sixth harmonic and so on.

Total number of conductors in the winding $= 4$ cond./slot $\times$ 4 slots/pole/phase $\times$ 4 poles $\times$ 3 phases,

$= 192$ conductors.

Coils in series per phase $T_s = \dfrac{192}{2 \times 3} = 32$

Distribution factor $$k_m = \frac{\sin \dfrac{g'\psi}{2}}{g' \sin \dfrac{\psi}{2}}$$

$g' = 4,\ \psi = \pi/12$

Hence, $k_m = \dfrac{\sin (4 \times \pi/24)}{4 \times \sin \pi/24} = 0{\cdot}958$

Coil span factor $k_e = \cos \frac{1}{2}\epsilon$

$= \cos 15° \qquad = 0{\cdot}966$

e.m.f. per phase $= 4.k_f.k_m.k_e.f.\Phi.T_s.10^{-8}$ volts,

$= 4 \times 1{\cdot}11 \times 0{\cdot}958 \times 0{\cdot}966 \times 50 \times 12 \times 10^{+6} \times 32 \times 10^{-8}$

$= 789{\cdot}1$ volts.

Terminal e.m.f., in star $= \sqrt{3} \times 789{\cdot}1$ volts,

$=$ **1367 volts.**

18. *What are the causes of harmonics in the voltage and current waves of electrical machinery and what means are taken in design to reduce them?*

The phase voltage of a 750-*kW*, 2,200-*volt*, 3-*phase*, 50-*cycle alternator has a* 5 *per cent third harmonic. What is the circulating current on normal voltage if the machine is delta connected? The resistance and reactance per phase are* 0·25 *ohm and* 0·7 *ohm respectively. Express the loss due to the circulating current as a percentage of full-load output.*

(*I.E.E., Pt. II, May,* 1939)

Causes of harmonics in the waveform are:

(a) Non-sinusoidal waveform of the field flux.
(b) Variation in the reluctance of the air-gap due to the slotting of the stator core.

Design measures to reduce the harmonics are:

(a) (i) Shaping the poles by chamfering the pole tips.
  (ii) Skewing the poles.
  (iii) Short-chording the armature winding by making the coil-span less than a full pole-pitch.
  (iv) Distributing the armature winding.
(b) (i) Increasing the number of slots per pole.
  (ii) Skewing the poles through one slot-pitch.
  (iii) Making the number of slots per pole pair an odd number. This eliminates tooth ripples completely from the waveform.

Third harmonics in the e.m.f. waveform are in time-phase in all three phases. When the phases are delta connected these harmonics become

additive around the closed circuit and produce a circulating current.

In the problem given, phase e.m.f. $= 2{,}200$ volts.
Hence, third harmonic per phase $= 0{\cdot}05 \times 2{,}200$ volts,
$= 110$ volts.

Reactance per phase at third harmonic frequency $= 3 \times 0{\cdot}7 = 2{\cdot}1$ ohms.
Impedance per phase at this frequency $= \sqrt{0{\cdot}25^2 + 2{\cdot}1^2}$
$= 2{\cdot}11$ ohms.

Circulating current $= \dfrac{110}{2{\cdot}11}$
$=$ **52·1 amperes.**

Copper loss in three phases $= 3 \times 52{\cdot}1^2 \times 0{\cdot}25$ watts,
$= 2035$ watts.

Percentage of full-load output $= \dfrac{2035}{750{,}000} \times 100$
$=$ **0·271 per cent.**

19. *Each winding of a* 3-*phase,* 50-*cycle alternator has an e.m.f. wave consisting of a fundamental with a maximum value of* 1,000 *volts, a* 20 *per cent third harmonic and a* 10 *per cent fifth harmonic. Calculate the R.M.S. value of the line voltage when the windings are connected* (*a*) *in star,* (*b*) *in delta.*

Find also the circulating current in delta connection if the reactance per phase of the machine at 50 *cycles per second is* 12 *ohms.*

(*C. and G. Final, Pt. I,* 1944)

With both the star and delta connections the third harmonic components of the three phases cancel out at the line terminals because they are co-phasal. Thus the line e.m.f. is composed of the fundamental and fifth harmonic components only.

(a) **Star.** Maximum value of fundamental $= 1000$ volts,
" " fifth harmonic $= 100$ volts.
Hence, " " phase e.m.f. $= \sqrt{1000^2 + 100^2}$
$= 1005$ volts.
R.M.S. value of phase e.m.f. $= 1005 \div \sqrt{2}$
R.M.S. " line e.m.f. $= \sqrt{3} \times 1005 \div \sqrt{2}$
$=$ **1231 volts.**

b) **Delta.** In this case the line e.m.f. is the same as the phase e.m.f.
i.e. **R.M.S. value of line e.m.f.** $= 1005 \div \sqrt{2}$
$=$ **711 volts.**

In delta the third harmonic components are additive around the closed circuit and the value of the circulating current is determined by the reactance per phase at the third harmonic frequency.

Third harmonic e.m.f. per phase $= 1/\sqrt{2} \times 0{\cdot}20 \times 1000$ volts,
$= 141{\cdot}4$ volts.
Reactance per phase at this frequency $= 3 \times 12$ ohms,
$= 36$ ohms.

Circulating current $= \dfrac{141{\cdot}4}{36}$
$=$ **3·928 amperes.**

(ii) Voltage regulation.

20. *Explain what is meant by the "synchronous impedance" of an alternator. Find the synchronous impedance and reactance in an alternator in which a given field current produces an armature current of* 250 *amperes on short-circuit and a generated e.m.f. of* 1,500 *volts on open-circuit. The armature resistance is* 2 *ohms. Hence calculate the terminal p.d. when a load of* 250 *amperes at* 6,600 *volts and lagging power factor* 0·8 *is switched off.*

(*C. and G. Final, Pt. I*, 1939)

$$\text{Synchronous impedance} = \frac{\text{Generated e.m.f. on open-circuit}}{\text{Current on short-circuit}}$$

for the same value of field current in each case.

$$\text{i.e. } \textbf{Synchronous impedance} = Z = \frac{1500 \text{ volts}}{250 \text{ amperes}} = \textbf{6 ohms.}$$

$$\textbf{Synchronous reactance} = X = \sqrt{Z^2 - R^2}$$

$$= \sqrt{6^2 - 2^2} = \textbf{5·65 ohms.}$$

The terminal p.d. when the load is switched off is the same as the generated e.m.f. while the machine is on load provided that the field excitation is not altered. The generated e.m.f. on load is found by adding the p.d.s across the internal resistance and reactance to the terminal p.d. on load.

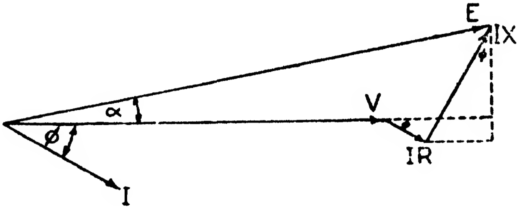


Fig. 10

Referring to the vector diagram, Fig. 10.

$$\text{Resistance p.d.} = IR = 250 \times 2 = 500 \text{ volts.}$$

$$\text{Reactance p.d.} = IX = 250 \times 5{\cdot}65 = 1{,}413 \text{ volts.}$$

$$E \sin \alpha = IX.\cos \varnothing - IR.\sin \varnothing$$

$$= 1413 \times 0{\cdot}8 - 500 \times 0{\cdot}6 = 830 \text{ volts.}$$

$$E \cos \alpha = V + IR.\cos \varnothing + IX.\sin \varnothing$$

$$= 6600 + 500 \times 0{\cdot}8 + 1413 \times 0{\cdot}6$$

$$= 7848 \text{ volts.}$$

$$E = \sqrt{(E\sin\alpha)^2 + (E\cos\alpha)^2}$$

$$= \sqrt{(830)^2 + (7848)^2} = 7892 \text{ volts.}$$

Therefore, **terminal e.m.f. when load is switched off = 7892 volts.**

21. *The following test results were obtained on a* 6,600-*volt alternator:*

Open-circuit voltage	3100	4900	6600	7500	8300
Exciting current, amperes	16	25	37½	50	70

Full-load current was circulated on short-circuit with a field excitation of 20 *amperes. Calculate the full-load regulation when the power factor is* 0·8 *lagging, using (a) the ampere-turn method, (b) the synchronous impedance method. Neglect resistance and leakage reactances. Discuss the relative accuracy of the methods.* (*C. and G. Final, Pt. II, Sect. A,* 1940)

(a) **Ampere-turn method**

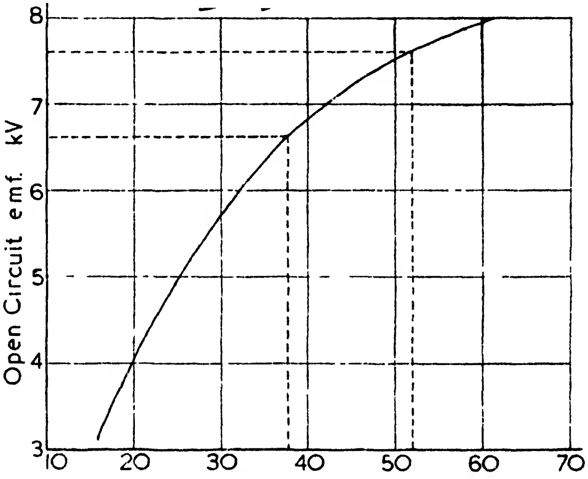


Fig. 11

The open-circuit characteristic (Fig. 11) is plotted from the data given in the question. From it the exciting current to give normal e.m.f. (6600V) on open circuit is found to be 37·5 amperes. It is also given that the excitation required to circulate full-load current on short-circuit is 20 amperes. On short-circuit the field excitation is balanced by the armature reaction ampere-turns, therefore on full-load it is assumed that the armature-reaction is equivalent to a field excitation of 20 amperes. (This assumption is not strictly accurate because the behaviour of the machine on load is not exactly the same as on short-circuit.) To give the normal terminal p.d. of 6600 volts on full-load the field excitation must be increased by 20 amperes combined vectorially with the normal value of 37·5 amperes, in order that the effect of armature reaction may be counteracted.

In Fig. 12 $I_o = 37{\cdot}5$ amperes, $I_a = 20$ amperes, and I_t is the total exciting current required.

It will be observed that I_a is drawn parallel with the load current vector. This is because the armature reaction flux is in phase with the armature current.

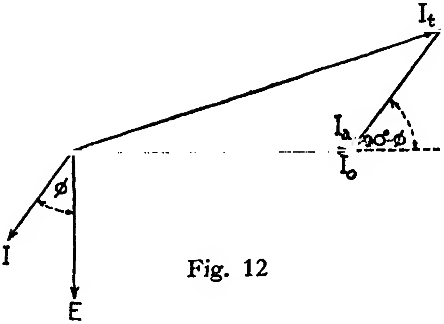


Fig. 12

From the vector diagram,

$$I_t^2 = I_0^2 + I_a^2 + 2I_oI_a \cos(90 - \phi)$$
$$= 37{\cdot}5^2 + 20^2 + 2 \times 37{\cdot}5 \times 20 \times 0{\cdot}6 = 2706{\cdot}25$$
$$I_t = 52{\cdot}02 \text{ amperes.}$$

This is the excitation required on full-load to give a terminal p.d. of 6600 volts. If this load is thrown off with the excitation unaltered the terminal p.d. will rise to 7600 volts (obtained from the open-circuit characteristic).

Therefore,

$$\textbf{percentage regulation} = \frac{\text{No-load p.d.} - \text{Full-load p.d.}}{\text{Full-load p.d.}} \times 100$$
$$= \frac{7600 - 6600}{6600} \times 100$$
$$= \mathbf{15{\cdot}16 \text{ per cent.}}$$

(b) Synchronous reactance method

Call the normal e.m.f. of 6600 volts 100 per cent, and let 100 per cent field excitation be that which is required to produce it on open-circuit, i.e. 37·5 amperes.

100 per cent load current is produced on short-circuit by an excitation of 20 amperes. Hence if 100 per cent excitation were applied on short-circuit the short-circuit current would be $\dfrac{100 \times 37{\cdot}5}{20} = 187{\cdot}5$ per cent.

Under conditions of 100 per cent field excitation the synchronous reactance is therefore given by: $Z_s = \dfrac{\text{Open-circuit e.m.f.}}{\text{Short-circuit current}}$

$$= \frac{100}{187{\cdot}5} = 0{\cdot}533 \text{ or } 53{\cdot}3 \text{ per cent.}$$

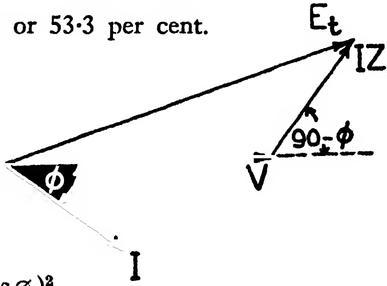


Fig. 13

The vector diagram is now drawn in Fig. 13 with the reactance drop of $IZ_s = 53{\cdot}3$ per cent of the normal e.m.f. added vectorially to V which represents a terminal p.d. of 100 per cent. Then E_t is the total e.m.f. generated on full-load and power factor 0·8 lagging. From the vector diagram,

$$E_t^2 = (100 + 53{\cdot}3 \sin\phi)^2 + (53{\cdot}3 \cos\phi)^2$$
$$= (100 + 53{\cdot}3 \times 0{\cdot}6)^2 + (53{\cdot}3 \times 0{\cdot}8)^2$$
$$E_t = 138{\cdot}7 \text{ per cent.}$$

Hence, **percentage regulation** when full-load is thrown off is

(138·7 — 100) = **38·7 per cent.**

The results obtained by the two methods are seen to vary widely. The regulation by the ampere-turn method is probably somewhat lower and by the synchronous reactance method higher than the true value. The first method is more likely to be accurate. Owing to the non-linear shape of the open-circuit characteristic the synchronous reactance is not a constant but varies widely with the amount of field excitation. The result has been worked for 100 per cent excitation but some other value of field excitation, say a higher one, would give a different value for the synchronous reactance and the percentage regulation would have come out lower.

22. *A* 5000-*kVA*, 6600-*volt*, 3-*phase*, *star-connected alternator has the open-circuit characteristic given below and requires* 4000 *ampere-turns to produce full-load current on short-circuit. The reactance drop is* 8 *per cent and the resistance drop is* 2 *per cent of normal voltage. Find the ampere-turns required to give normal voltage on no-load and when delivering full-load at* 0·8 *lagging power factor:*

Terminal voltage (*V*)	3100	4900	6600	7500	8300
Field excitation (*A-T*)	3200	5000	7500	10000	14000

(*C. and G. Final, Pt. II* 1943)

(a) **On no-load** the ampere-turns required to give normal e.m.f. is obtained direct from the open-circuit characteristic.

For a no-load e.m.f. of 6600 volts the field excitation required is **7500 ampere-turns.**

(b) **Full-load, 0·8 power factor lagging.** The vector diagram is drawn in Fig. 14.

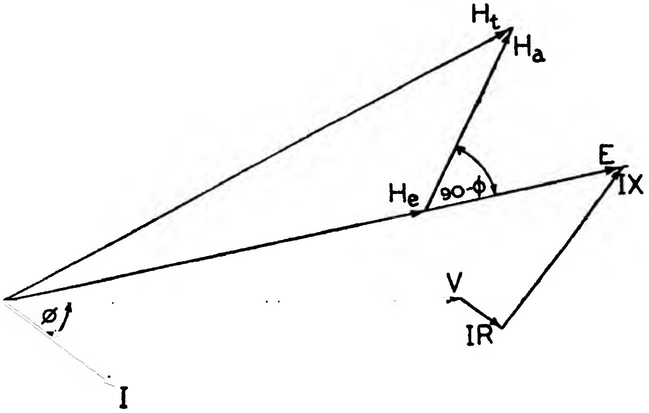


Fig. 14

In this diagram,

V = 6600 volts,

IR = 2 per cent of 6600 volts,

= 132 volts.

$IX = 8$ per cent of 6600 volts,
$= 528$ volts.

The e.m.f. to be generated to give the required terminal p.d. is

$$E = \sqrt{(V + IR\cos\phi + IX\sin\phi)^2 + (IX\cos\phi - IR\sin\phi)^2}$$
$$= \sqrt{(6600 + 132 \times 0{\cdot}8 + 528 \times 0{\cdot}6)^2 + (528 \times 0{\cdot}8 - 132 \times 0{\cdot}6)^2}$$
$$= 7030 \text{ volts.}$$

The open-circuit characteristic (Fig. 15) now needs to be plotted and from it the excitation required to generate 7030 volts is found to be 8500 ampere-turns.

i.e. $H_e = 8500$ ampere-turns,
$H_a = 4000$ ampere-turns (equivalent to the armature reaction m.m.f. on full-load)

Hence,
$$H_t = \sqrt{(H_e + H_a\cos(90 - \phi))^2 + (H_a\sin(90 - \phi))^2}$$
$$= \sqrt{(8500 + 4000 \times 0{\cdot}6)^2 + (4000 \times 0{\cdot}8)^2}$$
$$= 11350 \text{ ampere-turns.}$$

Therefore, the total excitation required on full-load, power factor 0·8 lagging is **11350 ampere-turns.**

Note.—Strictly, H_a includes the ampere-turns required to overcome the armature impedance on short-circuit which should be subtracted from the excitation H_a to give the value of the armature-reaction ampere-turns. There is no data given, however, for finding the amount of excitation to be deducted from H_a which has therefore been taken as the armature-reaction ampere-turns.

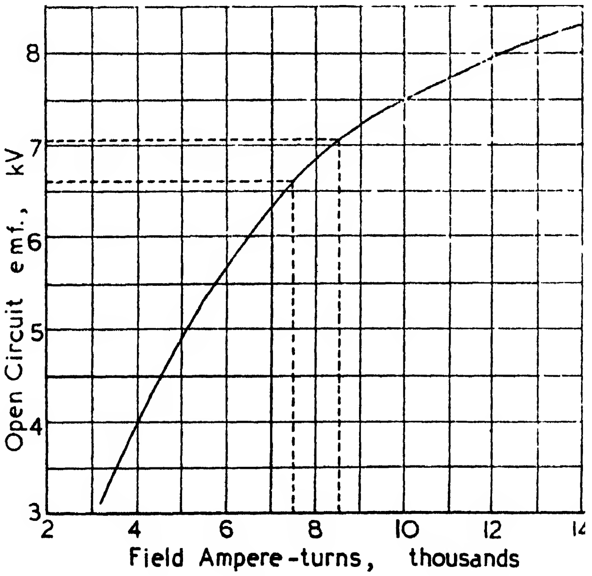


Fig. 15

23. *A salient-pole, 3-phase, star-connected alternator rated at 5000-kVA, 6600 volts, has a resistance of* 1·5 *per cent and a leakage reactance of* 10 *per cent. The short-circuit armature-reaction for full-load current is equivalent to* 60 *amperes field current. The armature cross-reaction per armature ampere-turn is one-half of the direct reaction. The open-circuit characteristic is as follows:*

Field current, amperes	32	50	75	100	140
Terminal p.d. volts	3100	4900	6600	7500	8300

Find the percentage regulation on full-load at power factor 0·8 (*lagging.*)
(*I.E.E., Pt. II, May,* 1940)

The vector diagram for full-load 0·8 power factor lagging is shown in Fig. 16. OV is drawn to represent normal terminal p.d. (6600 volts) taken as 100 per cent. Then to the same scale the resistance and leakage reactance drops of 1·5 and 10 per cent are added in phase and in quadrature respectively with the current to give OE.

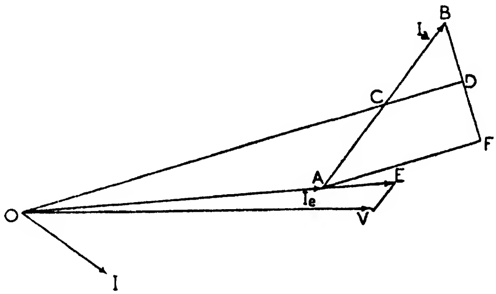


Fig. 16

OE = 107·4 per cent = 7088 volts, for which the corresponding field excitation I_e is 86·5 amperes.

The excitation vector diagram is now imposed on the e.m.f. diagram, with OA = I_e drawn in line with E, and AB = I_a = 60 amperes, the excitation equivalent to full-load armature reaction on short-circuit, drawn in quadrature with the load current I. To be quite correct both I_e and I_a should be shown 90° ahead of these positions because the e.m.f. is in quadrature with the main flux and the armature reaction flux is in phase with the load current I. The diagram is drawn as shown purely for convenience and to save space.

AB is now divided at C such that AC = k.AB where k is the ratio of the cross-reaction to the direct-reaction per ampere-turn. In this case k = 0·5 so that C bisects AB.

A line is now drawn through OC and produced; a perpendicular from B is drawn to this line meeting it at D. Then OD is the excitation required and is made up of the vector sum of OA, AF (to balance the direct reaction) and FD (to balance the cross-reaction).

By measurement, OD = I_t = 133 amperes.

The open-circuit characteristic shows that the corresponding e.m.f. generated is 8180 volts.

$$\textbf{Percentage regulation} = \frac{8180-6600}{6600} \times 100$$
$$= \textbf{23·9 per cent.}$$

24. *Define the "synchronous impedance" of a* 3*-phase synchronous generator. Explain the advantages and limitations of this conception. Find the regulation by the armature-reaction method of a* 1000*-kVA,* 2000*-volt,* 50*-cycle,* 3*-phase generator having the following open-circuit test figures:*

Open-circuit terminal e.m.f., per cent	35	90	100	110	120	128
Excitation, per cent	25	80	100	125	160	200

An excitation of 80 *per cent was required for full-load current on short-circuit, and* 200 *per cent for full-load current at rated voltage and zero power factor.* (*I.E.E., Pt. II, Nov.,* 1937)

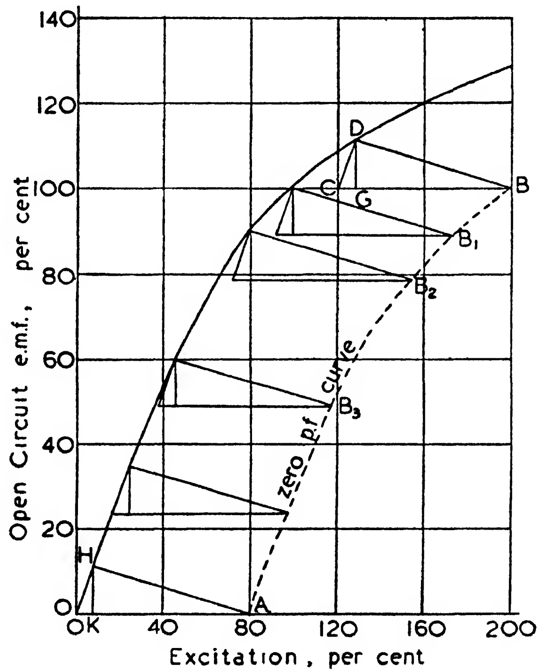


Fig. 17

From the above data the open-circuit characteristic is plotted (Fig. 17). On the same axes another curve is derived showing how the terminal p.d. varies with excitation on full-load current at zero power factor. Two points are given on this curve, viz. A and B. OA = 80 per cent, the short-circuit excitation; and B is fixed by 100 per cent p.d. being given at 200 per

cent excitation. The remainder of the curve may now be plotted as follows:

BC is drawn horizontally such that CB = OA. Then a line is drawn through C parallel to the commencement of the open-circuit characteristic near the origin, cutting the o.c.c. at D. BD is joined and a perpendicular DG dropped on BC. The triangle BGD is imposed at various points on the o.c.c. and the corresponding points B_1 B_2 B_3, etc., are points on the zero power factor curve.

Referring to the triangle OAH at the origin, KA is the excitation necessary to balance the armature-reaction and OK is the excitation necessary to generate HK which is the p.d. across the leakage reactance.

From this diagram drawn to scale,

Leakage reactance drop (HK) = 11·5 per cent.

Armature-reaction excitation (AK) = 71 per cent.

The vector diagram (Fig. 18) is now drawn to find the regulation for full-load, power factor 0·8 lagging, this power factor being assumed as it is not given in the question.

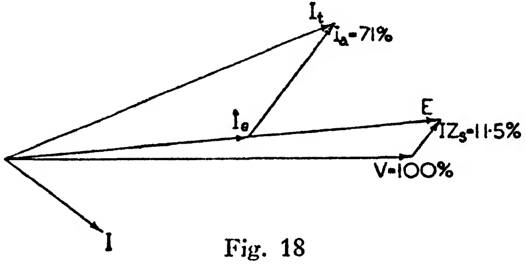


Fig. 18

The e.m.f. generated, E, is measured and found to be 108 per cent: the corresponding field excitation I_e is 120 per cent, which is drawn in line with E as described in Problem 23.

I_a is drawn perpendicular to I to represent the excitation equivalent to the armature reaction, and I_t is the total excitation required.

By measurement, I_t = 176 per cent and the open-circuit characteristic shows the corresponding e.m.f. to be 122·5 per cent.

Percentage regulation = (122·5 — 100)
= **22·5 per cent.**

Note.—It has been assumed, in effect, that the machine is of the non-salient-pole type, so that the armature reaction on full-load is taken to be the same as on short-circuit. This assumption would not be justified for a salient-pole machine and the armature-reaction would have to be split up into its cross-reaction and direct-reaction components as described in Problem 23. To do this the ratio of cross- to direct-reaction would need to be known. This depends on the ratio of pole arc to pole-pitch which is not given.

25. *Tests on a* 15000-*kVA*, 11000-*volt*, 3-*phase*, 50-*cycle*, *star-connected alternator gave the following results:*

Field AT per pole, thousands	10	18	24	30	40	45	50
Open-circuit line e.m.f., kV	4·9	8·4	10·1	11·5	12·8	13·3	13·65
Full-load current, zero power factor test, line p.d., kV	—	0	—	—	—	10·2	—

Find the armature reaction ampere-turns, the leakage reactance, and the regulation for full-load at 0·8 *power factor lagging. Neglect resistance.*

(*I.E.E., Pt. II, May,* 1937)

This problem is solved in the same manner as Problem 24. Actually there is no need to draw the full-load zero power factor curve in full since all the data required can be obtained from the triangle BCD (Fig. 19.)

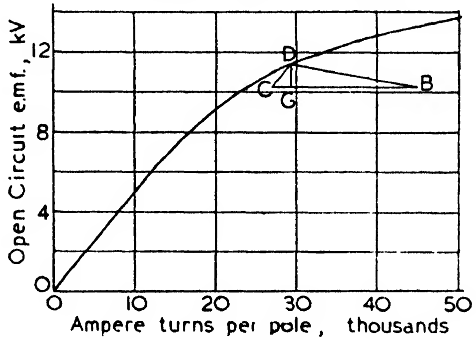


Fig. 19

Measurements from this diagram give:

Armature-reaction ampere-turns = GB = **15700 A.T./pole.**

Full-load reactance drop = DG = 1150 volts.

Full-load current $= \frac{15000}{\sqrt{3} \times 11}$ amperes.

Leakage reactance per phase $= \frac{1150}{\sqrt{3}} \div \frac{15000}{\sqrt{3} \times 11}$ ohms.

= **0·84 ohm.**

Alternatively, percentage leakage reactance $= \frac{1150}{11000} \times 100$

= **10·45 per cent.**

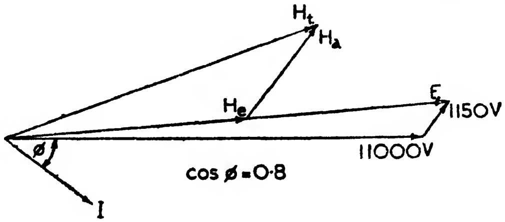


Fig. 20

The full-load regulation is found by drawing the vector diagram to scale (Fig. 20).

From this diagram, E = 11750 volts, for which the excitation H_e = 32000 A.T./pole.

Now, H_a = 15700 A.T./pole.

Therefore excitation required = H_t = 44000 A.T./pole.

The corresponding e.m.f. generated (from the o.c.c.) = 13200 volts.

Therefore, **percentage regulation** $= \dfrac{13200 - 11000}{11000} \times 100$

$= \mathbf{20}$ **per cent.**

(iii) Design calculations.

26. *Obtain an expression for the "output coefficient" of a* 3*-phase alternator, carefully defining each term in the expression. A turbo-alternator running at* 3000 *r.p.m. can be built for a maximum peripheral speed of* 150 *m. per sec., and a core length of* 2·5 *m. Estimate its maximum possible rating, using suitable values for the specific loadings.* (*I.E.E., Pt. II, May,* 1937)

The "output coefficient" of a 3-phase alternator is given by:

$$G = \frac{\text{kVA}}{D^2LN} = 10{\cdot}45\,\bar{B}.ac.10^{-11}$$

where $\bar{B}$ = the specific magnetic loading or mean flux density in gauss.

ac = the specific electric loading in ampere-conductors per cm. of armature periphery.

D = the armature diameter or stator bore in cm.

L = the core length in cm.

N = the speed in rev. per sec.

In this problem,

$$\pi Dn = 150 \text{ m. per sec.}$$

$$\frac{\pi \times D \times 3000}{60} = 150 \qquad D = 3/\pi \text{ m.}$$

Therefore, kVA rating $= 10{\cdot}45 \times \bar{B}.ac.10^{-11} \times \left(\dfrac{300}{\pi}\right)^2 \times 250 \times 50$

Assuming provisionally a mean flux density of 5500 gauss, and a specific electric loading of 500 ampere-conductors per cm.,

$$\text{kVA rating} = 10{\cdot}45 \times 5500 \times 500 \times 10^{11} \times \left(\frac{300}{\pi}\right)^2 \times 250 \times 50$$

$$= 32740 \text{ kVA.}$$

i.e. **Maximum kVA rating = 30000 kVA (approximately).**

27. *Develop an expression for the output of a synchronous generator in terms of the specific magnetic and electric loadings, the stator bore, and length, and the speed. Define the loadings. Taking the mean gap-density over the pole-pitch as* 5500 *gauss, the electric loading as* 250 *ampere-conductors per cm., and a peripheral speed not exceeding* 35 *m. per sec., determine suitable stator dimensions for a* 500*-kVA,* 50*-cycle,* 3*-phase alternator to run at* 375 *r.p.m.* (*I.E.E., Pt. II, Nov.,* 1938)

$$\text{kVA output} = 10{\cdot}45 \times \bar{B} \times ac \times 10^{-11} \times D^2 \times L \times N,$$

i.e. $500 = 10{\cdot}45 \times 5500 \times 250 \times 10^{-11} \times D^2L \times \dfrac{375}{60}$

whence, $D^2L = 0{\cdot}557 \times 10^6$ cu.cm.

Also, $\pi DN = 35$ m. per sec.

$$D = \frac{35 \times 100}{\pi \times 6{\cdot}25} \text{ cm.}$$

$$= 178{\cdot}2 \text{ cm.}$$

A suitable core diameter would therefore be 170 cm. which would keep the peripheral speed lower than the maximum allowable figure of 35 m. per sec.

Then $$L = \frac{0{\cdot}557 \times 10^6}{170 \times 170}$$
$$= 19{\cdot}28 \text{ cm., say } 19 \text{ cm.}$$

Therefore the **stator dimensions** will be: **Bore, 170 cm.**
Length, 19 cm.

28. *Obtain figures for the core-length, stator bore, number of slots and turns for a* 1000-*kVA*, 3000-*volt*, 50-*cycle*, 3-*phase*, 20-*pole, synchronous generator. Explain carefully the various limitations and give reasons for the choice of the several specific values used.* (*I.E.E., Pt. II, Nov.*, 1937)

The design details are calculated from the expression:

$$\text{kVA rating} = 10{\cdot}45 \times \bar{B} \times ac \times 10^{-11} \times D^2 \times L \times N,$$

where the symbols have the meanings given in Problem 26.

For a 50-cycle, 20-pole alternator, $N = 5$ r.p.s.

The values which have now to be decided are for $\bar{B}$, ac, D and L.

$$1000 = 10{\cdot}45 \times \bar{B} \times ac \times 10^{-11} \times D^2L \times 5$$

Hence, $\bar{B} \times ac \times D^2L = 1{\cdot}912 \times 10^{12}$

The dimensions of the armature thus depend upon the product of $\bar{B}$ and ac.

$\bar{B}$ is limited by considerations of the ampere-turns which will be required on the field winding, and by the iron losses.

ac is limited by considerations of heating, and the effect which leakage reactance and armature reaction would have on the voltage regulation. Initial values must be allotted to $\bar{B}$ and ac in the light of previous experience and modified afterwards if found necessary.

For a slow speed alternator such as the machine under consideration, the following values may be assumed:

$\bar{B} = 5500$ gauss, ac $= 300$ ampere-conductors per cm.

Using these values in the above expression we get

$$5500 \times 300 \times D^2L = 1{\cdot}912 \times 10^{12}$$
$$D^2L = 1{\cdot}16 \times 10^6 \text{ cu.cm.}$$

The length of a pole-pitch is

$$Y = \frac{\pi D}{\text{number of poles}} = \frac{\pi D}{20}$$

It is usual in a case like this to allow a core length of about 10 per cent greater than a pole-pitch.

i.e. $$L = 1{\cdot}1\ Y = \frac{1{\cdot}1\ \pi D}{20}$$
$$= 0{\cdot}173\ D$$

Therefore, $D^2L = D^2 \times 0{\cdot}173D = 0{\cdot}173\ D^3 = 1{\cdot}16 \times 10^6$

$$D = 188{\cdot}5 \text{ cm. say } 190 \text{ cm.}$$

A limitation is imposed on the diameter by the necessity for keeping the peripheral speed below about 35 m. per sec. in machines of this kind. If the diameter is made 190 cm.,

$$\text{peripheral speed} = \pi \times 190 \times 5 \text{ cm./sec. at the rated speed of 5 r.p.s.}$$

$$= 29{\cdot}85 \text{ m. per sec.}$$

As this is well within the maximum permissible limit there is no need to alter the diameter from the calculated value of 190 cm.

Using this value for D,

$$L = \frac{1{\cdot}16 \times 10^6}{190^2} = 32 \text{ cm.}$$

Hence, the **core dimensions** are:—**Diameter, 190 cm.**
Length, 32 cm.

$$\text{Pole-pitch} \qquad Y = \frac{\pi \times 190}{20} = 29{\cdot}85 \text{ cm.}$$

$$\text{Pole area} \qquad YL = 29{\cdot}85 \times 32 \text{ sq.cm.} \doteq 955 \text{ sq.cm.}$$

$$\text{Flux per pole} \qquad = YL \times \bar{B}$$

$$\text{i.e.} \qquad \Phi = 955 \times 5500 = 5{\cdot}25 \times 10^6 \text{ maxwells}$$

$$\text{e.m.f. per phase} \qquad = 4{\cdot}44\, k_m\, f\, \Phi\, T_s\, 10^{-8} \text{ volts}$$

where T_s = turns in series per phase.

Also, the e.m.f. per phase, if it is assumed that the winding is star-connected, will be $3000/\sqrt{3}$. A value of 0·97 will also be assumed for the distribution factor k_m.

$$\text{Then,} \qquad T_s = \frac{3000 \times 10^8}{\sqrt{3} \times 4{\cdot}44 \times 0{\cdot}97 \times 50 \times 5{\cdot}25 \times 10^6}$$

$$= 153 \text{ turns.}$$

$$\text{Number of conductors per phase} = Z_{ph} = 2 \times 153 = 306$$

$$\text{Number of conductors per slot} = \frac{Z_{ph}}{\text{slots per pole per phase} \times \text{no. of poles}}$$

$$\text{i.e.} \qquad u = \frac{306}{g' \times 20}$$

$$\text{Therefore} \qquad ug' = 15{\cdot}3$$

where u = number of conductors per slot
g' = number of slots per pole per phase.

It is usual to make g' a whole number and u must be a whole number. In the interests of good waveform g' should be 3 or more. It is evident that Z_{ph} needs to be modified in order that ug' may be a whole number.

Suppose $Z_{ph} = 300$ then ug' 15 and there are two possibilities,

(a) $u = 5$ conductors per slot, $g' = 3$ slots per pole per phase,

(b) $u = 3$ „ „ „ $g' = 5$ „ „ „ „

Increasing the number of slots reduces the leakage inductance and gives better heat conduction away from the conductors, but also increases the cost of insulation. A useful working arrangement is to allow about 1000 ampere-conductors per slot.

$$\text{Phase current} = \frac{1000 \times 1000}{\sqrt{3} \times 3000} = 192{\cdot}5 \text{ amperes.}$$

With 5 conductors per slot the number of ampere-conductors per slot $= 192{\cdot}5 \times 5 = 962{\cdot}5$ which is quite suitable.

Therefore $g' = 3$ slots per pole per phase.

Total number of slots $= 3 \times 20 \times 3 =$ **180**

Number of turns per phase $= Z_{ph}/2 = 150$

Total number of turns $= 150 \times 3 =$ **450**

29. *A* 200-*kVA*, 50-*cycle per sec.,* 1000 *r.p.m.,* 6·6-*kV, star-connected synchronous generator is to have a double layer winding, lap-connected with full-pitch coils, arranged in* 12 *slots per pole with a* 60° *phase-spread. Using suitable values of flux density and current density, obtain values for the main dimensions, the number of turns per phase in the winding, the number of armature slots and the number of conductors per slot.*

(*I.E.E., Pt. II, May,* 1943)

For a frequency of 50 cycles/sec. at a speed of 1000 r.p.m. the number of poles $= 120 \times 50/1000 = 6$.

Phase difference between adjacent coil e.m.f.s $= \pi/g = \pi/12$

$$\text{Distribution factor} \quad k_m = \frac{\sin \frac{g'\psi}{2}}{g' \sin \frac{\psi}{2}}$$

$$= \frac{\sin (4\pi/24)}{4 \times \sin \pi/24} = 0{\cdot}958$$

$$\text{e.m.f. per phase} = 4{\cdot}44\ k_m\ f\ \Phi\ T_s\ 10^{-8} \text{ volts}$$

$$\frac{6{\cdot}6 \times 1000}{\sqrt{3}} = 4{\cdot}44 \times 0{\cdot}958 \times 50 \times \Phi \times T_s \times 10^{-8}$$

Whence, $\Phi \times T_s = 1{\cdot}807 \times 10^9$

Now, kVA output $= 10{\cdot}45 \times \bar{B} \times ac \times 10^{-11} \times D^2LN$

i.e. $200 = 10{\cdot}45 \times \bar{B}.ac.10^{-11}.D^2L.1000/60$

Assume that $\bar{B} = 5000$ gauss, and $ac = 400$ ampere-conductors per cm.

Then $200 \times 60 = 10{\cdot}45 \times 5000 \times 400 \times 10^{-11} \times D^2L \times 1000$

Whence $D^2L = 57400$ cu.cm.

Assume a maximum peripheral speed of 35 m. per sec.

$$\pi Dn = 3500 \text{ cm. per sec.}$$

$$D = \frac{3500 \times 60}{\pi \times 1000} = 66{\cdot}8 \text{ cm., say 66 cm.}$$

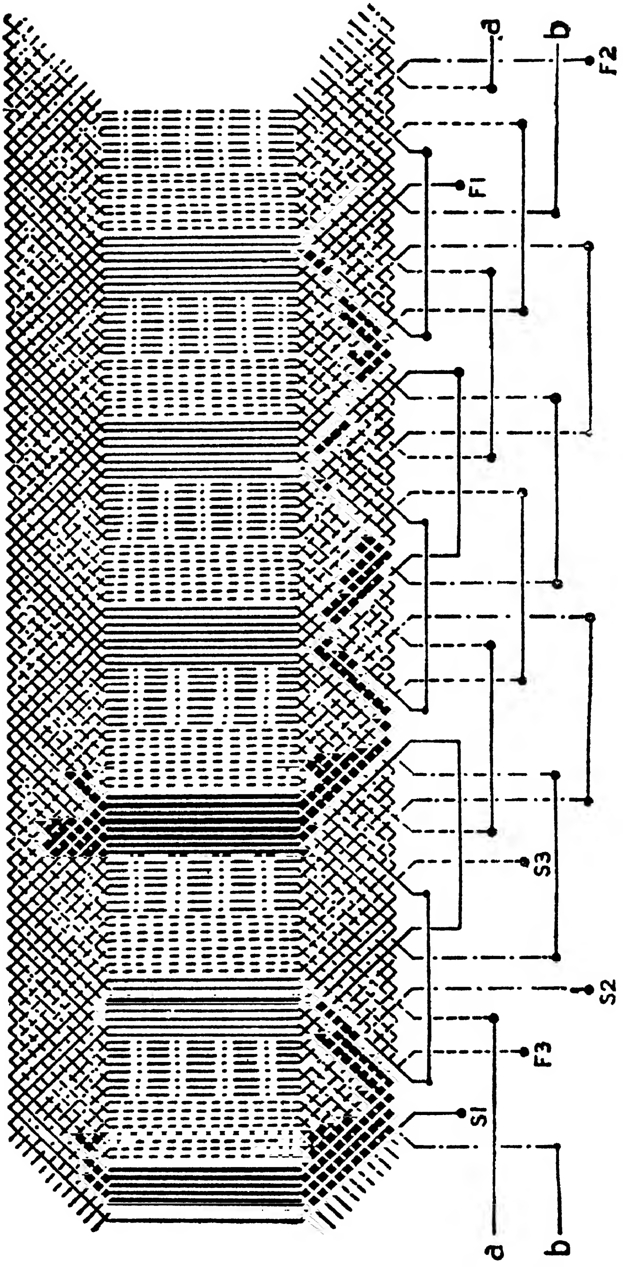


Fig. 21. For STAR connection join F_1, F_2, F_3. Line terminals are S_1, S_2 S_3.

Then $L = \frac{57400}{66^2} = 13{\cdot}18$ cm., say 13 cm.

Hence, the **armature dimensions** are: **Bore diameter, 66 cm.**
Length, 13 cm.

Pole-pitch $Y = \pi D/p = \frac{\pi \times 66}{6} = 11\pi$ cm.

Pole area $YL = 11\pi \times 13$ sq.cm. $= 449{\cdot}2$ sq.cm.

Flux per pole $\Phi = YL \times \bar{B} = 449{\cdot}2 \times 5000 = 2{\cdot}246 \times 10^6$ maxwells

Turns per phase $= \frac{1{\cdot}807 \times 10^9}{2{\cdot}246 \times 10^6} = 804$

Turns per pole per phase $= \frac{804}{6} = 134$

Conductors per pole per phase $= 268$

Hence, $g'u = 268$, where g' = number of slots per pole per phase
u = number of conductors per slot

Since there are to be 12 slots per pole, therefore
$g' = 12/3 = 4$ and $u = 67$

Now in a double layer winding there will be two coil sides per slot so that u must be a multiple of two. Make $u = 68$.
This gives 272 conductors per pole per phase and

Turns per phase $= \frac{272}{2} \times 6 = $ **816**

Number of armature slots $= 4 \times 3 \times 6 = 72$

Number of conductors per slot $= 68$

The **winding will therefore consist of 2 coil sides per slot, 24 coils per phase, each coil consisting of 34 turns.**

Fig. 21 shows a developed view of the winding.

Note.—The figure assumed for the maximum peripheral speed in these calculations is more usually assumed for slow speed machines. The output of this machine, however, 200-kVA, is so small that if the values usually taken for high speed machines were used, viz., maximum peripheral speed = 150 m. per sec., the diameter would have been greater and the core length too short to be practicable.

30. *Draw developed diagrams showing* (i) *a* 4-*pole, star-connected, single-layer winding comprising* 24 *conductors with the overhang in* 3 *planes*; (ii) *a mesh-connected,* 6-*pole, single-layer winding with* 36 *conductors and the overhang in* 2 *planes. In each case mark the phases and the terminals.*
(*I.E.E., Pt. II, May,* 1939)

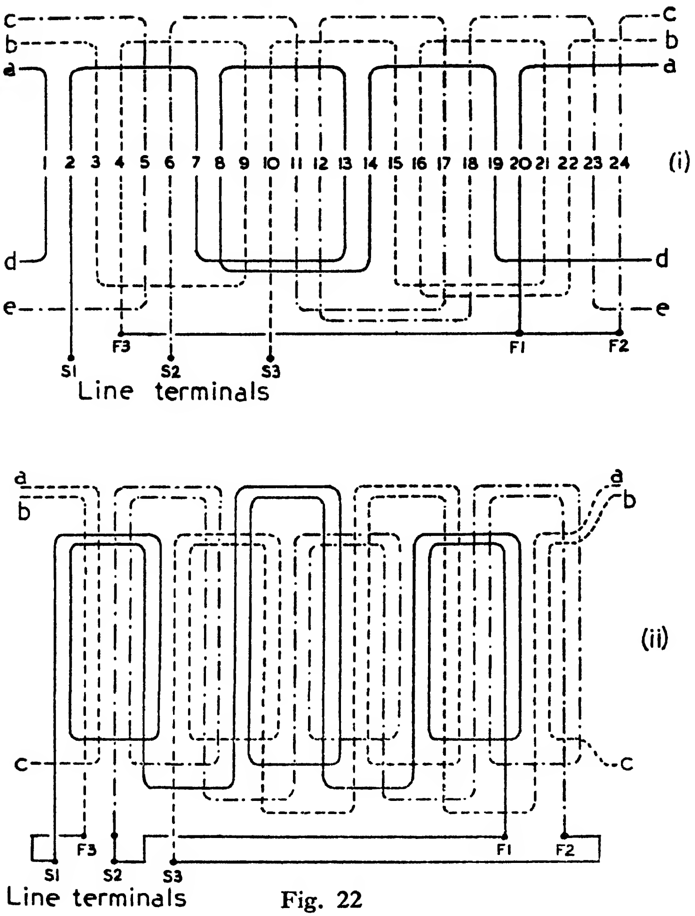


Fig. 22

In (i), F_1, F_2, F_3 are joined to form the neutral terminal.

In (ii), phase 3 must have a set of crank-ended coils to close the winding. This always happens in 6-pole machines with 2-plane overhangs.

(iv) Parallel operation.

31. *Two single-phase generators operate in parallel on a load impedance of* $\underset{\cdot}{Z}$ *ohms. Their e.m.f.s are* $\underset{\cdot}{E}_1$ *and* $\underset{\cdot}{E}_2$ *and their synchronous impedances* $\underset{\cdot}{Z}_1$ *and* $\underset{\cdot}{Z}_2$*. Deduce the terminal voltage in terms of these e.m.f.s and the admittances* $\underset{\cdot}{Y}$, $\underset{\cdot}{Y}_1$ *and* $\underset{\cdot}{Y}_2$.

Determine the terminal voltage and the kW output of each machine if $\underset{\cdot}{E}_1 = 100$, $\underset{\cdot}{E}_2 = 110$ *volts,* $\underset{\cdot}{Z} = 3 + j4$, *and* $\underset{\cdot}{Z}_1 = \underset{\cdot}{Z}_2 = 0{\cdot}2 + j1$ *ohms.*

(*London B.Sc. Eng., July,* 1945)

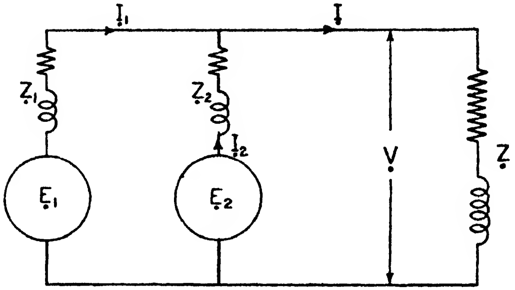


Fig. 23

Terminal voltage of first machine $= E_1 - I_1Z_1 = V$ (1)

Terminal voltage of second machine $= E_2 - I_2Z_2 = V$ (2)

Also
$$V = I\,Z = (I_1 + I_2)\,Z$$

Hence
$$I_1 + I_2 = \frac{V}{Z} = V\,Y \qquad (3)$$

From equation (1)
$$I_1 = \frac{E_1 - V}{Z_1} = (E_1 - V)\,Y_1 \qquad (4)$$

From equation (2)
$$I_2 = \frac{E_2 - V}{Z_2} = (E_2 - V)\,Y_2 \qquad (5)$$

Therefore
$$I_1 + I_2 = (E_1 - V)\,Y_1 + (E_2 - V)\,Y_2 \qquad (6)$$

From equations (3) and (6),
$$V\,Y = (E_1 - V)\,Y_1 + (E_2 - V)\,Y_2$$

i.e.
$$V\,(Y_1 + Y_2 + Y) = E_1Y_1 + E_2Y_2$$

or
$$V = \frac{E_1Y_1 + E_2Y_2}{Y_1 + Y_2 + Y}$$

With the values given in the question,
$$Y = \frac{1}{3 + j4} = \frac{1}{3 + j4} \times \frac{3 - j4}{3 - j4} = 0{\cdot}12 - j0{\cdot}16 \text{ mho.}$$

$$Y_1 = Y_2 = \frac{1}{0{\cdot}2 + j1} = \frac{1}{0{\cdot}2 + j1} \times \frac{0{\cdot}2 - j1}{0{\cdot}2 - j1} = 0{\cdot}1923 - j0{\cdot}9615 \text{ mho.}$$

Hence,
$$V = \frac{100\,(0{\cdot}1923 - j0{\cdot}9615) + 110\,(0{\cdot}1923 - j0{\cdot}9615)}{2\,(0{\cdot}1923 - j0{\cdot}9615) + 0{\cdot}12 - j0{\cdot}16}$$
$$= \frac{40{\cdot}38 - j201{\cdot}9}{0{\cdot}5046 - j2{\cdot}083}$$
$$= \frac{(40{\cdot}38 - j201{\cdot}9)\,(0{\cdot}5046 + j2{\cdot}083)}{0{\cdot}5046^2 + 2{\cdot}083^2}$$

$$= 96{\cdot}01 - j3{\cdot}841 \text{ volts}$$
$$= \mathbf{96{\cdot}10 \angle -2°18'} \textbf{ volts.}$$

From equation (4),

$$\dot{I}_1 = (\dot{E}_1 - \dot{V})\, \dot{Y}_1$$
$$= (100 - 96{\cdot}01 + j3{\cdot}841)\,(0{\cdot}1923 - j0{\cdot}9615)$$
$$= (3{\cdot}99 + j3{\cdot}841)\,(0{\cdot}1923 - j0{\cdot}9615)$$
$$= 4{\cdot}460 - j3{\cdot}097$$
$$= \mathbf{5{\cdot}43 \angle -34°48'} \textbf{ amperes.}$$

From equation (5),

$$\dot{I}_2 = (\dot{E}_2 - \dot{V})\, \dot{Y}_2$$
$$= (100 - 96{\cdot}01 + j3{\cdot}841)\,(0{\cdot}1923 - j0{\cdot}9615)$$
$$= (13{\cdot}99 + j3{\cdot}841)\,(0{\cdot}1923 - j0{\cdot}9615)$$
$$= 6{\cdot}384 - j12{\cdot}71$$
$$= \mathbf{14{\cdot}23 \angle -63°20'} \textbf{ amperes.}$$

Power output of first machine $= 96{\cdot}10 \times 5{\cdot}43 \times \cos(34°48' - 2°18')$
$= 443{\cdot}9$ watts or **0·44 kW.**

Power output of second machine $= 96{\cdot}10 \times 14{\cdot}23 \times \cos(63°20' - 2°18')$
$= 663{\cdot}0$ watts or **0·66 kW.**

32. *Explain what factors produce "phase-swinging" or "hunting" in an alternator running in parallel with other machines and show how this action may be reduced.*

Calculate the value of the synchronizing power in kilowatts for one mechanical degree of displacement at full load 0·8 *power-factor lagging for a* 3-*phase,* 2000-*kVA,* 6600-*volts,* 50 *c.p.s.,* 12-*pole machine having a synchronous reactance of* 25 *per cent and negligible resistance.*

(*London B.Sc. Eng., July,* 1945)

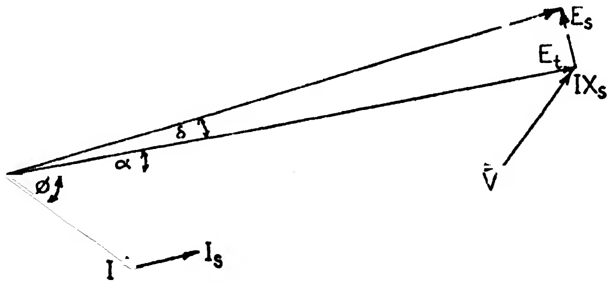


Fig. 24

The vector diagram is shown in Fig. 24 where

V = terminal p.d. = 6600 volts.

I = full load current

$$= \frac{2 \times 10^6}{\sqrt{3}\, V} \text{ amperes lagging by } 36°52' = \varnothing$$

Ix_s = voltage drop in the synchronous reactance x_s

E_t = the induced e.m.f. on full load leading V by the angle a

The percentage reactance drop is given by

$$X_s = \frac{x_s\ (\text{ohms}) \times I}{V} \times 100 \text{ per cent}$$

Also

$$\begin{aligned}\underset{\cdot}{E}_t &= \underset{\cdot}{V} + 0{\cdot}6\ Ix_s + j0{\cdot}8\ Ix_s \\ &= \underset{\cdot}{V}\left[1 + \frac{X_s}{100}(0{\cdot}6 + j0{\cdot}8)\right] \\ &= \underset{\cdot}{V}\left[1 + 0{\cdot}25\ (0{\cdot}6 + j0{\cdot}8)\right] \\ &= \underset{\cdot}{V}\ (1{\cdot}15 + j0{\cdot}2) \\ &= 1{\cdot}167\ \angle\underline{9^\circ 52'}\ \underset{\cdot}{V} \\ &= 7700\ \angle\underline{9^\circ 52'} \text{ volts.}\end{aligned}$$

Now suppose the vector $\underset{\cdot}{E}_t$ to be displaced by a small angle δ corresponding to one mechanical degree of displacement. For a 12-pole machine $\delta = 6^\circ$ and as a result of this displacement an additional internal e.m.f. $\underset{\cdot}{E}_s$ is created which causes the synchronizing current $\underset{\cdot}{I}_s$ to flow, where $\underset{\cdot}{I}_s$ lags $\underset{\cdot}{E}_s$ by 90°.

Then

$$\begin{aligned}E_s &= 2\ E_t \sin \tfrac{1}{2}\ \delta \\ &= 2 \times 7700 \times \sin 3^\circ \\ &= 806 \text{ volts.}\end{aligned}$$

$$\begin{aligned}I_s &= \frac{E_s}{x_s} \\ &= \frac{E_s\, I}{0{\cdot}25V} \\ &= \frac{E_s}{0{\cdot}25V} \times \frac{2 \times 10^6}{\sqrt{3}\ V} \text{ and lags } \underset{\cdot}{V} \text{ by } (a + \tfrac{1}{2}\delta)\end{aligned}$$

i.e. by 12°52′

Hence, **synchronizing power** $= \sqrt{3}\ V\ I_s \cos 12^\circ 52'$

$$\begin{aligned}&= \sqrt{3}\ V \times \frac{806}{0{\cdot}25V} \times \frac{2 \times 10^6}{\sqrt{3} \times 6600} \times 0{\cdot}9754 \\ &= 952800 \text{ watts or } \mathbf{952{\cdot}8\ kW.}\end{aligned}$$

33. *Explain with diagrams the changes in the armature current and power factor of an alternator connected to constant frequency bus-bars and constant voltage (a) when the steam supply is varied and the excitation is kept constant, (b) when the excitation is varied over a wide range and the steam supply is kept constant.*

A turbo-alternator having a reactance of 10 *ohms has an armature current of* 220 *amperes at unity power factor when running on* 11000-*volt constant frequency bus-bars. If the steam admission be unchanged and the e.m.f. raised by* 25 *per cent determine graphically or otherwise the new value of machine current and power factor.*

If this higher value of excitation were kept constant and the steam supply gradually increased, at what power output would the alternator break from synchronism? Find also the current and power factor to which this maximum load corresponds. State whether this power factor is lagging or leading.

(*C. and G. Final, Pt. IIB,* 1937)

It is assumed that the alternator is a three-phase one and that the reactance given is per phase. Hence the reactance between the terminals

$$= \sqrt{3} \times 10 = 17{\cdot}32 \text{ ohms.}$$

At unity power factor the reactance drop $IX = 220 \times 17{\cdot}32 = 3810$ volts. The vector diagram for this condition is shown in Fig. 25(a) and the e.m.f. of the alternator is

$$E = \sqrt{V^2 + (IX)^2} = \sqrt{11000^2 + 3810^2}$$
$$= 11640 \text{ volts.}$$

If the excitation is increased with an unchanged steam supply, i.e. with the same power input, the power output will not alter. The result is to give the armature current a lagging reactive component which exerts a demagnetizing effect on the main field and neutralizes the increase in excitation. The vector diagram is Fig. 25(b), which shows that the locus of the e.m.f. vector is the dotted line perpendicular to $I_R X$. I_R is the power component of the total current $= 220$ amperes as before and I_x is the reactive demagnetizing component.

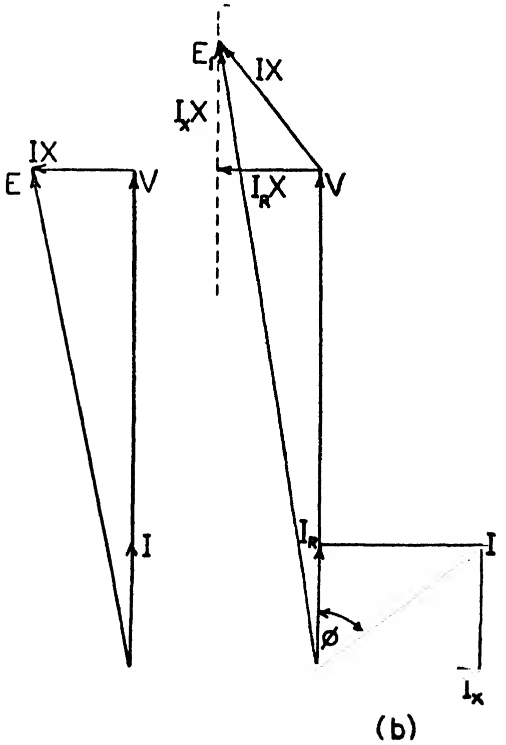


Fig. 25

With the increased excitation the e.m.f. is

$$E_1 = 1{\cdot}25\ E$$
$$= 1{\cdot}25 \times 11640$$
$$= 14550 \text{ volts.}$$

Then from Fig. 25(b)

$$V + I_xX = \sqrt{E_1^2 - (I_RX)^2}$$
$$11000 + I_xX = \sqrt{(14550)^2 - (3810)^2}$$
$$= 14040$$
$$I_xX = 3040$$
$$I_x = 175{\cdot}6 \text{ amperes.}$$

New value of machine current $= I = \sqrt{I_R^2 + I_x^2}$

$$= \sqrt{220^2 + 175{\cdot}6^2}$$
$$= \mathbf{281{\cdot}5 \text{ amperes.}}$$

New power factor $= \cos \varnothing = \frac{I_R}{I} = \frac{220}{281{\cdot}5}$

$$= \mathbf{0{\cdot}781 \text{ lagging.}}$$

When the excitation is kept constant and the steam supply is increased the locus of the e.m.f. vector is a circle (Fig. 26). A series of dotted lines may be drawn as shown, each of which represents the locus of the e.m.f. vector for a certain constant power output at varying excitation. Maximum power output conditions are reached when the circular e.m.f. locus is tangential to this family of loci, and if the steam supply is increased still further the alternator will break from synchronism.

Therefore maximum power output is being given when

$$I_RX = E_1 \text{ and } I_xX = V$$

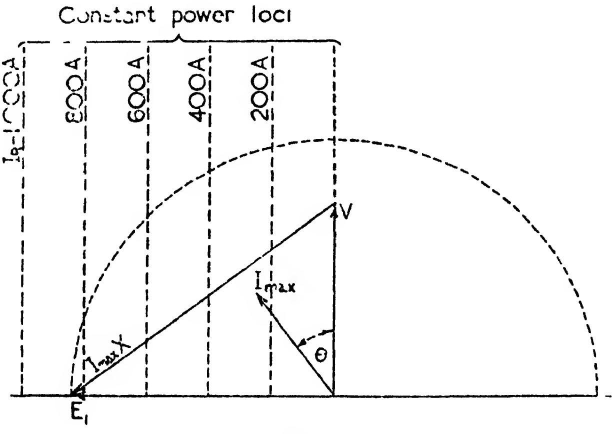


Fig. 26

Hence, $I_R = \frac{14550}{17{\cdot}32} = 840{\cdot}3$ amperes.

$I_x = \frac{11000}{17{\cdot}32} = 635{\cdot}2$ amperes.

$$I_{max} = \sqrt{840{\cdot}3^2 + 635{\cdot}2^2} = 1054 \text{ amperes.}$$

Phase angle for maximum output $= \theta = \text{arc tan } \frac{I_x}{I_R}$

$= 38° \ 5'$

Therefore, **maximum current output** $=$ **1054 amperes.**

Power factor at maximum output $= \cos\theta =$ **0·798 leading**

Maximum power output $= \frac{\sqrt{3} \times 11000 \times 1054 \times 0{\cdot}798}{1000}$ kW

$=$ **16000 kW.**

34. *Enumerate the conditions which must be fulfilled for the stable parallel operation of two 3-phase alternators. Show, by the aid of vector diagrams, how the parallel operation of two alternators is affected by (a) alteration in the steam supply to one alternator, (b) alteration in the excitation of one alternator.*

Two 20-MVA machines operate in parallel to supply a load of 35 MVA at 0·8 power factor (lagging). If the output of one machine is 25 MVA at 0·9 power factor (lagging) what is the output of the other machine, and at what power factor is it operating? Explain what adjustments should be made to equalize the load on the machines. (*I.E.E., Pt. II, May,* 1943)

Power factor of the combined load $= \cos\varnothing = 0{\cdot}8$, hence $\sin\varnothing = 0{\cdot}6$.

Total power output $= 35 \cos\varnothing$ MW $= 28$ MW.

Reactive MVA of total load $= 35 \sin\varnothing$ MVA_r $= 21$ MVA_r lagging.

Power output of first machine $= 25 \cos\varnothing_1 = 25 \times 0{\cdot}9$ MW $= 22{\cdot}5$ MW.

Reactive output of first machine $= 25 \sin\varnothing_1 = 25 \times 0{\cdot}4357$ MVA_r $= 10{\cdot}89$ MVA_r lagging

Power output of the other machine $= (28 - 22{\cdot}5)$ MW $= 5{\cdot}5$ MW.

Reactive output of the other machine $(21 - 10{\cdot}89)$ MVA_r $= 10{\cdot}11$ MVA_r lagging

Therefore,

output of the other machine $= \sqrt{5{\cdot}5^2 + 10{\cdot}11^2}$

$=$ **11·51 MVA.**

Phase angle of the other machine $= \text{arc tan } 10{\cdot}11/5{\cdot}5 = 61°27'$

Machine power factor $= \cos 61°27' =$ **0·478 lagging.**

To equalize the load on the machines the following adjustments should be made:

1. The governors controlling the steam supply to the turbines should be adjusted to decrease the steam supply to the first machine and to increase the supply to the other. This will tend to equalize the power outputs of the machines.
2. The excitation of the first machine should be slightly decreased and that of the other slightly increased. This will tend to equalize the reactive MVA outputs of the machines.

35. *Explain the procedure in starting a steam turbo-alternator and in paralleling the 3-phase alternator with machines which are already running. What extra precautions must be taken when the machine is paralleled the first time after being installed?*

Two 50-MVA, 3-phase alternators operate in parallel. The settings of the governors are such that the rise in speed from full load to no load is 2 per cent in one machine and 3 per cent in the other, the characteristics being straight lines in both cases. If each machine is fully loaded when the total load is 100 MW what will be the load on each machine when the total load is 60 MW? (*I.E.E., Pt. II, May,* 1942)

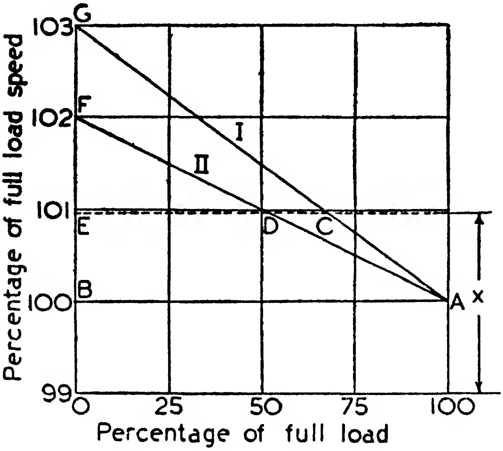


Fig. 27

For the machines to run in synchronism the speed must be the same for each. When each machine is fully loaded the operating point for each is represented by the point A, at 100 per cent of full-load speed.

The total load is now reduced to 60 MW and the speed rises to some value x per cent of full-load speed. The operating points are C for machine I and D for machine II.

From the diagram, by similar triangles,

$$\frac{ED}{AB} = \frac{EF}{BF}$$

$$\text{i.e. } ED = \frac{102 - x}{2} \times \text{100\% of full-load}$$

Hence,

$$ED = (51 - 0{\cdot}5x) \times 50 \text{ MW}$$
$$= 2550 - 25x \text{ MW.}$$

Also,

$$\frac{EC}{AB} = \frac{EG}{BG}$$

$$EC = \frac{103 - x}{3} \times 100 \text{ per cent of full-load}$$

$$= \frac{103 - x}{3} \times 50 \text{ MW} = \frac{5150 - 50x}{3} \text{ MW.}$$

But ED + EC = 60 MW.

$$2550 - 25x + \frac{5150 - 50x}{3} = 60$$

$$7650 - 75x + 5150 - 50x = 180$$

whence, $x = 100{\cdot}96$ per cent of full-load speed

Therefore, **output of first machine** $= 2550 - 25x$

$$= 2550 - 2524 \text{ MW}$$

$= \mathbf{26\ MW.}$

output of second machine $= \dfrac{5150 - 50x}{3}$

$$= \frac{5150 - 5048}{3} \text{ MW}$$

$= \mathbf{34\ MW.}$

36. *Two* 6600-*volt, star-connected alternators in parallel supply the following loads:—*

400 *kW at unity power factor,*
400 *kW at* 0·85 ,, ,, *lagging,*
300 *kW at* 0·80 ,, ,, ,,
800 *kW at* 0·70 ,, ,, ,,

The armature current of one machine is 100 *amperes at* 0·9 *power factor lagging. Determine the armature current, the output and the power factor of the other machine.* (*I.E.E., Pt. II, May,* 1938)

Total power supplied by both machines $= (400 + 400 + 300 + 800)$
$= 1900$ kW.

Power supplied by one machine $= \sqrt{3} \times 6600 \times 100 \times 0{\cdot}9 \times 10^{-3}$
$= 1028$ kW.

Hence, **power supplied by the other machine** $= 1900 - 1028 =$ **872 kW.**

Phase angle of first load $= \varnothing_1 = 0°$ $(\cos \varnothing_1 = 1)$
Phase angle of second load $= \varnothing_2 = 31°48'$ $(\cos \varnothing_2 = 0{\cdot}85)$
Phase angle of third load $= \varnothing_3 = 36°52'$ $(\cos \varnothing_3 = 0{\cdot}8)$
Phase angle of fourth load $= \varnothing_4 = 45°34'$ $(\cos \varnothing_4 = 0{\cdot}7)$
Reactive kVA of each load = Power in kW × tan. of phase angle.

Reactive kVA of first load $= 400 \times 0 = 0$ kVA$_r$
Reactive kVA of second load $= 400 \times 0{\cdot}62 = 248$ kVA$_r$
Reactive kVA of third load $= 300 \times 0{\cdot}75 = 225$ kVA$_r$
Reactive kVA of fourth load $= 800 \times 1{\cdot}02 = 816$ kVA$_r$
Reactive kVA of total load $= 1289$ kVA$_r$
Reactive kVA of one machine $= 1028 \times \tan (\text{arc} \cos 0{\cdot}9)$
$= 1028 \times 0{\cdot}4841$
$= 497{\cdot}5$ kVA$_r$

Reactive kVA of other machine $= 1289 - 4975$
$= 791{\cdot}5$ kVA$_r$

Phase angle of other machine $= \text{arc} \tan \dfrac{791{\cdot}5}{872}$
$= 42°14'$

Power factor of other machine $= \cos 42°14' =$ **0·74 lagging**

Armature current of other machine $= \dfrac{872000}{\sqrt{3} \times 6600 \times 0{\cdot}74}$
$=$ **103 amperes.**

(v) Efficiency, cooling systems, etc.

37. *Determine the efficiency of a* 6000-*kVA turbo-alternator from the following test figures taken at unity power factor and full-load: Volume of cooling air at outlet,* 12 *cubic metres per second*; *outlet temperature,* 40°*C.*;

inlet temperature, 15°*C.*; *barometer pressure,* 750 *millimetres. The specific heat of air at constant pressure is* 0·2375, *and* 1 *kilogram of dry air at* 0°*C. and* 760 *millimetres pressure occupies* 0·775 *cubic metre.*

Prove any formula used. (*C. and G. Final, Pt. IIA*, 1940)

By the combination of Boyle's and Charles' Laws,

Weight of outlet air per sec. $= \frac{12}{0{\cdot}775} \times \frac{273}{273 + 40} \times \frac{750}{760}$ kg.

$= 13{\cdot}33$ kg.

Heat absorbed per sec. $= 13{\cdot}33 \times 10^3 \times (40 - 15) \times 0{\cdot}2375$

$= 79140$ calories.

1 kW = 1000 joules per sec.

$= 1000/4{\cdot}18$ calories per sec.

Hence, losses dissipated by the cooling air $= \frac{79140 \times 4{\cdot}18}{1000}$ kW

$= 330{\cdot}9$ kW.

Output at unity power factor = 6000 kW.

Efficiency $\frac{\text{Output}}{\text{Output} + \text{losses}} \times 100$

$= \frac{6000}{6000 + 330{\cdot}9} \times 100$ per cent

= **94·77 per cent.**

38. *Describe the closed-circuit air-cooling system used for a turbo-alternator. Find the volume of cooling air, in m.³ per sec., required to dissipate the* 670 *kW of loss in a* 20000-*kVA alternator on full load at* 0·8 *power factor. The inlet and outlet temperatures are to be respectively* 15°*C. and* 35°*C. Determine also the amount of cooling water, in gallons per min., required to cool the air, assuming a rise of* 8°*C. in the water temperature.* (*Specific heat of air at constant pressure,* 0·2375: *volume of dry air,* 0·775 *m.³ per kg. at* 0°*C. and atmospheric pressure.*) (*I.E.E., Pt. II, May,* 1942)

Heat to be dissipated $= 670 \times \frac{1000}{4{\cdot}18}$ calories per sec.

$= 1{\cdot}603 \times 10^5$ calories per sec.

Weight of air required per sec. $= \frac{1{\cdot}603 \times 10^5}{(35 - 15) \times 0{\cdot}2375}$ gm.

$= 33{\cdot}75$ kg.

Volume of air required at 0°C. and atmospheric pressure $= 33{\cdot}75 \times 0{\cdot}775$ cu. m. per sec.

$= 26{\cdot}16$ cu. m. per sec.

Volume of air required at inlet $= 26{\cdot}16 \times \frac{273 + 15}{273}$ cu. m. per sec.

= **27·6 cu. m. per sec.**

Weight of cooling water per sec. $= \dfrac{1{\cdot}603 \times 10^6}{8}$ gm.

$= 0{\cdot}2 \times 10^5$ gm.

1 gallon of water weighs 10 lb. or $(10 \times 453{\cdot}6)$ gm.

Hence, **amount of cooling water required** $= \dfrac{0{\cdot}2 \times 10^5 \times 60}{4536}$ gallons per min.

$=$ **264·5 gallons per min.**

CHAPTER III

TRANSFORMERS

(i) Equivalent circuits, voltage regulation, etc.

39. *The test figures of a short-circuit test on the high-voltage side of a single-phase* 100-*kVA*, 10000/400-*volt transformer are: applied p.d.* 500 *volts, current* 10 *amperes, power input* 1 *kW.*

Calculate the low-voltage terminal p.d. when the load on the low-voltage side is 250 *amperes at* 0·8 *lagging power factor.*

(*C. and G. Final, Pt. I,* 1944)

When one winding of a transformer is short-circulated there is no flux in the core and no induced e.m.f. in the windings. To circulate a given current in the windings then needs an applied p.d. sufficient only to overcome the winding resistances and leakage reactances. To simplify the calculations the resistance and reactance of the winding which is short-circuited can be "referred to" the other winding where they may be added to the corresponding constants of that winding. This gives the "equivalent resistance" and "equivalent reactance" of the whole transformer referred to that winding. From these values the equivalent impedance may be found.

In this problem, referred to the high-voltage side,

$$\text{equivalent impedance} = \frac{500 \text{ volts}}{10 \text{ amperes}} = 50 \text{ ohms} = Z_o,$$

$$\text{equivalent resistance} = \frac{1000 \text{ watts}}{(10 \text{ amperes})^2} = 10 \text{ ohms} = R_o,$$

$$\text{equivalent reactance} = \sqrt{Z_o^2 - R_o^2}$$

$$= \sqrt{50^2 - 10^2} = 49 \text{ ohms} = X_o.$$

Fig. 28 shows the equivalent circuit with the low-voltage side on load at 250 amperes and 0·8 lagging power factor.

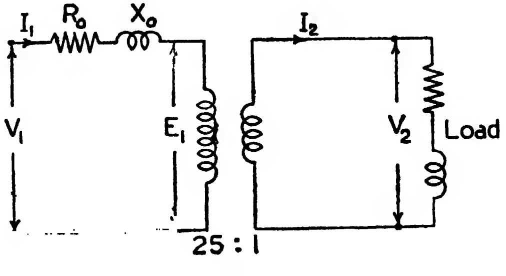


Fig. 28

If the magnetizing current in the primary be neglected,

$$\frac{\text{primary current for 250 amperes}}{\text{secondary current}} = \frac{250}{25} = 10 \text{ amperes} = I_1$$

$$\text{p.d. across } R_o = I_1 R_o = 10 \times 10 = 100 \text{ volts.}$$

$$\text{p.d. across } X_o = I_1 X_o = 10 \times 49 = 490 \text{ volts.}$$

These p.d.s are subtracted vectorially from V_1 the input p.d. to obtain E_1 the primary induced e.m.f.

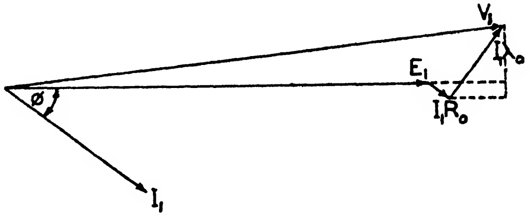


Fig. 29

Again if the magnetizing current be neglected, the primary current I_1 makes the same phase angle with the primary induced e.m.f. as the secondary current does with V_2.

From the vector diagram, Fig. 29

$$(E_1 + I_1R_o \cos \varnothing + I_1X_o \sin \varnothing)^2 + (I_1X_o \cos \varnothing - I_1R_o \sin \varnothing)^2 = V_1^2$$

$$(E_1 + 100 \times 0{\cdot}8 + 490 \times 0{\cdot}6)^2 + (490 \times 0{\cdot}8 - 100 \times 0{\cdot}6)^2 = 10{,}000^2$$

i.e. $$(E_1 + 374)^2 + (332)^2 = 10000^2$$

whence, $E_1 = 9620$ volts.

Therefore, $V_2 = 9620 \div 25 = 384{\cdot}8$ volts.

i.e. **Terminal p.d. on low-voltage side at this load = 384·8 volts.**

40. *Define the terms "equivalent resistance" and "equivalent reactance" as applied to transformers. Deduce an expression for the equivalent resistance, referred to the primary, of a single-phase transformer in which the actual resistances of the primary and secondary windings are R_1 and R_2 ohms respectively, and the numbers of turns on these windings are N_1 and N_2 respectively.*

A 3-phase transformer rated at 1000 *kVA,* 11/3·3 *kV has its primary windings star-connected and its secondary windings delta-connected. The actual resistances per phase of these windings are: primary* 0·375 *ohm, secondary* 0·095 *ohm; and the leakage reactances per phase are: primary* 9·5 *ohms, secondary* 2 *ohms. Calculate the voltage at normal frequency which must be applied to the primary terminals in order to obtain full-load current in the windings when the secondary terminals are short-circuited. Calculate also the power input under these conditions.* (*I.E.E., Pt. II, Nov.,* 1943)

Primary phase voltage $= \dfrac{11}{\sqrt{3}}$ kV.

Secondary phase voltage $= 3{\cdot}3$ kV.

Hence, phase turns ratio $= k = \dfrac{11}{\sqrt{3}} \times \dfrac{1}{3{\cdot}3} = 1{\cdot}925$

Resistance of the secondary per phase referred to primary $= R_2' = k^2R_2 = 1{\cdot}925^2 \times 0{\cdot}095$
$= 0{\cdot}352$ ohm.

Total resistance per phase referred to the primary $= R_o = R_1 + R_2'$
$= 0{\cdot}375 + 0{\cdot}352 = 0{\cdot}727$ ohms.

Reactance of secondary per phase referred to the primary $= X_2' = k^2X_2 = 1{\cdot}925^2 \times 2$ ohms
$= 7{\cdot}411$ ohms.

Total reactance per phase referred to the primary $= X_o = X_1 + X_2'$
$= 9{\cdot}5 + 7{\cdot}411 = 16{\cdot}91$ ohms.

Impedance per phase referred to the primary $= Z_o = \sqrt{R_o^2 + X_o^2}$
$= \sqrt{0{\cdot}727^2 + 16{\cdot}91^2}$
$= 16{\cdot}92$ ohms.

Full-load primary current $I_1 = \dfrac{1000000}{\sqrt{3} \times 11000} = 52{\cdot}5$ amperes.

Applied p.d. per phase required to circulate this current with the secondary on short-circuit is

$I_1Z_o = 52{\cdot}5 \times 16{\cdot}92$ volts.

Line p.d. necessary $= \sqrt{3} \times I_1Z_o$
$= \sqrt{3} \times 52{\cdot}5 \times 16{\cdot}92$ volts
$=$ **1.53 kV.**

Power input under these conditions $= 3I_1^2R_o$
$= 3 \times 52{\cdot}5^2 \times 0{\cdot}727$ watts
$=$ **6000 watts.**

41. *A 3-phase transformer, ratio* 33/6·6 *kV, delta/star,* 2 *MVA, has a primary resistance of* 8 *ohms per phase and a secondary resistance of* 0·08 *ohm per phase. The percentage impedance is* 7 *per cent. Calculate the secondary voltage and the efficiency at full-load,* 0·75 *power factor lagging.*

(*I.E.E., Pt. II, May,* 1941)

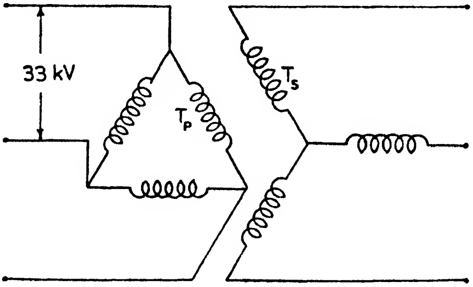


Fig. 30

Secondary current on full-load $= \dfrac{2 \times 10^6}{\sqrt{3} \times 6{\cdot}6 \times 10^3}$
$= 175$ amperes.

Turns ratio per phase $= \dfrac{T_p}{T_s} = \dfrac{33 \times \sqrt{3}}{6{\cdot}6} = 8{\cdot}65$
$= k$

Equivalent resistance per phase referred to the secondary $= R_2 + R_1'$

$$= R_2 + R_1 \frac{1}{k^2}$$

$$= 0{\cdot}08 + 8\left(\frac{1}{8{\cdot}65}\right)^2$$

$$= 0{\cdot}1867 \text{ ohm} = R_o'$$

Secondary impedance drop per phase $= \frac{7}{100} \times \frac{6600}{\sqrt{3}}$ volts,

$$= 266{\cdot}7 \text{ volts.}$$

Hence, secondary impedance per phase $= \frac{266{\cdot}7}{175}$ ohms,

$$= 1{\cdot}523 \text{ ohms} = Z_o'$$

Equivalent reactance per phase referred to the secondary $= \sqrt{Z_o'^2 - R_o'^2}$

$$= \sqrt{1{\cdot}523^2 - 0{\cdot}1867^2} = 1{\cdot}51 \text{ ohms.}$$

$$= X_o'$$

Fig. 31 shows the equivalent circuit per phase referred to the secondary:

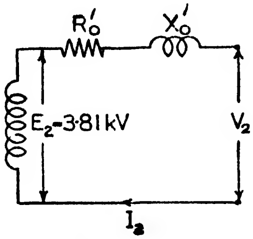


Fig. 31

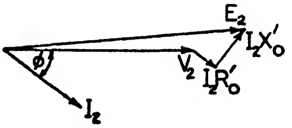


Fig. 32

From the vector diagram, Fig. 32, the secondary voltage on full-load is seen to be given by

$$3810 = \sqrt{(V_2 + I_2R_o' \cos\phi + I_2X_o' \sin\phi)^2 + (I_2X_o' \cos\phi - I_2R_o' \sin\phi)^2}$$

$$= \sqrt{(V_2 + 175 \times 0{\cdot}1867 \times 0{\cdot}75 + 175 \times 1{\cdot}51 \times 0{\cdot}6613)^2 + (175 \times 1{\cdot}51 \times 0{\cdot}75 - 175 \times 0{\cdot}1867 \times 0{\cdot}6613)^2}$$

$$V_2 = 3607 \text{ volts}$$

$V_2 \times \sqrt{3}$ = **6250 volts = secondary terminal p.d. on full-load, 0·75 p.f. lagging.**

Total power output on this load $= 2 \times 0{\cdot}75$ MW $= 1{\cdot}5$ MW.

Copper loss in 3 phases on full-load $= 3 \times 175^2 \times 0{\cdot}1867$ watts

$$= 17150 \text{ watts.}$$

Therefore, ignoring the iron losses,

full-load efficiency $= \frac{1500000}{1500000 + 17150} \times 100$ per cent

= **98·9 per cent.**

42. *Develop an equivalent diagram for the transformer and derive therefrom an expression for the voltage regulation.*

Draw a complete curve of percentage regulation to a base of power factor

for a transformer having a copper loss of 1·5 *per cent of the output and a reactance drop of* 4 *per cent of the voltage.* (*C. and G. Final Pt. II,* 1941)

Provided that the percentage regulation does not exceed about 20 per cent it may be expressed fairly accurately in terms of the percentage copper loss and the percentage reactance drop for full-load current as follows:

$$e = e_r \cos\varnothing + e_x \sin\varnothing$$

where e = the percentage regulation at full-load
e_r = the percentage copper loss of full-load output,
e_x = the percentage reactance drop at full-load current,
$\varnothing$ = the load phase angle at the secondary terminals.

In this problem,

$$e = 1{\cdot}5 \cos\varnothing + 4 \sin\varnothing$$

The table below shows the values for e at various power factors. For lagging power factors $\sin\varnothing$ is positive and for leading power factors it is negative.

$\cos\varnothing$	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0 lag
$\sin\varnothing$	1·0	0·995	0·98	0·954	0·917	0·866	0·80	0·714	0·60	0·436	0·0
e	4·0	4·13	4·22	4·26	4·27	4·21	4·10	3·91	3·60	3·09	1·5
$\cos\varnothing$	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0 lead
	—	—	—	—	—	—	—	—	—	—	
$\sin\varnothing$	1·0	0·995	0·98	0·954	0·917	0·866	0·80	0·714	0·60	0·436	0·0
	—	—	—	—	—	—	—	—	—	—	+
e	4·0	3·83	3·62	3·36	3·07	2·71	2·30	1·81	1·20	0·39	1·5

Fig. 33 shows the curve of percentage regulation plotted from this table for power factors ranging from zero lagging to zero leading.

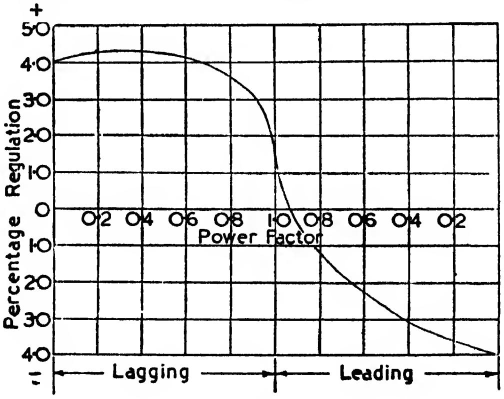


Fig. 33

(ii) Losses, efficiency, temperature-rise, etc.

43. *Derive from first principles an expression for the eddy current loss in transformer sheet steel. Apply the expression to the calculation of the loss in a transformer core weighing* 1000 *kg., built of* 0·42 *mm. plates of steel of resistivity* 80 *microhm-cm. Flux density* 12500 *lines per sq. cm.: frequency* 50 *cycles per sec.: specific gravity* 7·5. (*I.E.E., Pt. II, May,* 1940)

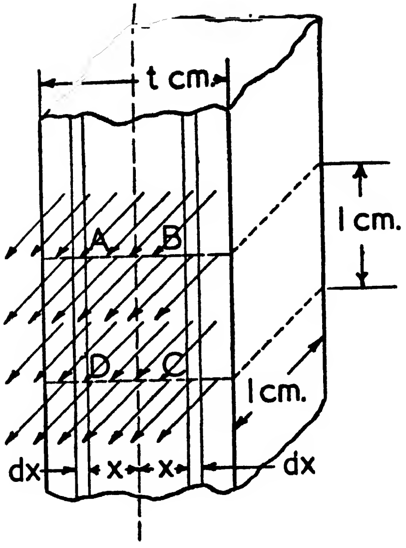


Fig. 34

Consider 1 sq. cm. of a sheet of the iron (Fig. 34), thickness t cm. If the maximum flux density normal to the surface shown is B_m and the frequency is f cycles per sec., the maximum flux linking the area ABCD is $2B_mx$ maxwells, where $x = \frac{1}{2}AB$.

The R.M.S. value of the e.m.f. induced is, in the loop formed by the strips AD and BC,

$$e = 4{\cdot}44\ f.2B_mx.10^{-8} \text{ volts.}$$

This induced e.m.f. causes a current to flow up one of these strips and down the other. The resistance of each strip is

$$\frac{\rho \times 1 \text{ cm.}}{(dx \times 1) \text{ sq. cm.}} \text{ i.e. } \frac{\rho}{dx},$$

where ρ is the resistivity of the iron in ohm.-cm.

Hence, current flowing in the strips $= \dfrac{\text{e.m.f.}}{\text{resistance}}$

$$= 4{\cdot}44f.2B_mx.10^{-8} \div \frac{2\rho}{dx}$$

$$= \frac{4{\cdot}44fB_mx.dx.10^{-8}}{\rho} \text{ amperes.}$$

Power loss due to this current $= \dfrac{4{\cdot}44^2f^2B_m^2x^2(dx)^2 10^{-16}}{\rho^2} \times \dfrac{2\rho}{dx}$

$$= \frac{2(4{\cdot}44)^2f^2B_m^2x^2dx}{\rho} \times 10^{-16} \text{ watts,}$$

Total loss dissipated in a sheet of width t cm. $= \dfrac{2(4{\cdot}44)^2f^2B_m^2 10^{-16}\int_0^{\frac{t}{2}} x^2dx}{\rho}$

$$= \frac{4{\cdot}44^2f^2B_m^2t^3 10^{-16}}{12\rho} \text{ watts.}$$

Weight of iron in the sheet considered = t cu. cm. × s where s is the specific gravity of the iron,
= ts gm.

Therefore, loss per kg. of iron $= \dfrac{4{\cdot}44^2f^2B_m^2t^3\ 10^{-16} \times 10^3}{12 \times ts}$ watts,

$$= \frac{16{\cdot}43\ f^2B_m^2t^2}{\rho s} \times 10^{-14} \text{ watts.}$$

In the transformer core given,

$$f = 50,\ B_m = 12500,\ t = 0{\cdot}042,\ s = 7{\cdot}5,\ \rho = 80 \times 10^{-6},$$

Hence, **eddy current loss** $= \dfrac{16{\cdot}43 \times 50^2 \times 12500^2 \times 0{\cdot}042^2 \times 10^{-14}}{80 \times 10^{-6} \times 7{\cdot}5}$ watts per kg.

= **188·7 watts,** in a core weighing 1000 kg.

44. *State and prove the conditions under which a transformer operates at its maximum efficiency.*

A 1100/230-*volt,* 150-*kVA, single-phase transformer has a core loss of* 1·4 *kW and a full-load copper loss of* 1·6 *kW. Determine* (*a*) *the kVA load for maximum efficiency,* (*b*) *the efficiency curve from* ¼ *to* 1¼ *of full-load at a power factor of* 0·8 *lagging.* (*H.N.C.*, 1938)

The load at which a transformer operates at its maximum efficiency is that load at which the copper losses are equal to the iron losses.

(a) Now, since the copper losses are proportional to the square of the kVA output, the kVA output is proportional to the square root of the copper losses.

Given that the copper loss is 1·6 kW at a load of 150 kVA,
kVA load at which the copper losses will be 1·4 kW (= iron losses) $= 150 \times \sqrt{\dfrac{1{\cdot}4}{1{\cdot}6}}$ kVA
= 140·3 kVA.

Hence, **load for maximum efficiency** = **140·3 kVA.**

(b) **Efficiency curve**

$$\text{The efficiency} = \frac{\text{Output}}{\text{Output} + \text{copper loss} + \text{iron loss}} \times 100 \text{ per cent.}$$

The iron loss is constant at 1·4 kW on all loads while the copper loss varies according to the square of the kVA output and is 1·6 kW when the output is 150 kVA, i.e. 120 kW at power factor 0·8.

The losses, output, and efficiencies at various loads are tabulated as follows:

Full-load ×	0·25	0·5	0·75	1·0	1·25
Output in kW	30	60	90	120	150
Copper loss, kW	0·1	0·4	0·9	1·6	2·5
Iron loss, kW	1·4	1·4	1·4	1·4	1·4
Input in kW	31·5	61·8	92·3	123	153·9
Efficiency, per cent	95·2	97·1	97·5	97·57	97·47

The efficiency curve is plotted from these figures in Fig. 35.

Note.—Maximum efficiency (at 140·3 kVA) $= \dfrac{140{\cdot}3 \times 0{\cdot}8}{140{\cdot}3 \times 0{\cdot}8 + 2{\cdot}8} \times 100$
= **97·57 per cent.**

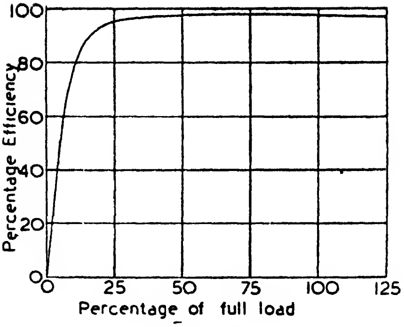


Fig. 35

45. *The full-load voltage drops in a transformer are 2 per cent and 4 per cent, due respectively to resistance and leakage reactance. The full-load copper loss is equal to the iron losses. Calculate (a) the efficiency on half-load at unity power factor, (b) the lagging power factor on the full-load output at which the voltage drop is a maximum, and (c) this maximum percentage voltage drop.*
(*C. and G. Final*, Pt. 1. 1943)

(a) Since the full-load voltage drop due to resistance is 2 per cent the full-load copper loss is 2 per cent of the output.

If full-load at unity power factor be denoted by 100 per cent,

Iron losses (constant) = 2 per cent (the same as the copper losses on full-load)

Copper loss on half-load = $\frac{1}{4}$ of 2 per cent
= 0·5 per cent.

Output on half-load = 50 per cent.

Hence, **efficiency on half-load** $= \frac{\text{Output}}{\text{Output} + \text{losses}} \times 100$

$$= \frac{50}{50 + 2 + 0{\cdot}5} \times 100$$

= **95·23 per cent.**

(b) The percentage voltage drop is given by:

$e = e_r \cos\varnothing + e_x.\sin\varnothing$ (*see* Problem 42)

By differentiating e with respect to $\varnothing$ and equating $\frac{de}{d\varnothing}$ to zero it can be shown that e is a maximum when

$$\tan\varnothing = \frac{e_x}{e_r}$$

Hence, maximum voltage drop occurs when $\varnothing = \text{arc} \tan \frac{4}{2}$
$= 63°26'$

and **lagging power factor for maximum voltage drop** = $\cos\varnothing$ = **0·447.**

(c) **Maximum voltage drop** $= e_r.\cos\varnothing + e_x.\sin\varnothing$
$= 2 \times \cos 63°26' + 4 \times \sin 63°26'$
$= 2 \times 0{\cdot}447 + 4 \times 0{\cdot}8945$
= **4·47 per cent.**

46. *Explain the term "all-day efficiency" as applied to transformers, and discuss its importance in the selection of a transformer for a given duty.*

It is required to select a transformer to supply a load which varies over each 24-hour period in the following manner: 100 *kW for* 5 *hours,* 200 *kW for* 5 *hours,* 300 *kW for* 12 *hours,* 360 *kW for* 2 *hours. Two transformers each rated at* 300 *kVA are available: one (A) has an iron loss of* 1·3 *kW and a full-load copper loss of* 3·7 *kW; the other (B) has an iron loss of* 2·5 *kW and a full-load copper loss of* 2·5 *kW. Calculate the annual cost of supplying the losses for each transformer if the cost of energy is* 0·8*d. per kWh. Thence determine which transformer should be selected, if the capital cost of transformer A is £*50 *more than that of transformer B and the annual charges for interest and depreciation are* 8 *per cent. Calculate also the full-load and maximum efficiencies of the transformer selected, and the load at which maximum efficiency occurs.* (*I.E.E., Pt. II, May,* 1943)

It is assumed that the load power factor is unity, in which case full-load for each transformer is 300 kW.

Transformer A

Energy dissipated per day as iron losses $= 24 \times 1{\cdot}3 = 31{\cdot}2$ kWh.
Copper loss on 100 kW output $= (100/300)^2 \times 3{\cdot}7 = 0{\cdot}411$ kW.
" " " 200 kW " $= (200/300)^2 \times 3{\cdot}7 = 1{\cdot}644$ kW.
" " " 300 kW " $= 3{\cdot}7$ kW.
" " " 360 kW " $= (360/300)^2 \times 3{\cdot}7 = 5{\cdot}328$ kW.

Hence, energy dissipated per day as copper losses $= 0{\cdot}411 \times 5 + 1{\cdot}644 \times 5 + 3{\cdot}7 \times 12 + 5{\cdot}328 \times 2$
$= 65{\cdot}33$ kWh.

Total energy losses per day $= 31{\cdot}2 + 65{\cdot}33 = 96{\cdot}53$ kWh.
Cost of energy losses per annum $= 96{\cdot}53 \times 365 \times 0{\cdot}8$d.
= 28180d.
= £117 8s.

Transformer B

Energy dissipated per day as iron losses $= 24 \times 2{\cdot}5 = 60$ kWh.
Copper loss on 100 kW output $= (100/300)^2 \times 2{\cdot}5 = 0{\cdot}278$ kW.
" " " 200 kW " $= (200/300)^2 \times 2{\cdot}5 = 1{\cdot}111$ kW.
" " " 300 kW " $= 2{\cdot}5$ kW.
" " " 360 kW " $= (360/300)^2 \times 2{\cdot}5 = 3{\cdot}6$ kW.

Hence, energy dissipated per day as copper losses $= 0{\cdot}278 \times 5 + 1{\cdot}111 \times 5 + 2{\cdot}5 \times 12 + 3{\cdot}6 \times 2$
$= 44{\cdot}145$ kWh.

Total energy losses per day $= 60 + 44{\cdot}145 = 104{\cdot}145$ kWh.
Cost of energy losses per annum $= 104{\cdot}145 \times 365 \times 0{\cdot}8$d.
= 30400d.
= £126 13s.

i.e. cost of energy losses in A is £9 5s. less than in B. As the capital cost of A is £50 more than that of B, the interest and depreciation charges on A will be 8 per cent of £50, i.e. £4 more than those on B.

Hence the total annual charges on A will be £9 5s. − £4 = £5 5s. less than those on B.

Therefore, **transformer A should be selected.**

Full-load losses of transformer A $= 1{\cdot}3 + 3{\cdot}7$ kW $= 5$ kW.

Full-load efficiency of transformer A $= \dfrac{300}{300 + 5} \times 100$

$= $ **98·35 per cent.**

Maximum efficiency occurs when the copper losses are equal to the iron losses, i.e. when they are 1.3 kW each.

Load at which this occurs $= \sqrt{\dfrac{1{\cdot}3}{3{\cdot}7}} \times 300$ kW

$= $ **177·8 kW.**

Maximum efficiency $= \dfrac{177{\cdot}8}{177{\cdot}8 + 2 \times 1{\cdot}3} \times 100$

$= $ **98·56 per cent.**

47. *The iron loss and the full-load copper loss of a* 500-*kVA works transformer are* 4 *kW and* 9 *kW respectively. The transformer is energized* 8 *hours per day and the maximum demand is* 500 *kW. The load factor of losses is* 45 *per cent, the tariff* £3 10*s. per kW and* 0·25*d. per unit. Calculate the present capitalized value of the losses if the life of the transformer is* 20 *years, and the interest rate is* 4 *per cent per annum.* (*I.E.E., Pt. II, May,* 1941)

The load factor of losses is defined as the ratio

$$\frac{\text{Average value of losses}}{\text{Maximum value of losses}}$$

The maximum value of the losses will occur when the transformer is working on its maximum demand, which in this case is 500 kW, i.e. full-load for the transformer.

Hence, maximum value of losses $= 4$ kW $+ 9$ kW $= 13$ kW.

Average value of losses over an 8-hour period $= 0{\cdot}45 \times 13$ kW $= 5{\cdot}85$ kW.

Note.—It has been assumed here that the load factor of losses given is for the **total** losses (iron + copper) and not for the copper loss alone.

Annual cost of losses = £3 10s. per kW of maximum demand + 0·25d. per kWh.

= £3 10s. × 13 + 0·25d. × energy loss per annum.

Energy loss per day = 5·85 × 8 kWh = 46·8 kWh.

„ „ „ annum = 46·8 × 365 = 17082 kWh.

Hence, annual cost of losses = £45 10s. + 17082 × 0·25d.

= £63 8s. 4d.

To find the capitalized value of this annual expenditure on losses over a period of 20 years we proceed as follows:

Suppose a sum £A falls due at the end of each year for n years, and that the rate of interest is r per cent.

At the end of the first year £A is due and the value of this sum in the remaining n — 1 years is

$$£A\,(1 + \frac{r}{100})^{n-1} = £AR^{n-1}$$

Similarly at the end of the second year another £A is due and the amount of this sum in the remaining n − 2 years is £AR^{n-2}, and so on.

Therefore, total amount

$$\text{in n years} = £\,(AR^{n-1} + AR^{n-2} + \ldots + AR^2 + AR + A)$$
$$= £A\,(1 + R + R^2 + \ldots \text{ to n terms})$$
$$= £A.\frac{R^n - 1}{R - 1}$$

In this problem, £A = £63·42, R = (1 + 0·04) = 1·04, n = 20.

Hence, **capitalized value of losses over 20 years** $= \dfrac{£63{\cdot}42\,(1{\cdot}04^{20} - 1)}{0{\cdot}04}$

$$= \frac{£63{\cdot}42\,(2{\cdot}188 - 1)}{0{\cdot}04}$$
$$= \mathbf{£1884.}$$

48. *Give a diagram of connections for the back-to-back load test of two* 300-*kVA*, 6600/420-*volt, mesh-star,* 3-*phase transformers, and explain the theory of the test. A* 200-*kVA transformer on full-load at unity power factor has an efficiency of* 98 *per cent. The full-load temperature-rise measured in the oil is* 45° *C., and the copper loss is three times the core loss. Estimate the efficiency at unity power factor and the final steady oil temperature-rise for* 20 *per cent overload.* (*I.E.E., Pt. II, May,* 1938)

On full-load at unity power factor,

$$\text{Input} = 200 \times \frac{100}{98} \text{ kW} = 204{\cdot}1 \text{ kW}.$$

Therefore, total losses = 204·1 − 200 = 4·1 kW.

Of the total loss the copper loss is $\frac{3}{4}$ and the iron loss $\frac{1}{4}$.

Hence, copper loss on full-load $= \frac{3}{4} \times 4{\cdot}1$ kW $= 3{\cdot}075$ kW.

iron " " $= \frac{1}{4} \times 4{\cdot}1$ kW $= 1{\cdot}025$ kW.

On 20 per cent overload at unity power factor,

Copper loss $= \left(\frac{120}{100}\right)^2 \times 3{\cdot}075 = 4{\cdot}428$ kW.

Iron loss $= 1{\cdot}025$ kW.

Total losses $= 4{\cdot}428 + 1{\cdot}025$ kW $= 5{\cdot}453$ kW.

Output at this load $= \left(\frac{120}{100}\right) \times 200$ kW $= 240$ kW.

Hence, **efficiency at 20 per cent overload** $= \dfrac{240}{240 + 5{\cdot}453} \times 100$

$$= \mathbf{97{\cdot}74 \text{ per cent.}}$$

The steady temperature-rise of the oil is proportional to the amount of losses being dissipated by the oil.

i.e. **Temperature-rise at 20 per cent overload** $= 45° \text{ C.} \times \dfrac{5{\cdot}453}{4{\cdot}1}$

$$= \mathbf{59{\cdot}8° \text{ C.}}$$

49. *Describe briefly, on a basis of the nomenclature of the B.S.I., the several methods of cooling power transformers.*

A transformer has a full-load copper loss equal to twice its core loss. In a temperature-rise test at full-load the following rises were observed: 12° *C. after* 1 *hour,* 21° *C. after* 2 *hours. Find* (*a*) *the final steady temperature-rise and time constant*; (*b*) *the steady temperature-rise after* 1 *hour at* 50 *per cent overload, from cold.* (*I.E.E., Pt. II, May,* 1942)

The temperature rise θ of a transformer t hours after switching on from cold is given by the expression

$$\theta = \theta_m (1 - e^{-t/\alpha})$$

where θ_m = the final steady temperature-rise,
e = 2·718 . . . the base of Napierian logarithms,
α = the heating time constant of the transformer.

(a) In this case, on full-load,

$$\theta = 12^\circ \text{ C. when } t = 1 \text{ hour,}$$
$$\text{and } \theta = 21^\circ \text{ C. when } t = 2 \text{ hours.}$$
$$\text{Hence, } 12 = \theta_m (1 - e^{-1/\alpha}) \qquad (1)$$
$$\text{and } 21 = \theta_m (1 - e^{-2/\alpha}) \qquad (2)$$

Divide (2) by (1),

$$\frac{21}{12} = \frac{\theta_m (1 - e^{-2/\alpha})}{\theta_m (1 - e^{-1/\alpha})} = 1 + e^{-1/\alpha}$$

$$\text{i.e.} \quad e^{-1/\alpha} = 0{\cdot}75$$
$$-1/\alpha = \log_e 0{\cdot}75 = -0{\cdot}2876$$
$$\alpha = \mathbf{3{\cdot}477 \text{ hours} = \text{time constant.}}$$

Substitute for α in (1) above,

$$12 = \theta_m (1 - 0{\cdot}75) = 0{\cdot}25\ \theta_m$$
$$\theta_m = \mathbf{48^\circ \text{ C.} = \text{final steady temperature-rise.}}$$

(b) Copper loss on full-load = 2 × core loss

„ „ 50 per cent

$$\text{overload} = \left\{\frac{150}{100}\right\}^2 \times 2 \times \text{core loss,}$$
$$= 4{\cdot}5 \times \text{core loss.}$$

$$\frac{\text{Total loss on 50 per cent overload}}{\text{Total loss on full-load}} = \frac{(4{\cdot}5 + 1) \times \text{core loss}}{(2 + 1) \times \text{core loss}} = \frac{5{\cdot}5}{3}$$

$$\text{Total loss on 50 per cent overload} = \frac{5{\cdot}5}{3} \times \text{total loss on full-load}$$

Hence, final temperature-rise on 50 per cent overload

$$= \text{final temperature-rise on full load} \times \frac{5{\cdot}5}{3}$$
$$= \frac{5{\cdot}5}{3} \times 48^\circ \text{ C.} = 88^\circ \text{ C.}$$

Therefore, after 1 hour on 50 per cent overload the temperature-rise is given by:

$$\theta = 88 (1 - e^{-1/\alpha})$$
$$= 88 (1 - 0{\cdot}75) = \mathbf{22^\circ \text{ C.}}$$

50. *The full-load temperature-rise of an oil-immersed transformer is 15° C. after 1 hour and 23° C. after 2 hours: (a) find the final steady temperature rise on full-load, (b) estimate the 1-hour rating for the same temperature rise if the full-load copper loss is twice the iron loss.*

(*C. and G. Final, Pt. II* 1940)

Using the expression $\theta = \theta_m (1 - e^{-t/a})$, where the symbols have their same meanings as in Problem 49, from the data given on full-load

$$15 = \theta_m (1 - e^{-1/a}) \qquad (1)$$

$$23 = \theta_m (1 - e^{-2/a}) \qquad (2)$$

Dividing (2) by (1),

$$1 + e^{-1/a} = \frac{23}{15}$$

$$e^{-1/a} = \frac{8}{15} \qquad (3)$$

Substitute (3) in (1),

$$15 = \theta_m \left(1 - \frac{8}{15}\right) = \frac{7}{15}\theta_m$$

$$\theta_m = \frac{15 \times 15}{7} = 32{\cdot}14° \text{ C.}$$

i.e. **Final steady temperature rise on full-load = 32·14° C.**

If, on overload, the temperature rise was 32·14° C. after 1 hour, the final temperature-rise would be given by

$$32{\cdot}14 = \theta_{m1} (1 - e^{-1/a})$$

$$= \theta_{m1} \left(1 - \frac{8}{15}\right) = \frac{7}{15}\theta_{m1}$$

Hence, $$\frac{\theta_{m1}}{\theta_m} = \frac{15}{7}$$

and therefore, $$\frac{\text{losses on 1-hour rating}}{\text{losses on full-load}} = \frac{15}{7}$$

Let the 1-hour rating be x times full-load.

Copper loss on 1-hour rating $= x^2 \times$ copper loss on full-load
$= 2x^2 \times$ iron loss.

$$\frac{\text{losses on 1-hour rating}}{\text{losses on full-load}} = \frac{(2x^2 + 1) \times \text{iron loss}}{(2 + 1) \times \text{iron loss}} = \frac{2x^2 + 1}{3}$$

Then, $$\frac{2x^2 + 1}{3} = \frac{15}{7}$$

Whence, $x = 1{\cdot}647$

Therefore, **1-hour rating will be 64·7 per cent overload.**

51. *A single-phase transformer is on full-load for $1\frac{1}{2}$ hours, no load for 1 hour, and 25 per cent overload for 1 hour. Calculate the temperature-rise at the end of the period if the temperature-rises of 20° C. and 35° C. occur after 1 hour and 2 hours respectively on full-load. The transformer starts from cold in both cases. Take the full-load copper loss as 2·5 times the core loss.*

Let the core loss be P_0 watts,

Full-load copper loss $= 2{\cdot}5\ P_0$ watts.

Hence, copper loss on 25 per cent overload $= \left(\frac{125}{100}\right)^2 \times 2{\cdot}5\ P_o = 3{\cdot}906\ P_o$ watts.

$$\begin{aligned} \text{Total loss on full-load} &= 3{\cdot}5\ P_o \text{ watts,} \\ \text{,, \quad ,, \quad no-load} &= P_o \text{ watts,} \\ \text{,, \quad ,, \quad 25 per cent overload} &= 4{\cdot}906\ P_o \text{ watts.} \end{aligned}$$

The maximum temperature-rise from cold on full-load is found from the expression

$$\theta = \theta_m (1 - e^{-t/a}), \text{ where } a \text{ is the time constant.}$$

Given that $\theta = 20°$ C. when t = 1 hour and $\theta = 35°$ C. when t = 2 hours,

$$20 = \theta_m (1 - e^{-1/a}) \qquad (1)$$

$$35 = \theta_m (1 - e^{-2/a}) \qquad (2)$$

From (1) and (2), $1 + e^{-1/a} = \frac{35}{20} = 1{\cdot}75$

Therefore, $e^{-1/a} = 0{\cdot}75$ and $a = 3{\cdot}475$

From (1), $20 = \theta_m (1 - 0{\cdot}75)$

Whence $\theta_m = 80°$ C.

Therefore the maximum temperature-rise on full-load is 80° C. and since the temperature rise is proportional to the losses being dissipated,

no-load maximum temperature-rise $= 80 \times \frac{P_o}{3{\cdot}5\ P_o} = 22{\cdot}86°$ C.

Maximum temperature-rise on 25 per cent overload $= 80 \times \frac{4{\cdot}906\ P_o}{3{\cdot}5\ P_o} = 112{\cdot}1°$ C.

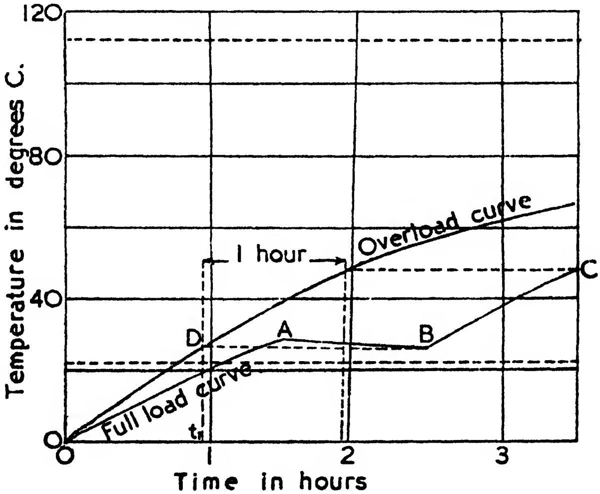


Fig. 36

At the end of $1\frac{1}{2}$ hours on full-load the temperature-rise is

$$\begin{aligned}\theta_A &= 80\ (1 - e^{-1.5/3.475})\\ &= 80\ (1 - e^{-0.4317}) = 80\ (1 - 0\cdot 65)\\ &= 28^\circ\ \text{C.}\end{aligned}$$

This condition is represented by the point A (Fig. 36). The transformer is now allowed to cool and it is assumed that the cooling time constant is the same as the heating time constant. If the transformer remained on no-load the temperature-rise would drop exponentially to a value of 22·86° C. The equation for the curve AB (t being measured from the point A) is therefore

$$\begin{aligned}\theta_B &= (28 - 22\cdot 86)\ e^{-t/3.475} + 22\cdot 86\\ &= 5\cdot 14\ e^{-t/3.475} + 22\cdot 86\end{aligned}$$

After 1 hour on no-load the temperature will have fallen to

$$\begin{aligned}\theta_B &= 5\cdot 14\ e^{-1/3.475} + 22\cdot 86\\ &= 5\cdot 14 \times 0\cdot 75 + 22\cdot 86 = 26\cdot 72^\circ\ \text{C.}\end{aligned}$$

The temperature at this point is represented by the point B. Now the transformer is switched on to 25 per cent overload and the temperature-rise follows the curve BC. If the transformer had been on 25 per cent overload from the start it would have reached the temperature-rise corresponding to B at the instant D, t_1 hours after switching on. The value of t_1 is found from

$$26\cdot 72 = 112\cdot 1\ (1 - e^{-t_1/3.475})$$

$$\text{i.e.}\qquad 1 - e^{-t_1/3.475} = \frac{26\cdot 72}{112\cdot 1} = 0\cdot 2383$$

$$e^{-t_1/3.475} = 0\cdot 7617$$

$$-\frac{t_1}{3\cdot 475} = \log_e 0\cdot 7617 = -\ 0\cdot 2723$$

$$t_1 = 0\cdot 946 \text{ hours.}$$

The temperature-rise reached after 1 hour on 25 per cent overload is the same as if the transformer had been on 25 per cent overload from the start and for a period of (0·946 + 1) hours, i.e. 1·946 hours.

Therefore at the point C the temperature-rise is

$$\begin{aligned}\theta_C &= 112\cdot 1\ (1 - e^{-1.946/3.475})\\ &= 112\cdot 1\ (1 - e^{-0.56})\\ &= 112\cdot 1\ (1 - 0\cdot 5712) = 48\cdot 06^\circ\ \text{C.}\end{aligned}$$

i.e. **Temperature rise at the end of the run = 48·06° C.**

52. *A transformer operates for $1\frac{1}{2}$ hours at 25 per cent overload and for 1 hour on no-load. The time constant is $3\frac{1}{2}$ hours, the maximum temperature-rise on full-load is 80° C., and the full-load copper loss is twice the iron loss. If this load cycle is repeated continuously, find the temperature limits within which the transformer operates. Prove any formula used.*

(C. *and* G. *Final, Pt. II,* 1941)

On full-load the copper loss = $2P_o$ where P_o is the iron loss.
On 25 per cent overload, the copper

$$\begin{aligned}\text{loss} &= \left\{\frac{125}{100}\right\}^2 \times 2P_o\\ &= 3\cdot 125\ P_o \text{ watts.}\end{aligned}$$

$$\text{Hence, } \frac{\text{total loss on 25 per cent overload}}{\text{total loss on full-load}} = \frac{(3{\cdot}125 + 1)\ P_o}{(2 + 1)\ P_o}$$
$$= 1{\cdot}375$$

$$\text{Therefore, } \frac{\text{maximum temperature-rise on overload}}{\text{maximum temperature-rise on full-load}} = 1.375$$

because the maximum temperature-rise is proportional to the losses being dissipated.

Then, maximum temperature-rise on 25 per cent overload $= 80°\ \text{C.} \times 1{\cdot}375$
$= 110°\ \text{C.}$

Similarly, the losses on no-load are one-third of the losses on full-load, therefore

$$\text{maximum temperature-rise on no-load} = 80°\ \text{C.} \times \frac{1}{3}$$
$$= 26{\cdot}67°\ \text{C.}$$

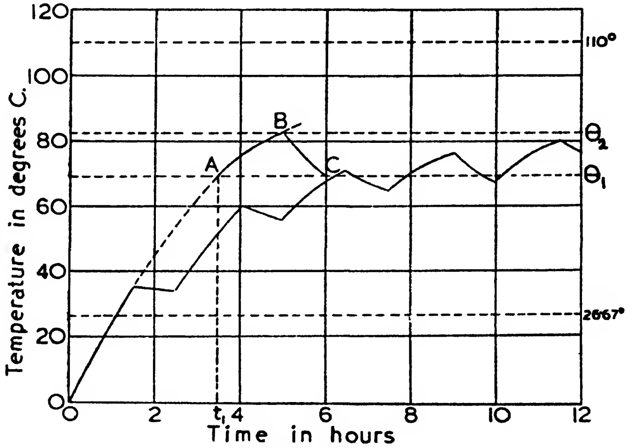


Fig. 37

Fig. 37 shows the first few cycles of temperature variation. The temperature-rise on 25 per cent overload follows the curve

$$\theta_h = 110\ (1 - e^{-t/3.5})$$

On no-load the temperature-rise drops along the curve

$$\theta_c = (\theta_m - 26{\cdot}67)\ e^{-t/3.5} + 26{\cdot}67$$

where θ_m is the maximum temperature-rise reached during the previous load period.

Eventually the transformer operates between the temperature limits θ_1 and θ_2 such that the rise of temperature from θ_1 to θ_2 during $1\frac{1}{2}$ hours on overload is equal to the drop from θ_2 to θ_1 during the 1 hour on no-load.

If the transformer had remained on overload from the start it would have reached the temperature-rise θ_1 in t_1 hours, where

$$\theta_1 = 110\ (1 - e^{-t_1/3.5}) \qquad (1)$$

and it would have reached a value θ_2 1½ hours later, i.e.

$$\begin{aligned} \theta_2 &= 110\,(1 - e^{-(t_1 + 1.5)/3.5}) \\ &= 110\,(1 - e^{-1.5/3.5} \times e^{-t_1/3.5}) \\ &= 110\,(1 - 0{\cdot}6515\ e^{-t_1/3.5}) \\ &= 110 - 71{\cdot}67\ e^{-t_1/3.5} \qquad (2) \end{aligned}$$

From B the transformer cools to C such that

$$\begin{aligned} \theta_1 &= (\theta_2 - 26{\cdot}67)\ e^{-1/3.5} + 26{\cdot}67 \\ &= (\theta_2 - 26{\cdot}67)\ 0{\cdot}7515 + 26{\cdot}67 \\ &= 0{\cdot}7515\ \theta_2 + 6{\cdot}63 \qquad (3) \end{aligned}$$

From (3), and (1),

$$\begin{aligned} \theta_2 &= \frac{\theta_1 - 6{\cdot}63}{0{\cdot}7515} = \frac{110 - 110\ e^{-t_1/3.5} - 6{\cdot}63}{0{\cdot}7515} \\ &= 137{\cdot}6 - 146{\cdot}4\ e^{-t_1/3.5} \qquad (4) \end{aligned}$$

Equating (2) and (4),

$$137{\cdot}6 - 146{\cdot}4\ e^{-t_1/3.5} = 110 - 71{\cdot}67\ e^{-t_1/3.5}$$

Hence,

$$\begin{aligned} 74{\cdot}73\ e^{-t_1/3.5} &= 27{\cdot}6 \\ e^{-t_1/3.5} &= 0{\cdot}3693 \\ \frac{-t_1}{3{\cdot}5} &= \log_e 0{\cdot}3693 = -\,0{\cdot}9963 \\ t_1 &= 3{\cdot}5 \times 0{\cdot}9963 = 3{\cdot}487 \text{ hours.} \end{aligned}$$

Substituting for t_1 in (1),

$$\begin{aligned} \theta_1 &= 110\,(1 - e^{-3.487/3.5}) \\ &= 110\,(1 - e^{-0.9963}) = 110\,(1 - 0{\cdot}369) \\ &= 69{\cdot}4^\circ \text{ C.} \end{aligned}$$

From (3)

$$\begin{aligned} \theta_2 &= \frac{\theta_1 - 6{\cdot}63}{0{\cdot}7515} = \frac{62{\cdot}77}{0{\cdot}7515} \\ &= 83{\cdot}5^\circ \text{ C.} \end{aligned}$$

Therefore **the transformer operates between temperature-rise limits of 69·4° C. and 83·5° C.**

(iii) Parallel operation.

53. *Two 250-kVA transformers supplying a network are connected in parallel on both primary and secondary sides. The voltage ratios are the same, their resistance drops are* 1·5 *per cent and* 0·9 *per cent and their reactance drops are* 3·33 *per cent and* 4·0 *per cent respectively. Calculate the kVA loading of each transformer and its power factor when the total load on the transformers is* 500 *kVA at* 0·707 *power factor lagging.* (*C. and G. Final, Pt. II,* 1941)

Let $\underset{\cdot}{P}$ = the total kVA load on the two transformers
$\underset{\cdot}{P}_1$ = the kVA loading of the first transformer,
$\underset{\cdot}{P}_2$ = „ „ „ second „
$\underset{\cdot}{Z}_1$ = the percentage impedance of the first transformer
$\underset{\cdot}{Z}_2$ = „ „ „ second „

Then, $\underset{\cdot}{P}_1 = \underset{\cdot}{P}\cdot\dfrac{\underset{\cdot}{Z}_2}{\underset{\cdot}{Z}_1 + \underset{\cdot}{Z}_2}$ and $\underset{\cdot}{P}_2 = \underset{\cdot}{P}\cdot\dfrac{\underset{\cdot}{Z}_1}{\underset{\cdot}{Z}_1 + \underset{\cdot}{Z}_2}$

Note that these equations are **vector** expressions.

Now, $\underset{\cdot}{Z}_1 = (1{\cdot}5 + j3{\cdot}33)$ per cent $= 3{\cdot}655\angle\underline{65^\circ 46'}$ per cent
$\underset{\cdot}{Z}_2 = (0{\cdot}9 + j4{\cdot}0)$ per cent $= 4{\cdot}1\angle\underline{77^\circ 18'}$ per cent.

$$\text{Hence, } \underset{\cdot}{Z}_1 + \underset{\cdot}{Z}_2 = (1{\cdot}5 + j3{\cdot}33) + (0{\cdot}9 + j4{\cdot}0)$$
$$= 2{\cdot}4 + j7{\cdot}33 = 7{\cdot}713\angle\underline{71^\circ52'} \text{ per cent.}$$

$$\text{Total load} \quad \underset{\cdot}{P} = 500\angle\underline{-45^\circ} \text{ kVA.}$$

$$\text{Therefore,} \quad \underset{\cdot}{P}_1 = 500\angle\underline{-45^\circ}\cdot\frac{4{\cdot}1\angle\underline{77^\circ18'}}{7{\cdot}713\angle\underline{71^\circ52'}}$$
$$= \frac{500 \times 4{\cdot}1}{7{\cdot}713}\angle\underline{(-45^\circ + 77^\circ18' - 71^\circ52')} \text{ kVA}$$
$$= 265{\cdot}8\angle\underline{-39^\circ34'} \text{ kVA}$$
$$= 265{\cdot}8 \text{ kVA at power factor } 0{\cdot}771 \text{ lagging.}$$

$$\underset{\cdot}{P}_2 = 500\angle\underline{-45^\circ}\cdot\frac{3{\cdot}655\angle\underline{65^\circ46'}}{7{\cdot}713\angle\underline{71^\circ52'}} \text{ kVA}$$
$$= \frac{500 \times 3{\cdot}655}{7{\cdot}713}\angle\underline{(-45^\circ + 65^\circ46' - 71^\circ52')} \text{ kVA}$$
$$= 236{\cdot}9\angle\underline{-51^\circ6'} \text{ kVA}$$
$$= 236{\cdot}9 \text{ kVA at power factor } 0{\cdot}628 \text{ lagging.}$$

Therefore the loadings are:

Transformer 1: 265·8 kVA at power factor 0·771 lagging.
Transformer 2: 236·9 kVA at power factor 0·628 lagging.

54. *Two single-phase transformers rated at* 500-*kVA and* 400-*kVA respectively are connected in parallel to supply a load of* 1000 *kVA at* 0·8 *lagging power factor. The resistance and reactance of the first transformer are* 2·5 *per cent and* 6 *per cent respectively, and of the second transformer* 1·6 *per cent and* 7 *per cent respectively. Calculate the kVA loading and the power factor at which each transformer operates.* (*C. and G. Final, Pt. II*, 1943)

In this problem the percentage resistances and reactances refer to different kVA ratings so before the respective percentage impedances can be used to ascertain the share which each transformer takes of the total load they must be expressed in terms of the same kVA rating, e.g. 500 kVA.

Hence, with the same nomenclature as in Problem 53, for the first transformer,

$$\underset{\cdot}{Z}_1 = (2{\cdot}5 + j6{\cdot}0) \text{ per cent} = 6{\cdot}5\angle\underline{67^\circ23'} \text{ per cent.}$$

For the second transformer, 1·6 per cent resistance on a 400 kVA rating is equivalent to $\frac{5}{4} \times 1{\cdot}6$, i.e. 2 per cent resistance on a 500 kVA rating. Similarly, 7 per cent reactance becomes equivalent to $\frac{5}{4} \times 7$, i.e. 8.75 per cent on the higher rating.

$$\text{Then,} \quad \underset{\cdot}{Z}_2 = (2{\cdot}0 + j8{\cdot}75) \text{ per cent} = 8{\cdot}974\angle\underline{77^\circ6'} \text{ per cent.}$$

Both percentage impedances are now expressed in terms of the same rating, 500 kVA.

$$\underset{\cdot}{Z}_1 + \underset{\cdot}{Z}_2 = (2{\cdot}5 + j6{\cdot}0) + (2{\cdot}0 + j8{\cdot}75)$$
$$= (4{\cdot}5 + j14{\cdot}75) = 15{\cdot}42\angle\underline{73^\circ2'} \text{ per cent.}$$

$$\text{Total load} \quad \underset{\cdot}{P} = 1000\angle\underline{-36^\circ54'} \text{ kVA.}$$

$$\text{Hence,} \quad \underset{\cdot}{P}_1 = 1000\angle\underline{-36^\circ54'}\cdot\frac{8{\cdot}974\angle\underline{77^\circ6'}}{15{\cdot}42\angle\underline{73^\circ2'}}$$

$$= \frac{1000 \times 8{\cdot}974}{15{\cdot}42} \angle\underline{(-36°54' + 77°6' - 73°2')}$$

$$= 582\angle\underline{-32°50'} \text{ kVA}$$

$$P_2 = 1000\angle\underline{-36°54'} \cdot \frac{6{\cdot}5\angle\underline{67°23'}}{15{\cdot}42\angle\underline{73°2'}}$$

$$= \frac{1000 \times 6{\cdot}5}{15{\cdot}42} \angle\underline{(-36°54' + 67°23' - 73°2')}$$

$$= 421{\cdot}5\angle\underline{-42°33'} \text{ kVA.}$$

Therefore, the loadings and power factors of the respective transformers are (note that in each case the power factor is the cosine of the load phase angle):

Transformer 1: 582 kVA at power factor 0·84 lagging,
Transformer 2: 421·5 kVA at power factor 0·737 lagging.

(iv) Design calculations.

55. *Calculate approximate overall dimensions for a* 200-*kVA*, 6600/440-*volt*, 50-*cycle*, 3-*phase*, *core-type transformer. The following data may be used: e.m.f. per turn*, 9 *volts; maximum flux density*, 13000 *lines per sq. cm.; current density*, 2·5 *amperes per sq. mm.; window space factor* 0·3; *overall height = overall width.* (*C. and G. Final, Pt. II*, 1937)

The kVA output of a 3-phase transformer is given by the expression:—

$$\text{kVA} = 3{\cdot}33\ fA_iA_wB_m\delta k_w \cdot 10^{-9}$$

where f = the frequency in cycles per second,
A_i = the net area of the core cross-section, in sq. cm.
A_w = the net window area in sq. cm.,
B_m = the maximum flux density in lines per sq. cm.
δ = the current density in amperes per sq. mm.,
k_w = the window space factor.

Applying this expression to the data given,

$$200 = 3{\cdot}33 \times 50 \times A_iA_w \times 13000 \times 2{\cdot}5 \times 0{\cdot}3 \times 10^{-9}$$

Whence, $A_iA_w = 1{\cdot}232 \times 10^5 \text{ cm.}^4$

The value of A_i can be found from the e.m.f. per turn, thus

$$E_t = 4{\cdot}44\ fB_mA_i \cdot 10^{-8} \text{ volts,}$$

i.e. $9 = 4{\cdot}44 \times 50 \times 13000 \times A_i \times 10^{-8}$

$$A_i = 311{\cdot}8 \text{ sq. cm.}$$

For a core of this size a 3-stepped construction would be used (Fig. 38). With such a core the relation between the net core area and the diameter of the circumscribing circle is

$$A_i = 0{\cdot}6d^2$$

Therefore, $311{\cdot}8 = 0{\cdot}6d^2$

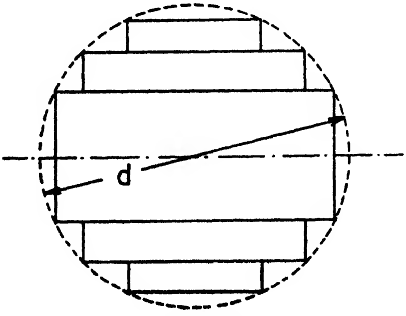


Fig. 38

and $d = 22{\cdot}79$ cm., say **23 cm.**

The revised value of A_i with this value of d is $0{\cdot}6 \times 23^2 = 317$ sq. cm.

$$\text{Hence,} \quad A_w = 1{\cdot}232 \times \frac{10^5}{317} = 388{\cdot}7 \text{ sq. cm., say } 390 \text{ sq. cm.}$$

Let w be the window width, in cm.
h ,, ,, height ,,

Then $w = \dfrac{390}{h}$

Overall width $W = 2w + 2d + 0{\cdot}9d$
$= 2w + 66{\cdot}7$

Overall height $H = h + 2 \times 0{\cdot}9d$
$= h + 41{\cdot}4$

Since the overall width = the overall height, i.e.

$$W = H$$

$$2w + 66{\cdot}7 = h + 41{\cdot}4$$

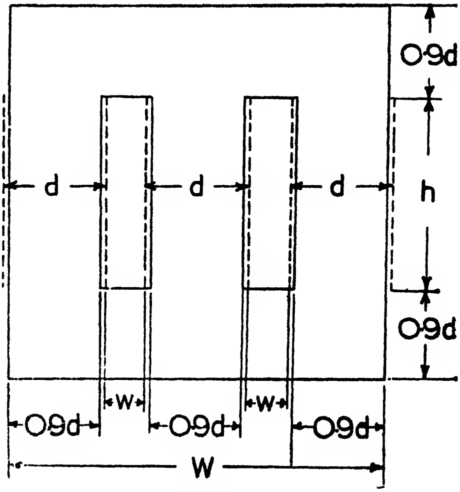


Fig. 39

$$\text{i.e.} \quad \frac{780}{h} + 66{\cdot}7 = h + 41{\cdot}4$$

$$h^2 - 25{\cdot}3h - 780 = 0$$

and $h = 43$ cm. approximately.

$$w = \frac{390}{43} = 9{\cdot}10 \text{ cm.}$$

Then $W = H = 43 + 41{\cdot}4 = 84{\cdot}4$ cm.

The **approximate overall dimensions** of the **core** of this transformer are therefore: **Width = Height = 84·4 cm.**

To find the overall dimensions of the complete transformer would involve calculating the numbers of turns on the respective windings and the cross-sectional area of the conductors. To do this the method of connection of the windings, i.e. star or mesh, must be known.

56. *Design a core for a* 1-*phase*, 300-*kVA*, 50-*cycle per sec.*, *core-type power transformer. The e.m.f. per turn may be taken as* $0{\cdot}6\sqrt{kVA}$ *and the window space factor as* 0·3. *State clearly all assumptions and design values.*
(*I.E.E.*, *Pt. II*, *Nov.*, 1943)

e.m.f. per turn $= 0{\cdot}6\sqrt{\text{kVA}}$

i.e. $E_t = 0{\cdot}6\sqrt{300} = 10{\cdot}4$ volts.

Now $E_t = 4{\cdot}44\, fB_m.A_i.10^{-8}$ volts

(see Problem 55 for the meaning of these symbols)

$$10{\cdot}4 = 4{\cdot}44 \times 50 \times 13000 \times A_i \times 10^{-8}$$

assuming that the maximum flux density in the core is 13000 lines per sq. cm.

Therefore $A_i = 360$ sq. cm. = net core area.

Also kVA output $= 2{\cdot}22\ fA_iA_wB_m\delta k_w.10^{-9}$ for a single-phase transformer.

Assume that the current density δ is 2·5 amperes per sq. mm.

$$300 = 2{\cdot}22 \times 50 \times 360 \times A_w \times 13000 \times 2{\cdot}5 \times 0{\cdot}3 \times 10^{-9}$$

whence $A_w = 777$ sq. cm. = net window area.

Assume that the core has a 3-step construction, Fig. 38.

The diameter of the circumscribing circle is found from

$$A_i = 0{\cdot}6d^2$$
$$360 = 0{\cdot}6d^2$$
$$d = 24{\cdot}5 \text{ cm.}$$

Assume that the spacing of the core centres is D = 35 cm.

Then, window area = window height × (D − d)

$A_w =$ „ „ × (35 − 24·5)

$$\text{Therefore, window height} = \frac{777}{10{\cdot}5} = 74 \text{ cm.}$$

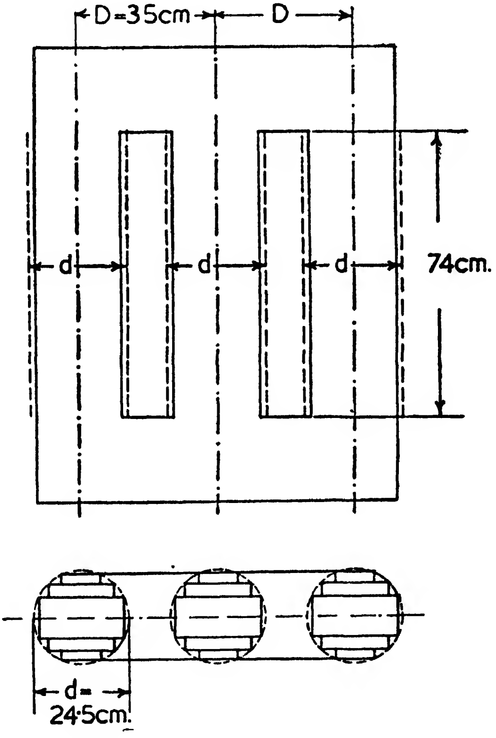


Fig. 40

The **dimensions of the core** (Fig. 40) **are therefore:**

Diameter of circumscribing circle = **24·5 cm.**
Net core area = **360 sq. cm.**
Pitch of core centres = **35 cm.**
Height of window = **74 cm.**
Net window area = **777 sq. cm.**
Overall width = **92 cm.**
Overall height = **96 cm.**

57. *The window in the core of a* 2200/440-*volt*, 15-*kVA*, 50-*cycle, single-phase transformer has a gross available area of* 340 *sq. cm., the space factor being* 0.35. *Assuming a maximum core density of* 10 *kilogauss and a current density in the conductors of* 2·1 *amperes per sq. mm., estimate the sectional area of the iron in the limb and the diameter of the circumscribing circle around the square core section. Find also the numbers of primary and secondary turns aud the corresponding conductor cross-sections.*

(*I.E.E., Pt. II, May*, 1938)

$$\text{kVA output} = 2{\cdot}22\ fB_mA_iA_w\delta k_w.10^{-9}$$

Now, $f = 50$ cycles per sec., $B_m = 10000$ gauss, $A_w = 340$ sq. cm., $\delta = 2{\cdot}1$ amperes per sq. mm., $k_w = 0{\cdot}35$, kVA $= 15$

Hence, $15 = 2{\cdot}22 \times 50 \times 10000 \times A_i \times 340 \times 2{\cdot}1 \times 0{\cdot}35 \times 10^{-9}$

and $A_i =$ **54·1 sq. cm. = net cross-section of iron core.**

For a square cross-section the relation between A_i and the diameter of the circumscribing circle is

$A_i = 0{\cdot}45d^2$ and this allows for 10 per cent of the gross core area being taken up by the insulation between the laminations.

Hence, $54{\cdot}1 = 0{\cdot}45d^2$, and

$d =$ **11 cm. = diameter of the circumscribing circle.**

e.m.f. per turn $E_t = 4{\cdot}44\ fB_mA_i.10^{-8}$ volts,

$= 4{\cdot}44 \times 50 \times 10000 \times 54{\cdot}1 \times 10^{-8}$ volts,

$= 1{\cdot}2$ volts.

Therefore, **number of secondary turns** $= \frac{440}{1{\cdot}2} =$ **367**

Number of primary turns $= 367 \times \frac{2200}{440}$ = **1835**

Secondary current $= \frac{15 \times 10^3}{440}$ = 34·1 amperes.

Primary current $= \frac{15 \times 10^3}{2200}$ = 6·8 amperes.

Using a current density of 2·1 amperes per sq. mm.,

Secondary conductor cross-section $= \frac{34{\cdot}1}{2{\cdot}1} = 16{\cdot}25$ sq. mm.

Primary conductor cross-section $= \frac{6{\cdot}8}{2{\cdot}1} =$ **3·24 sq. mm.**

Suitable **wire gauges** would be: **Primary, 14 S.W.G., Sec. 7 S.W.G.**

58. *Obtain suitable values for the number of turns and the cross-section of the conductors and core of a* 100-*kVA*, 6600/440-*volt*, 3-*phase transformer*, 50-*cycles per sec., mesh-star, core-type. Assume a current density of* 2·5 *amperes per sq. mm., a window factor of* 0·3, *and a flux density of* 12000 *lines per sq. cm. in the core. Make the window area approximately* 1½ *times the core area.*

(*I.E.E., Pt. II, May*, 1939)

$$\text{kVA} = 3{\cdot}33\ fB_mA_iA_w\delta k_w.10^{-9}$$

From the data given, $f = 50$ cycles per sec., $B_m = 12000$ gauss,
$A_w = 1{\cdot}5A_i$, $\delta = 2{\cdot}5$ amperes per sq. mm., $k_w = 0{\cdot}3$,
$\text{kVA} = 100$.

Hence, $100 = 3{\cdot}33 \times 50 \times 12000 \times 1{\cdot}5A_i^2 \times 2{\cdot}5 \times 0{\cdot}3 \times 10^{-9}$

and A_i = **211 sq. cm. = net core cross-section**

e.m.f. per turn $E_t = 4{\cdot}44\ fB_mA_i.10^{-8}$ volts,
$= 4{\cdot}44 \times 50 \times 12000 \times 211 \times 10^{-8}$ volts,
$= 5{\cdot}62$ volts.

Primary e.m.f. per phase = 6600 volts, secondary e.m.f. per phase $= \dfrac{440}{\sqrt{3}}$ volts.

Hence, **secondary turns per phase** $= \dfrac{440}{\sqrt{3}} \times \dfrac{1}{5{\cdot}62}$
$= 45{\cdot}2$ say **45 turns**

Primary turns per phase $= \dfrac{6600}{5{\cdot}62} =$ **1170 turns.**

Primary phase current $= 100 \times \dfrac{10^3}{(3 \times 6600)} = 5{\cdot}05$ amperes.

Secondary phase current $= 100 \times \dfrac{10^3}{(\sqrt{3} \times 440)} = 131{\cdot}2$ amperes.

Therefore with a current density of 2·5 amperes per sq. mm.,

Cross-section of primary conductors $= \dfrac{5{\cdot}05}{2{\cdot}5} =$ **2·02 sq. mm.**

Cross-section of secondary „ $= \dfrac{131{\cdot}2}{2{\cdot}5} =$ **52·5 sq. mm.**

For the **primary 16 S.W.G. copper wire** would be suitable (2·075 sq. mm. bare).

For the **secondary** a suitable conductor would be **two paper-covered copper strips,** each **7 mm.** × **3·75 mm. (bare)** in parallel.

59. *Outline the method of obtaining the core and window area of a small power transformer.*

Estimate the no-load current of a 6600/440-*volt*, 50-*cycle, single-phase, core-type transformer with the following particulars:*

Mean length of magnetic path = 270 *cm.*
Gross cross-sectional area = 140 *sq. cm.*
Maximum flux density = 11000 *gauss*
Specific core-loss at 50-*frequency and* 11000 *gauss* = 2·1 *W per kg.*
A.-T. per cm. for transformer steel at 1100 *gauss* = 6·5

The effect of the joints is equivalent to that of an air-gap of 1 *mm. in the magnetic circuit.*

(*C. and G. Final, Pt. II*, 1942)

The no-load current will be calculated from its components, the magnetizing current and the core-loss current, which are in quadrature time phase.

Magnetizing current

Ampere-turns required for the iron path $= 270 \times 6{\cdot}5$
$= 1755$

Ampere-turns required for the air-gap $= 0{\cdot}8B_m \times$ gap-length,
$= 0{\cdot}8 \times 11000 \times 0{\cdot}1$
$= 880$

Total ampere-turns required $= 1755 + 880$
$= 2635$

The number of turns on the primary is found from

$$E_1 = 4{\cdot}44\ fB_mA_iT_1.10^{-8} \text{ volts,}$$

where A_i = the net cross-section of the core.

As the core has a gross cross-section of 140 sq. cm. and allowing 10 per cent reduction in the active area on account of the insulation between the laminations,

$$A_i = 0{\cdot}9 \times 140 = 126 \text{ sq. cm.}$$

Hence, $6600 = 4{\cdot}44 \times 50 \times 11000 \times 126 \times T_1 \times 10^{-8}$
and $T_1 = 2144$ turns.

$$\text{Magnetizing current} = \frac{\text{A.-T.}}{\sqrt{2}T_1}$$

$$= \frac{2635}{\sqrt{2} \times 2144} = 0{\cdot}869 \text{ amperes.}$$

Note.—The factor $\sqrt{2}$ has been introduced here because the ampere-turns had been worked out on maximum flux density figures and the R.M.S. value of the current is required.

Core-loss current

Core volume $= 270 \times 140$ cu. cm. $= 37800$ cu. cm.

Assuming a specific gravity of 7·5 for the core,
core weight $= 37800 \times 7{\cdot}5 \times 10^{-3}$ $= 283{\cdot}5$ kg.

At the specific core loss given,
core loss $= 283{\cdot}5 \times 2{\cdot}1$ watts $= 595{\cdot}4$ watts.

At a primary voltage of 6600,
core loss current $= \frac{595{\cdot}4}{6600}$ amperes $= 0{\cdot}09$ amperes.

Hence, **no-load current** $= \sqrt{0{\cdot}869^2 + 0{\cdot}09^2}$ $=$ **0·874** „

(v) The "Scott" connection (for 3- to 2-phase conversion).

60. *Two electric furnaces are supplied with single-phase current at* 80 *volts from a* 3-*phase*, 11000-*volt system by means of two single-phase Scott-connected transformers, with similar secondary windings. When the load on one transformer is* 500 *kW and on the other is* 800 *kW, what current will flow in each of the* 3-*phase lines at unity power factor?*

(*I.E.E., Pt. II, Nov.*, 1938)

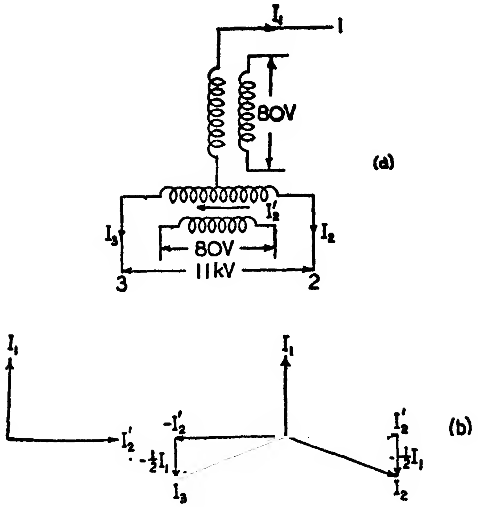


Fig. 41

The arrangement of the windings and loads is shown in Fig. 41(a)

$$\text{Transformation ratio of main transformer} = \frac{11000}{80} = 137{\cdot}5$$

$$\text{Transformation ratio of teaser transformer} = \frac{0{\cdot}866 \times 11000}{80} = 119$$

$$\text{Primary current of the main transformer} = \frac{800000}{(80 \times 137{\cdot}5)} = 72{\cdot}7 \text{ amperes.}$$

$$\text{Primary current of the teaser transformer} = \frac{500000}{(80 \times 119)} = 52{\cdot}5 \text{ amperes.}$$

These currents are labelled I_2^1 and I_1 respectively in Fig. 41(b). From this diagram it will be seen that

Current in line 1 = I_1 = 52·5 amperes.

At the tapping point on the main winding where it is joined to the teaser transformer primary the current divides equally through the two halves of the main winding and flows in opposite directions through it.

In each half of the main primary however, its own current I_2^1 is 90° out of phase with $\frac{1}{2}I_1$. This is seen from the vector diagram of the currents (Fig. 41(b)).

Hence, **line currents in lines 2 and 3** $= \sqrt{(I_2^1)^2 + (\frac{1}{2}I_1)^2}$

$= \sqrt{72{\cdot}7^2 + 26{\cdot}25^2}$

= 77·2 amperes.

61. *Explain with connection and vector diagrams, the transformer connections to obtain a 2-phase supply from a 3-phase supply.*

Two 100-*volt, single-phase furnaces take loads of* 600 *kW and* 900 *kW respectively at a power factor of* 0·71 *and are supplied from* 6600-*volt,* 3-*phase mains through a Scott-connected transformer combination. Calculate the currents in the* 3-*phase lines, neglecting transformer losses.*

(*C. and G. Final, Pt. II,* 1938)

Fig. 42 shows the connection and vector diagrams.

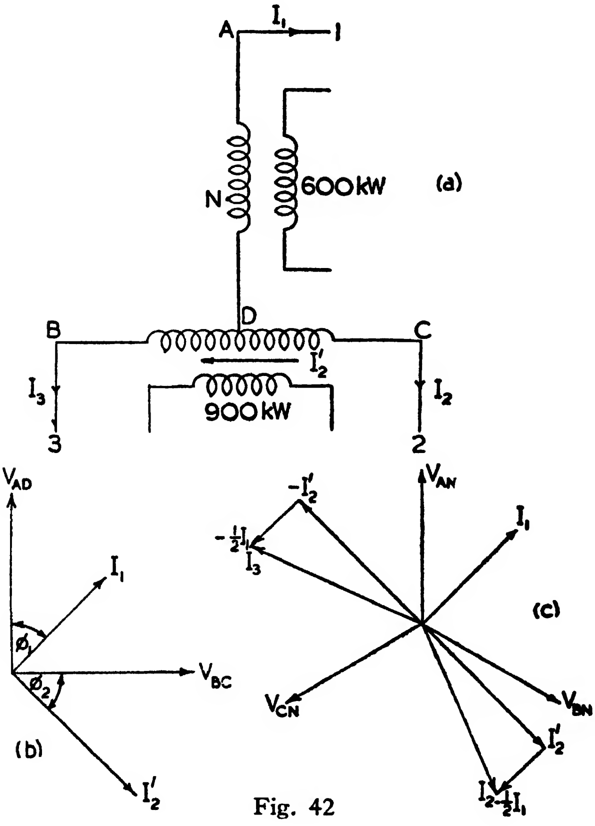


Fig. 42

Diagram (a) shows the connections; (b) shows the individual load currents in the primaries and their phase relations to the voltages across the main primary and the teaser primary windings; (c) shows how the line currents are obtained vectorially.

Main primary voltage $= 6600$ volts.

Due to the load of 900 kW on the main secondary,

$$\text{main primary current } I_2' = \frac{900000}{(6600 \times 0{\cdot}71)} = 192{\cdot}1 \text{ amperes.}$$

teaser primary voltage $= 0{\cdot}866 \times 6600$ volts.

Due to the load of 600 kW on the teaser secondary,

$$\text{teaser primary current } I_1 = \frac{600000}{(0{\cdot}866 \times 6600 \times 0{\cdot}71)} = 147{\cdot}9 \text{ amperes.}$$

Hence, **current in primary line 1 = 147·9 amperes.**

The current in each of the lines 2 and 3 consists of the main primary current I_2' and one-half of the teaser primary current I_1 vectorially combined. As the power factor is the same on both secondaries the two components of the line currents are in quadrature.

$$\begin{aligned} \text{i.e.} \quad I_2 = I_3 &= \sqrt{(I_2')^2 + (\tfrac{1}{2}I_1)^2} \\ &= \sqrt{(192{\cdot}1)^2 + (73{\cdot}95)^2} \\ &= 205{\cdot}8 \text{ amperes.} \end{aligned}$$

Hence, **currents in lines 2 and 3 are both equal to 205·8 amperes.**

CHAPTER IV

INDUCTION MOTORS

(i) Slip, speed, torque, losses, etc.

62. *A* 500-*volt,* 6-*pole,* 50-*cycle per second,* 3-*phase induction moto develops* 20 *h.p. inclusive of mechanical losses when running at* 995 *r.p.m., the power factor being* 0·87. *Calculate* (*a*) *the slip,* (*b*) *the rotor copper losses,* (*c*) *the total input if the stator losses are* 1500 *watts,* (*d*) *the line current,* (*e*) *the number of cycles per minute of the rotor e.m.f.*

Synchronous speed of the motor in r.p.m.

$$= \frac{120 \times \text{frequency}}{\text{number of poles}} = \frac{120 \times 50}{6} = 1000 \text{ r.p.m.}$$

(a) **Slip**

$$s = \frac{\text{Synchronous speed} - \text{shaft speed}}{\text{Synchronous speed}} = \frac{1000 - 995}{1000} = \mathbf{0{\cdot}005}$$

(b) Rotor copper loss

$$= \text{Slip} \times \text{Power input to rotor circuit,}$$

$$= s \times (\text{mechanical power developed} + \text{rotor copper loss})$$

hence,

rotor copper loss

$$= \frac{s}{1-s} \times \text{mechanical power developed}$$

$$= \frac{0{\cdot}005}{1 - 0{\cdot}005} \times 20 \times 746$$

$$= \mathbf{75 \text{ watts.}}$$

(c) Total input

$$= \text{Power input to rotor} + \text{stator losses}$$

Power input to rotor

$$= \frac{1}{s} \times \text{Rotor copper losses}$$

$$= \frac{1}{0{\cdot}005} \times 75 \text{ watts} = 15000 \text{ watts.}$$

Hence, **total input**

$$= 15000 + 1500 = \mathbf{16500 \text{ watts.}}$$

(d) **Line current**

$$= \frac{16500}{(\sqrt{3} \times 500 \times 0{\cdot}87)}$$

$$= \mathbf{21{\cdot}8 \text{ amperes.}}$$

(e) **Frequency of rotor e.m.f.**

$$= \text{Slip} \times \text{supply frequency}$$

$$= 0{\cdot}005 \times 50 \text{ cycles per second,}$$

$$= 0{\cdot}25 \text{ cycles per second,}$$

$$= \mathbf{15 \text{ cycles per minute.}}$$

63. *Why is a plain induction motor unsuitable for wide speed control? State and briefly explain what modifications can be made to secure speed control. Estimate the copper loss in the rotor circuit of an induction motor running at* 50 *per cent of synchronous speed with a useful output of* 55 *h.p. and mechanical losses totalling* 2 *h.p. If the stator losses total* 3·5 *kW at what efficiency is the motor operating?* (*I.E.E., Pt. II, Nov.,* 1938)

If the motor is running at 50 per cent of synchronous speed,

Slip $s = 0{\cdot}5$

Mechanical power developed $= (1 - s) \times$ power input to rotor circuit,

i.e. $(1 - 0{\cdot}5) \times$ power input to rotor $= (55 + 2)$ h.p.

$= 42500$ watts $= 42{\cdot}5$ kW.

Hence, power input to rotor $= 85$ kW.

At 50 per cent slip half of this input is copper loss in the rotor circuit.

Therefore, **rotor copper loss = 42·5 kW.**

Total power input to motor $=$ Input to rotor circuit $+$ stator loss

$= (85 + 3{\cdot}5)$ kW

$= 88{\cdot}5$ kW.

$$\textbf{Efficiency of motor} = \frac{\text{Output}}{\text{Input}} \times 100 \text{ per cent}$$

$$= \frac{55 \times 746 \times 100}{88500} \text{ per cent}$$

$$= \textbf{46·4 per cent.}$$

64. *Derive an expression for the ratio of the maximum torque to the full-load torque of an induction motor in terms of the standstill reactance, the slip, and the rotor resistance.*

A 50-*frequency,* 8-*pole induction motor has a full-load slip of* 4 *per cent. The rotor resistance is* 0·001 *ohm per phase and the standstill reactance is* 0·005 *ohm per phase. Find the ratio of the maximum to the full-load torque and the speed at which maximum torque occurs.*

(*C. and G. Final, Pt. I,* 1942)

The relation between the maximum torque and the full-load torque of an induction motor is as follows:

$$T_f = 2T_m . \frac{s\alpha}{\alpha^2 + s^2}$$

where $T_f =$ full-load torque,

$T_m =$ maximum torque,

$s =$ the slip at full-load,

$\alpha =$ the ratio of rotor resistance to standstill reactance per phase, i.e. $\frac{R_2}{X_2}$

In this problem, $s = 0{\cdot}04$, $R_2 = 0{\cdot}001$ ohm, $X_2 = 0{\cdot}005$ ohm,

hence, $$\alpha = \frac{0{\cdot}001}{0{\cdot}005} = 0{\cdot}2$$

Therefore,
$$\frac{T_m}{T_f} = \frac{(a^2 + s^2)}{2sa}$$
$$= \frac{(0{\cdot}2^2 + 0{\cdot}04^2)}{2 \times 0{\cdot}2 \times 0{\cdot}04}$$
$= 2{\cdot}6 =$ **ratio of maximum to full-load torque.**

The slip for maximum torque is
$$s_m = \frac{R_2}{X_2} = \frac{0{\cdot}001}{0{\cdot}005} = 0{\cdot}2$$

Synchronous speed for this motor $= \frac{120f}{p}$ r.p.m.
$$= \frac{120 \times 50}{8} = 750 \text{ r.p.m.}$$

Shaft speed for maximum torque $= (1 - s_m) \times$ synchronous speed
$= (1 - 0{\cdot}2) \times 750$ r.p.m.,
$= 600$ r.p.m.

Hence, **maximum torque is developed at a speed of 600 r.p.m.**

(ii) Starting methods, starting torque, etc.

65. *Calculate the steps in a 5-section rotor startor for a 3-phase induction motor from the following data: maximum starting current = full-load current: full-load slip* = 0·018*: rotor resistance per phase* = 0·015 *ohm.*

Prove any formula used. (*C. and G. Final, Pt. II,* 1937)

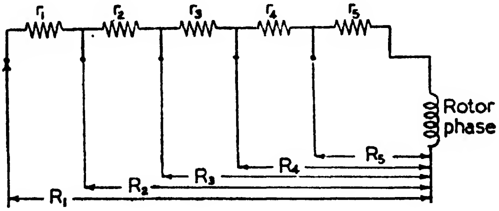


Fig. 43

The equivalent circuit of each phase of the rotor circuit can be regarded as a constant reactance X_2 in series with a resistance which is inversely proportional to the slip, i.e. $\frac{R}{s}$. The e.m.f. induced in this circuit is a constant E_2.

At any slip s the rotor current is therefore

$$I_2 = \frac{E_2}{\sqrt{\left\{\frac{R}{s}\right\}^2 + X_2^2}} \qquad (1)$$

The current on full-load is

$$I_{2f} = \frac{E_2}{\sqrt{\left\{\frac{0{\cdot}015}{0{\cdot}018}\right\}^2 + X_2^2}} \qquad (2)$$

This is also the current at starting with a total rotor resistance R_1 (*see* Fig. 43) and the slip $= s_1 = 1$.

Hence,
$$I_{2f} = \frac{E_2}{\sqrt{\left\{\frac{R_1}{s_1}\right\}^2 + X_2^2}} \qquad (3)$$

Comparing (2) and (3) we see that $\frac{R_1}{1} = \frac{0{\cdot}015}{0{\cdot}018}$

Therefore $R_1 = \frac{5}{6}$ ohm $= 0{\cdot}833$ ohm.

On the second step of the startor also the current $= I_{2f}$

Therefore
$$I_{2f} = \frac{E_2}{\sqrt{\left\{\frac{R_2}{s_2}\right\}^2 + X_2^2}}$$

and comparing this expression with (1) and (2),

$$\frac{R_2}{s_2} = \frac{R_1}{1} = \frac{0{\cdot}015}{0{\cdot}018}.$$

In a similar manner for the other steps,

$$\frac{R_1}{1} = \frac{R_2}{s_2} = \frac{R_3}{s_3} = \frac{R_4}{s_4} = \frac{R_5}{s_5} = \frac{0{\cdot}015}{0{\cdot}018}$$

Therefore,

$$\frac{R_2}{R_1} = \frac{R_3}{R_2} = \frac{R_4}{R_3} = \frac{R_5}{R_4} = \frac{0{\cdot}015}{R_5} = \frac{s_2}{1} = \frac{s_3}{s_2} = \frac{s_4}{s_3} = \frac{s_5}{s_4} = \frac{0{\cdot}018}{s_5}$$

Hence, $R_2 = \beta R_1$, $R_3 = \beta R_2 = \beta^2 R_1$, $R_4 = \beta R_3 = \beta^3 R_1$, $R_5 = \beta R_4 = \beta^4 R_1$ and $0{\cdot}015 = \beta R_5 = \beta^5 R_1$

$$\text{i.e.} \quad 0{\cdot}015 = \beta^5 \times \frac{0{\cdot}015}{0{\cdot}018}$$

$$\beta = \sqrt[5]{0{\cdot}018} = 0{\cdot}448$$

Then,
$$R_2 = \beta R_1 = 0{\cdot}448 \times 0{\cdot}833 = 0{\cdot}373 \text{ ohm},$$
$$R_3 = \beta R_2 = 0{\cdot}448 \times 0{\cdot}373 = 0{\cdot}167 \text{ ohm},$$
$$R_4 = \beta R_3 = 0{\cdot}448 \times 0{\cdot}167 = 0{\cdot}075 \text{ ohm},$$
$$R_5 = \beta R_4 = 0{\cdot}448 \times 0{\cdot}075 = 0{\cdot}0335 \text{ ohm}.$$

The resistances of the various sections of the startor are therefore:

$r_1 = R_1 - 0{\cdot}015 =$ **0·818 ohm.**

$r_2 = R_2 - 0{\cdot}015 =$ **0·358 ohm.**

$r_3 = R_3 - 0{\cdot}015 =$ **0·152 ohm.**

$r_4 = R_4 - 0{\cdot}015 =$ **0·060 ohm.**

$r_5 = R_5 - 0{\cdot}015 =$ **0·0185 ohm.**

66. *A small induction motor has a short-circuit current equal to 5 times the full-load current. Find the starting torque as a percentage of full-load torque if the motor is started by (a) direct switching to the supply, (b) a star-delta startor, (c) an autotransformer, (d) a resistance in the stator circuit. The starting current in (c) and in (d) is to be limited to 2·5 times the full-load current and the full-load slip is 4 per cent. Derive any formulæ used.*

(*C. and G. Final, Pt. II*, 1942)

In any induction motor running at a slip s,

rotor copper loss = s × Power input to rotor circuit,

i.e. $I_2^2R_2 = s \times$ Power input to rotor circuit,

where I_2 is the rotor current and R_2 is the resistance of the rotor circuit.

Now, power input to rotor in mechanical units $= 2\pi N_sT$,

where T is the torque developed and N_s is the speed of rotation of the stator field.

Hence, rotor power input = T × a constant.

i.e.
$$T = \text{a constant} \times \frac{I_2^2R_2}{s} \qquad (1)$$

Let
I_s = the starting current per phase,
I_f = the full-load current per phase
T_s = the starting torque (at $s = 1$),
T_f = the full-load torque (at $s = 0{\cdot}04$),

Therefore, from (1),
$$\frac{T_s}{T_f} = \left\{\frac{I_s}{I_f}\right\}^2 \times \frac{0{\cdot}04}{1} \qquad (2),$$

(a) With direct switching to the supply, $I_s = 5I_f$,

Substituting in (2),
$$T_s = T_f \times (5)^2 \times 0{\cdot}04 = T_f$$

Hence, **starting torque** = **100 per cent of full-load torque.**

(b) With a star-delta startor, $I_s = \frac{1}{\sqrt{3}}$ of the short-circuit current,

i.e. $I_s = \frac{5}{\sqrt{3}} I_f$

From (2),
$$T_s = T_f \times \left\{\frac{5}{\sqrt{3}}\right\}^2 \times 0{\cdot}04 = \tfrac{1}{3}\, T_f$$

Therefore, **starting torque** = **33⅓ per cent of full-load torque.**

(c) With autotransformer starting, $I_s = 2{\cdot}5\ I_f$,

From (2), $T_s = T_f \times (2{\cdot}5)^2 \times 0{\cdot}04 = 0{\cdot}25\ T_f$

Therefore, **starting torque** = **25 per cent of full-load torque.**

It is assumed here that the *motor* current is to be limited to 2·5 times full load current, i.e., the input transformer ratio would be 2 : 1 and the *line* current on starting would be only 1·25 times full load current.

Alternatively, if the starting *line* current is to be limited to 2·5 times full load current, the transformer ratio must be $\sqrt{2} : 1$ and the motor starting current is 3·535 times the full load current. In this case the starting torque is 50 per cent of full load torque.

(d) With a resistance in the stator circuit, the effect is the same as with the autotransformer; both reduce the input voltage to the stator and as the starting current is the same as in (c),

starting torque = 25 per cent of full-load torque.

67. *Obtain an expression for the condition of maximum torque of an induction motor.*

Sketch the torque-slip curves for several values of rotor circuit resistance and indicate the condition for maximum torque to be obtained at starting.

If the motor has a rotor resistance of 0·02 *ohm and a standstill reactance of* 0·1 *ohm, what must be the value of the total resistance of a startor for the the rotor circuit for maximum torque to be exerted at starting?*

C. and G. Final, Pt. I, 1943)

The expression for the torque developed at any slip s is

$$T = A \times \frac{sR_2}{R_2^2 + X_2^2 s^2}$$

where R_2 and X_2 are the resistance and standstill reactance of the rotor circuit, and A is a constant.

To find the condition for maximum torque, differentiate T with respect to R_2 and equate to zero.

$$\frac{dT}{dR_2} = A.\frac{sR_2.2R_2 - s\,(R_2^2 + s^2X_2^2)}{(R_2^2 + s^2X_2^2)^2} = 0$$

i.e. $$2sR_2^2 = s\,(R_2^2 + s^2X_2^2)$$

$$R_2^2 = (sX_2)^2 \text{ or } s = \frac{R_2}{X_2}$$

i.e. **For maximum torque to be developed the slip must be equal to the ratio of the rotor circuit resistance to its standstill reactance.**

Fig. 44 shows the torque-slip curves for various values of rotor resistance in terms of the standstill reactance. Maximum torque occurs at starting (s = 1) when $R_2 = X_2$.

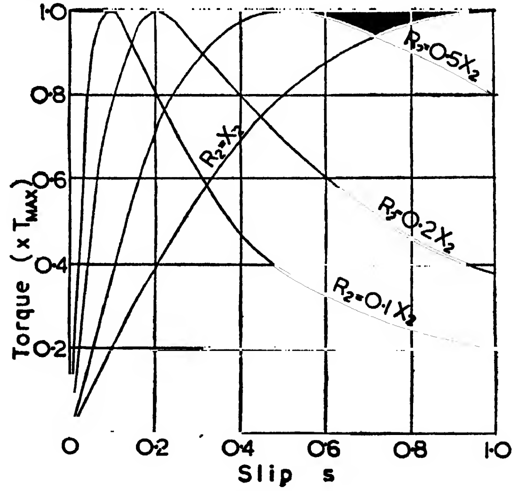


Fig. 44

If the motor has a rotor resistance of 0·02 ohm and a standstill reactance of 0·1 ohm, total rotor circuit resistance required at starting for maximum torque = X_2 = 0·1 ohm.

Hence, **startor resistance required** = 0·1 — 0·02 ohm

= **0·08 ohm.**

68. *For what induction-motor drives is it usual to employ star-delta starters in preference to any other type of startor?*

Find the ratio of starting to full-load current for a 15-*h.p.*, 415-*volt*, 3-*phase induction motor with star-delta startor, given that the full-load efficiency is* 0·88, *the full-load power factor is* 0·85, *the short-circuit current is* 40 *amperes at* 210 *volts, and the magnetizing current is negligible.*

(*I.E.E., Pt. II, May*, 1939)

$$\text{Full-load current} = \frac{15 \times 746}{(\sqrt{3} \times 415 \times 0{\cdot}85 \times 0{\cdot}88)}$$
$$= 20{\cdot}82 \text{ amperes.}$$

Input phase voltage with startor

$$\text{in star position} = \frac{415}{\sqrt{3}} \text{ volts.}$$

Since short-circuit current is 40 amperes with 210 volts input, i.e., a phase voltage of 210 volts with windings in delta, the phase current is $\frac{40}{\sqrt{3}}$ amperes.

Starting (i.e. short-circuit) current with

$$\text{startor in star position} = \frac{40}{\sqrt{3}} \times \frac{415}{\sqrt{3}} \times \frac{1}{210}$$
$$= 27{\cdot}21 \text{ amperes.}$$

$$\text{Therefore, } \frac{\textbf{starting current}}{\textbf{full-load current}} = \frac{27{\cdot}21}{20{\cdot}82}$$
$$= \mathbf{1{\cdot}31}$$

69. *Estimate in lb.-ft. the starting torque exerted by a* 25-*h.p.*, 420-*volt*, 6-*pole*, 50-*cycle*, 3-*phase induction motor, when an external resistance of* 1 *ohm is inserted in each rotor phase. Stator impedance,* 0·25 + *j*0·75 *ohm: rotor impedance,* 0·173 + *j*0·52 *ohm. Stator/rotor phase-voltage ratio,* 420/350*: connection, star/star.* (*I.E.E., Pt. II, May*, 1938)

Equivalent resistance of motor

$$\text{referred to rotor circuit} = 0{\cdot}173 + \left(\frac{350}{420}\right)^2 \times 0{\cdot}25$$
$$= 0{\cdot}346 \text{ ohm per phase.}$$

Equivalent reactance of motor

$$\text{referred to rotor circuit} = 0{\cdot}52 + \left(\frac{350}{420}\right)^2 \times 0{\cdot}75,$$
$$= 1{\cdot}04 \text{ ohms per phase.}$$

With 1 ohm external resistance added to each phase of the rotor circuit, equivalent impedance referred to

$$\text{rotor circuit} = \sqrt{(1 + 0{\cdot}346)^2 + (1{\cdot}04)^2}$$
$$= 1{\cdot}7 \text{ ohms.}$$

$$\text{Short-circuit rotor current} \quad I_2 = \frac{350}{\sqrt{3} \times 1{\cdot}7} \text{ amperes}$$
$$= 119 \text{ amperes.}$$

Rotor copper loss per phase on short-circuit $= 119^2 \times 1{\cdot}173 = 16610$ watts.

Now, rotor copper loss $=$ slip $\times$ power input to rotor circuit.

On short-circuit the slip $= 1$, therefore

power input to rotor $=$ Rotor copper loss,
$= 16610$ watts per phase,
$= 49830$ watts for 3 phases.

Let $T_s =$ the starting torque in lb.-ft.
$N_s =$ the synchronous speed in r.p.m.

Then, power input to rotor circuit $= 2\pi N_s T_s$ ft.-lb. per min.

$$= 2\pi N_s T_s \times \frac{746}{33000} \text{ watts.}$$

i.e.
$$2\pi N_s T_s \times \frac{746}{33000} = 49830$$

With 6 poles and a supply frequency of 50 cycles per second,

$$N_s = 120 \times \frac{50}{6} \text{ r.p.m.}$$
$$= 1000 \text{ r.p.m.}$$

Hence, **starting torque**
$$T_s = \frac{49830 \times 33000}{(2\pi \times 1000 \times 746)}$$
$$= \mathbf{351 \text{ lb.-ft.}}$$

70. *Why has a standard cage induction motor an inherently low starting torque? Briefly describe types of cage motor designed for higher starting torques without excessive starting currents.*

A cage motor takes 175 *per cent of full-load line current and develops* 35 *per cent of full-load torque at starting when operated by a star-delta switch. What would be the starting torque and current if an auto-transformer startor with* 80 *per cent tapping were employed?* (*I.E.E., Pt. II, Nov.*, 1942)

Let $T_f =$ the full-load torque,
$T_s =$ the starting torque,
$I_f =$ the full-load phase current,
$I_s =$ the starting phase current.
} with normal input voltage.

Now, torque $= k \times \frac{I_2^2}{s}$ where k is a constant, I_2 the rotor current, and s the slip.

If it is assumed that the rotor current is proportional to the input current on the stator side, then

$$\frac{T_s}{T_f} = \left\{\frac{I_s}{I_f}\right\}^2 \times \frac{s_f}{1} \text{ (since at starting the slip} = 1) \qquad (1)$$

With star-delta starting, $T_s = 0{\cdot}35\ T_f$, and $I_s = 1{\cdot}75\ \sqrt{3}\ I_f$.

Hence, $0{\cdot}35 = (1{\cdot}75 \times \sqrt{3})^2 \times s_f$

and $s_f = 0{\cdot}0381 =$ slip at full-load.

With autotransformer starting the input phase voltage is 0·8 of the normal value.

When the input voltage is $\frac{1}{\sqrt{3}}$, i.e. 0·5773 of the normal value (with windings in star) the line current is 175 per cent of full-load line current, therefore with 0·8 of the normal input voltage and the windings in delta,

Motor phase current $= 175 \times \frac{0{\cdot}8}{0{\cdot}5773}$ per cent of full-load line current,

$= 242{\cdot}5$ per cent of full-load line current.

Motor line current $= 242{\cdot}5 \times \sqrt{3}$

$=$ **420 per cent of full-load current.**

Substituting this value of motor current in equation (1) gives the starting torque,

$$T_s = \left\{\frac{I_s}{I_f}\right\}^2 \times 0{\cdot}0381 \times T_f$$
$$= (4{\cdot}2)^2 \times 0{\cdot}0381 \times T_f$$
$$= 0{\cdot}672\ T_f$$

i.e. **Starting torque = 67·2 per cent of full-load torque.**

The starting current in the primary of the autotransformer is 0·8 of the motor line current, i.e., 336 per cent of the full load motor line current.

71. *Make a list of the methods available for starting induction motors with cage rotors, and give notes on the operating features of each.*

A 3-phase slip-ring induction motor gives a reading of 55 *volts across slip-rings on open-circuit when at rest with normal stator voltage applied. The rotor is star-connected and has an impedance of* (0·7 + *j*5) *ohms per phase. Find the rotor current when the machine is* (*a*) *at standstill with the slip-rings joined to a star-connected startor with a phase impedance of* (4 + *j*3) *ohms:* (*b*) *running normally with a* 5 *per cent slip.*

(*I.E.E., Pt. II, Nov.,* 1943)

(a) Total impedance of rotor circuit per phase at standstill $= (0{\cdot}7 + j5) + (4 + j3)$

$= (4{\cdot}7 + j8)$ ohms,

$= 9{\cdot}277$ ohms.

e.m.f. induced into rotor circuit at standstill $= 55$ volts between rings,

$= \frac{55}{\sqrt{3}}$ volts per phase.

Therefore, **rotor current** $= \frac{55}{\sqrt{3}} \div 9{\cdot}277$ amperes

$=$ **3·424 amperes.**

(b) When running normally at 5 per cent slip, the rotor induced e.m.f. has only 5 per cent of its value at standstill. Also the frequency of this e.m.f. is only 5 per cent of what it was at standstill, i.e. 5 per cent of the supply frequency. Therefore the reactance of the rotor circuit is reduced to 5 per cent of its standstill value. Hence,

Reactance of rotor circuit per phase $= 0{\cdot}05 \times 5$ ohms,

$= 0{\cdot}25$ ohm.

Impedance of rotor circuit per phase $= 0{\cdot}7 + j0{\cdot}25$ ohms

$= 0{\cdot}743$ ohms

Induced e.m.f. per phase $= 0{\cdot}05 \times \frac{55}{\sqrt{3}}$

$=$ **1·588 volts.**

Therefore, **rotor current** $= \frac{1{\cdot}588}{0{\cdot}743}$ amperes

$= \mathbf{2{\cdot}136}$ **amperes.**

72. *A 3-phase induction motor takes a starting current at normal voltage of 5 times full-load value, and its full-load slip is 4 per cent. What autotransformer ratio would enable the motor to be started with not more than twice full-load current drawn from the supply? What would be the starting torque under this condition, and how would it compare with that obtained using a stator resistance startor under the same limitations of line current? If it were decided that a plain motor gave insufficient starting torque, what alternative type of cage motor could be used? Give details.*

(*I.E.E., Pt. II, Nov.,* 1943)

Let I_{sc} = the starting current at normal voltage,
I_f = the full-load current,
I_s = the line current using the autotransformer,
k = the step-down ratio of the transformer.

From the data given,

$$I_{sc} = 5\ I_f \qquad (1)$$

With the autotransformer the voltage input to the motor is reduced in the ratio k and therefore the motor input current will be reduced in the same ratio, i.e.

Motor input current $= \frac{I_{sc}}{k}$

The supply line current is still further reduced in the same ratio.

Supply line current $= \frac{I_{sc}}{k^2}$ and this must be limited to twice full-load current. Hence,

$$\frac{I_{sc}}{k^2} = 2\ I_f$$

$$= \frac{2}{5}\ I_{sc} \text{ from (1)}$$

Hence, $$k^2 = \frac{5}{2}$$

$$k = \sqrt{2{\cdot}5}$$

$$= 1{\cdot}58$$

i.e. **autotransformer ratio required is 1·58: 1 step-down.**

Motor input current $= \frac{I_{sc}}{1{\cdot}58}$

$$= \frac{5}{1{\cdot}58}\ I_f$$

$$= 3{\cdot}16\ I_f$$

Hence, $\frac{\text{starting torque}}{\text{full-load torque}}$

$$= \left\{\frac{3{\cdot}16\ I_f}{I_f}\right\}^2 \times s_f \text{ where } s_f = \text{full-load slip}$$

$$= 10 \times 0{\cdot}04$$
$$= 0{\cdot}4$$

i.e. **The starting torque is 40 per cent of full-load torque.**

With a stator resistance startor, if the line current were limited to twice full-load current the motor input current would also be twice full-load current.

Hence, motor input current $= 2\ I_f$

$$\text{Then, } \frac{\text{starting torque}}{\text{full-load torque}} = \left\{\frac{2\ I_f}{I_f}\right\}^2 \times 0{\cdot}04$$
$$= 0{\cdot}16$$

Therefore **the starting torque with an autotransformer will be 2·5 times that with a stator resistance startor with the same input line current.**

73. *A* 4-*pole,* 200-*volt,* 50-*c.p.s. star connected,* 3-*phase induction motor has a primary leakage impedance of* 0·8 + *j*2 *ohms, and an equivalent secondary leakage impedance at standstill of* 1·2 + *j*1 *ohms per phase. The magnetizing current may be assumed to be negligible. Find the maximum torque in lb.-ft. the motor can develop and the slip at which it occurs.*

(*London B.Sc. Eng., July,* 1945)

Fig. 45 shows the equivalent circuit per phase of the motor when it is running with a slip s.

$$E_1 = \frac{200}{\sqrt{3}} \text{ volts.}$$

$$I_1 = \frac{E_1}{0{\cdot}8 + j2 + \frac{1{\cdot}2}{s} + j1}$$

$$= \frac{E_1}{\left\{0{\cdot}8 + \frac{1{\cdot}2}{s}\right\} + j3}$$

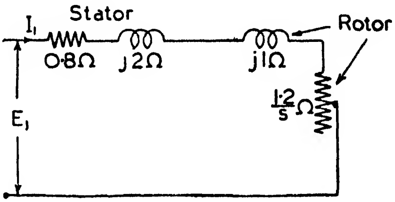


Fig. 45

The input power to the rotor circuit is given by $I_1^2 \times \frac{1{\cdot}2}{s}$ watts which is also a measure of the torque developed in synchronous watts.

$$\text{Hence, torque developed per phase } T = \frac{E_1^2 \times \frac{1{\cdot}2}{s}}{\left\{0{\cdot}8 + \frac{1{\cdot}2}{s}\right\}^2 + 3^2} \text{ synchronous watts,}$$

The maximum value of the torque is obtained at a slip which makes $\frac{dT}{ds} = 0$.

Differentiating T,

$$\frac{dT}{ds} = \frac{E_1^2\left\{-\frac{1{\cdot}2}{s^2}\right\}\left\{\left(0{\cdot}8 + \frac{1{\cdot}2}{s}\right)^2 + 3^2\right\} - E_1^2\left\{\frac{1{\cdot}2}{s}\right\} \times 2\left\{0{\cdot}8 + \frac{1{\cdot}2}{s}\right\}\left\{-\frac{1{\cdot}2}{s^2}\right\}}{\left\{\left(0{\cdot}8 + \frac{1{\cdot}2}{s}\right)^2 + 3^2\right\}^2}$$

This is zero when

$$\left\{0{\cdot}8+\frac{1{\cdot}2}{s}\right\}^2+9=2\left\{0{\cdot}8+\frac{1{\cdot}2}{s}\right\}\left\{\frac{1{\cdot}2}{s}\right\}$$

i.e.
$$0{\cdot}64+\frac{1{\cdot}92}{s}+\frac{1{\cdot}44}{s^2}+9=\frac{1{\cdot}92}{s}+\frac{2{\cdot}88}{s^2}$$

$$\frac{1{\cdot}44}{s^2}=9{\cdot}64$$

$$s=\sqrt{\frac{1{\cdot}44}{9{\cdot}64}}$$

$$=0{\cdot}387$$

Therefore, **maximum torque will occur at a slip of 0·387.** Substituting s = 0·387 in the torque expression we get

$$\text{maximum torque per phase}=\frac{\left\{\frac{200}{\sqrt{3}}\right\}^2\times\frac{1{\cdot}2}{0{\cdot}387}}{\left\{0{\cdot}8+\frac{1{\cdot}2}{0{\cdot}387}\right\}^2+9}\text{ synch. watts}$$

$$=1707\text{ synchronous watts.}$$

Now if T_m is the maximum torque per phase in lb.-ft. and N_s is the synchronous speed in r.p.m.

$$\frac{2\pi\ N_sT_m}{33000}=\frac{1707}{746}$$

But N_s is 1500 r.p.m. for a 4-pole motor running on a 50-c.p.s. supply.

Therefore,
$$\frac{2\pi\times1500\ T_m}{33000}=\frac{1707}{746}$$

whence
$$T_m=8\text{ lb.-ft. per phase}$$

The maximum torque the motor can develop is 24 lb.-ft.

(iii) The circle diagram.

74. *The following test data refer to a* 3-*phase,* 500-*volt, induction motor:*
No-load: 500 *volts,* 18 *amperes,* 1200 *watts.*
Locked rotor: 250 *volts,* 100 *amperes,* 11000 *watts.*
Determine the maximum power output.

(*I.E.E., Pt. II, May,* 1939)

Power factor on no-load $=\frac{1200}{(\sqrt{3}\times500\times18)}=0{\cdot}077$

No-load phase angle $=85°35'$

Power factor with locked rotor (short-circuit) $=\frac{11000}{(\sqrt{3}\times250\times100)}=0{\cdot}2541$

Short-circuit phase angle $=75°17'$

With normal applied voltage on short-circuit (i.e. double the actual voltage applied) the input current would be 200 amperes and the power taken would be 44000 watts.

From the above data the circle diagram (Fig. 46) is now constructed as follows:

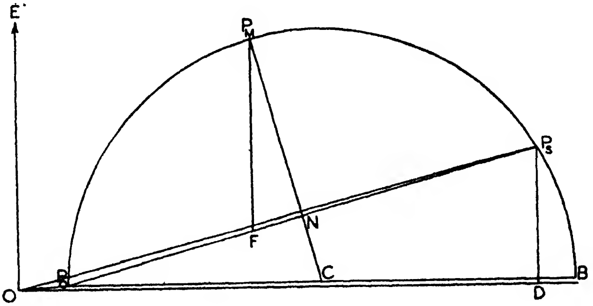


Fig. 46

The current scale assumed is 25 amperes per inch.

OE is drawn vertically to represent the input voltage. Lagging OE by an angle 85°35′ is OP_0 to represent I_0 the no-load current. Also lagging OE by 75°17′ is OP_s to represent I_s the short-circuit current with full-input voltage.

A horizontal line P_0B is drawn through P_0 and P_0P_s are joined. The line P_0P_s is bisected at right angles by NC meeting P_0B at C. Then C is the centre of the semicircle drawn to pass through P_0 and P_s. This semicircle is the locus of the current vector for all load conditions from no-load to short-circuit.

The power output under any given load condition is represented to scale by the vertical intercept between the semicircle and the line P_0P_s. The power scale is obtained by reference to the data that on short-circuit the power input is 44000 watts.

i.e. P_sD (= 2·03 inches) = 44000 watts.

Therefore the power scale is $\frac{44000}{2 \cdot 03}$ watts per inch, i.e. 21650 watts per inch.

The operating point for maximum power output is found by producing CN to cut the semicircle at P_m. A perpendicular line is dropped from P_m on to P_0P_s meeting it at F.

Then P_mF represents to scale the maximum power output.

By measurement from the diagram, P_mF = 2·87 inches.

Hence, **maximum power output** $= 2 \cdot 87 \times \frac{21650}{746}$ h.p.

= 83·3 h.p.

75. *Draw the circle diagram of a* 10 *h.p.*, 200-*volt*, 50-*cycle*, 3-*phase, slip-ring induction motor with a star-connected stator and rotor, a winding ratio of unity, a stator resistance of* 0·38 *ohm per phase and a rotor resistance of* 0·24 *ohm per phase. The following are test readings:*

No-load: 200 *volts*, 7·7 *amperes, power factor* 0·195.

Short-circuit: 100 *volts*, 47·6 *amperes, power factor* 0·454.

Find (*a*) *the starting torque, and* (*b*) *the maximum torque in synchronous watts*; (*c*) *the maximum power factor*; (*d*) *the slip for maximum torque*; (*e*) *the maximum output.* (*I.E.E., Pt. II, Nov.*, 1937)

No-load phase angle = arc cos 0·195 = 78°45′ = $\angle P_0OE$ (FIG. 47)
Short-circuit phase angle = arc cos 0·454 = 63° = $\angle P_sOE$

If normal voltage (200 volts) were applied the short-circuit current would be 47·6 × 2 amperes = 95·2 amperes.

The circle diagram is drawn in the manner described in Problem 74 to an assumed scale of 10 amperes per inch. In this diagram OP_0 represents 7·7 amperes and OP_s represents 95·2 amperes.

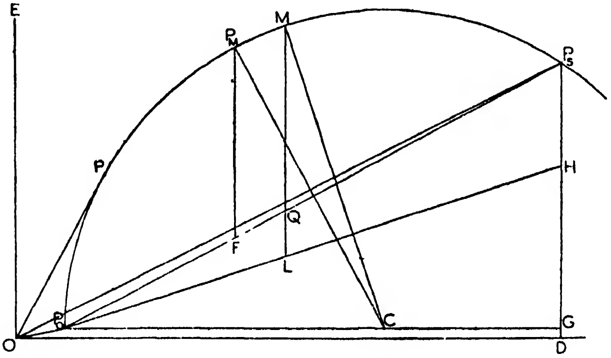


Fig. 47

The power scale is obtained as follows:

$$\text{Power input on short-circuit} = \sqrt{3} \times 200 \times 95{\cdot}2 \times 0{\cdot}454 \text{ watts},$$
$$= 14970 \text{ watts}.$$

This power input is represented on the diagram by the line P_sD which is 4·32 in. in length.

$$\text{Hence, 1 in. represents } \frac{14970}{4{\cdot}32} \text{ watts} = 3464 \text{ watts}.$$

The torque line P_0H is found by subdividing P_sG, the copper loss on short-circuit, in the ratio

$P_sH : P_sG$: : rotor loss : total loss
: : rotor resistance : rotor + stator resistance.
: : 0·24 ohm : 0·62 ohm

$$\text{Hence, } P_sH = 4{\cdot}17 \times \frac{0{\cdot}24}{0{\cdot}62} \text{ in.} = 1{\cdot}61 \text{ in.}$$

Then the torque developed on any load is represented by the intercept between the semicircle and the line P_oH. The length of the intercept multiplied by the power scale gives the torque in synchronous watts, a "synchronous watt" being defined as that torque which develops a power of 1 watt at the synchronous speed of the machine.

(a) **Starting torque** $= P_sH = 1{\cdot}61$ in.
$= 1{\cdot}61 \times 3464$ synchronous watts,
$=$ **5590 synchronous watts.**

(b) The maximum torque condition is found by drawing CM perpendicular to P_oH and dropping a vertical line to cut P_oH at L. Then ML represents the maximum torque.

Maximum torque $=$ ML $= 3{\cdot}6$ in.
$= 3{\cdot}6 \times 3464$ synchronous watts,
$=$ **12500 synchronous watts.**

(c) The maximum power factor is found by drawing the line OP tangential to the semicircle.

Maximum power factor $= \cos \angle \text{POE} = \cos 28° =$ **0·883**

(d) The slip for maximum torque is given from the diagram by the ratio $\frac{QL}{ML}$. Since QL $= 0{\cdot}71$ in. and ML $= 3{\cdot}6$ in. by measurement,

slip for maximum torque $= \frac{0{\cdot}71}{3{\cdot}6} =$ **0·194**

(e) The condition for maximum power output is found by drawing the line CP_m perpendicular to P_oP_s and dropping the vertical from P_m to meet P_oP_s at F. Then P_mF represents the maximum power output.

Maximum power output $= P_mF = 2{\cdot}98$ in.
$= 2{\cdot}98 \times 3464$ watts
$= 2{\cdot}98 \times \frac{3464}{746}$ h.p.
$=$ **13·8 h.p.**

76. *The following figures were obtained from tests on a* 5-*h.p.*, 200-*volt*, 50-*cycle*, 4-*pole*, 3-*phase*, *star-connected induction motor:*

No-load: 200 *volts*, 5·0 *amperes*, *total input* 350 *watts.*

Short-circuit: 100 *volts*, 20 *amperes*, *total input* 1700 *watts.*

Draw the circle diagram and obtain for full-load conditions the line current, power factor, slip, efficiency, speed, and torque (in terms of maximum torque). The stator copper loss at standstill is 55 *per cent of the total copper loss.*

(*I.E.E.*, *Pt. II*, *May*, 1937)

No-load power factor $= \frac{350}{\sqrt{3} \times 200 \times 5} = 0{\cdot}2021$

No-load phase angle $= 78°20'$

Short-circuit power factor $= \frac{1700}{(\sqrt{3} \times 100 \times 20)} = 0{\cdot}4918$

Short-circuit phase angle $= 60°32'$

Short-circuit current with full-voltage input $= 20 \times \frac{200}{100}$ amperes

$= 40$ amperes.

From the above data the circle diagram is drawn as described in Problem 74. The current scale asssumed is 5 amperes per inch and the power scale is 1732 watts per inch (Fig. 48).

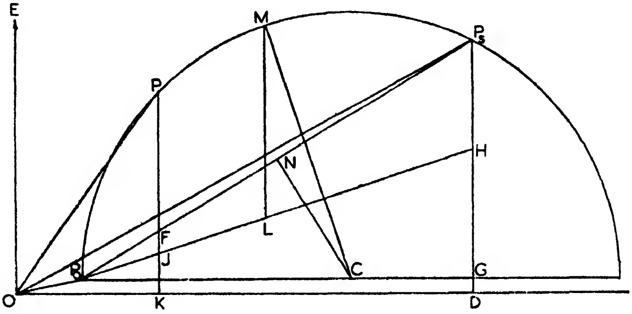


Fig. 48

On the scale employed the intercept between the semicircle and the line P_0P_s must be $\frac{3730}{1732}$ inches long, i.e. 2·15 inches for full-load output, because full-load is 5 h.p. (= 3730 watts).

PF = 2·15 inches therefore OP represents the line current and ∠ POE the phase angle on full-load. By measurement, OP = 3·7 inches and ∠ POE = 34·5°. Therefore

full-load line current $= 3{\cdot}7 \times 5$ = **18·5 amperes,**

full-load power factor = cos 34·5° = **0·824 lagging.**

The total stator + rotor copper loss on short circuit is represented by P_sG. This is subdivided at H such that GH (stator copper loss) = 0·55 of P_sG (total copper loss). H is joined to P_0. Then

Intercept between the semicircle and P_0P_s represents power output,
" " " " " P_0H " torque,
" " " " " P_0H " rotor power input,
" " " " " OD " total power input,
" " " P_0P_s " P_0H " rotor copper loss.

Now since rotor copper loss = slip × power input to rotor, on full-load,

$$\textbf{Slip} = \frac{FJ}{PJ} = \frac{0{\cdot}32}{2{\cdot}47} = \mathbf{0{\cdot}13}$$

$$\textbf{Efficiency} = \frac{PF}{PK} = \frac{2{\cdot}15}{3{\cdot}05} = 0{\cdot}705 = \mathbf{70{\cdot}5 \text{ per cent}}$$

$$\textbf{Speed} = (1 - s) \times \text{synchronous speed}$$

$$= (1 - 0{\cdot}13) \times \frac{120 \times 50}{4} = \mathbf{1305 \text{ r.p.m.}}$$

The maximum torque condition is found by drawing a line CM perpendicular to P_0H cutting the semicircle at M. A vertical line is drawn from M to cut P_0H at L. Then ML represents maximum torque while PJ represents full-load torque. PJ = 2·47 inches, and ML = 2·98 inches.

$$\text{Hence, } \frac{\text{full-load torque}}{\text{maximum torque}} = \frac{2{\cdot}47}{2{\cdot}98} = 0{\cdot}829$$

Full-load torque = 0·829 × maximum torque.

77. *The no-load and short-circuit tests on a* 30-*h.p.*, 500-*volt*, 4-*pole*, 50-*cycle, cage-rotor induction motor gave the following figures (for line values of voltage and current and two-wattmeter method of power measurement):*

No-load: 500 *volts*, 8·2 *amperes*, − 1·40 *and* + 2·83 *kW.*

Short-circuit: 140 *volts*, 45·0 *amperes*, − 1·50 *and* + 4·70 *kW.*

Draw the circle diagram and obtain from it (a) the stator current and power factor for full-load, (b) the maximum torque, and (c) the starting torque, in terms of full-load torque. The equivalent rotor resistance may be taken as equal to the stator resistance. (*I.E.E., Pt. II, May*, 1940)

No-load power input = −1·40 + 2·83 = 1·43 kW.

$$\text{No-load power factor} = \frac{1430}{(\sqrt{3} \times 500 \times 8{\cdot}2)} = 0{\cdot}2014$$

i.e. No-load phase angle = 78°23′

Short-circuit power input = −1·50 + 4·70 = 3·20 kW.

$$\text{Short-circuit power factor} = \frac{3200}{(\sqrt{3} \times 140 \times 45)} = 0{\cdot}2933$$

i.e. Short-circuit phase angle = 72°56′

$$\text{Short-circuit current with normal input voltage} = \frac{45 \times 500}{140}$$

$$= 160{\cdot}7 \text{ amperes.}$$

From this data the circle diagram is drawn as described in Problem 74. The diagram (Fig. 49) is assumed to be drawn to a current scale of 20 amperes per inch, and a power scale of 17320 watts per inch (P_sD = 2·36 inches, representing 41000 watts, the total power input on short-circuit with normal voltage applied)

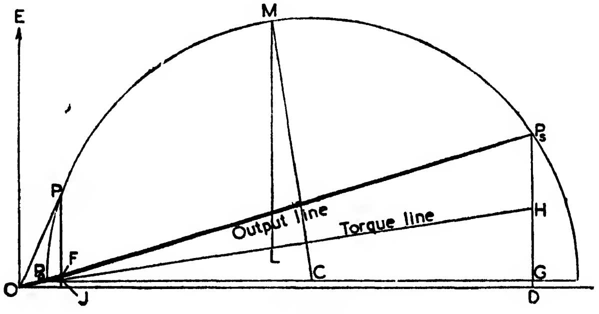


Fig. 49

(a) Full-load output = 30 h.p. = 30 × 746 watts = 22380 watts.

Thus for full-load the intercept between the semicircle and P_0P_s must be $\frac{22380}{17320}$ i.e. 1·29 inches long.

A point P is found such that PF = 1·29 inches, then P is the operating point for full-load.

$$\textbf{Stator full-load current} = OP = 1{\cdot}57 \text{ inches (by measurement)}$$
$$= 1{\cdot}57 \times 20 \text{ amperes}$$
$$= \mathbf{31{\cdot}4 \text{ amperes.}}$$

$$\textbf{Stator power factor on full-load} = \cos \angle POE$$
$$= \cos 24° = \mathbf{0{\cdot}914}$$

(b) Maximum torque is represented by ML and full-load torque by PJ.

By measurement, PJ = 1·32 inches and ML = 3·42 inches.

$$\text{Therefore, } \textbf{maximum torque} = \frac{3{\cdot}42}{1{\cdot}32} \times \text{full-load torque}$$
$$= \mathbf{2{\cdot}59 \times \text{full-load torque.}}$$

(c) Similarly the starting torque is represented by P_sH and P_sH = 1·14 inches.

$$\text{Therefore, } \textbf{starting torque} = \frac{1{\cdot}14}{1{\cdot}32} \times \text{full-load torque}$$
$$= \mathbf{0{\cdot}864 \times \text{full-load torque.}}$$

78. *The following particulars apply to a 3-phase,* 1000-*h.p.,* 3000-*volt, induction motor with stator and rotor windings star connected:*

Turns per phase, stator 210; *rotor* 50;

Resistance per phase, stator 0·35 *ohm; rotor* 0·020 *ohm.*

Leakage reactance per phase, stator 2·0 *ohms; rotor* 0·058 *ohm.*

Total iron loss 6·0 *kW; friction and windage* 4·0 *kW.*

Magnetizing current 50 *amperes.*

Draw clearly to scale the circle diagram and determine the current, power-factor, efficiency and slip at full-load, also the starting torque with full-voltage applied (expressed as a percentage of full-load torque).

(*London B.Sc. Eng., July,* 1945)

$$\text{Rotor resistance referred to stator} = 0{\cdot}02 \times \left\{\frac{210}{50}\right\}^2$$
$$= 0{\cdot}353 \text{ ohm.}$$

$$\text{Rotor reactance referred to stator} = 0{\cdot}058 \times \left\{\frac{210}{50}\right\}^2$$
$$= 1{\cdot}023 \text{ ohms.}$$

$$\text{Total no-load losses} = 6 \text{ kW} + 4 \text{ kW}$$
$$= 10 \text{ kW.}$$

In-phase component of no-load current $= \dfrac{10000}{\sqrt{3} \times 3000}$ amperes,

$= 1{\cdot}925$ amperes.

The equivalent circuit per phase referred to the stator may now be represented by Fig. 50.

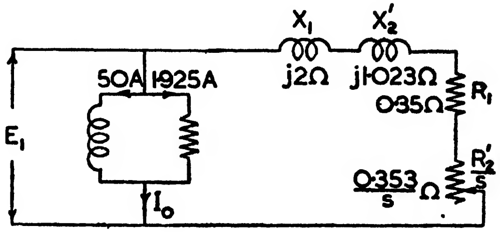


Fig. 50

The circle diagram (Fig. 51) is drawn by the following construction:

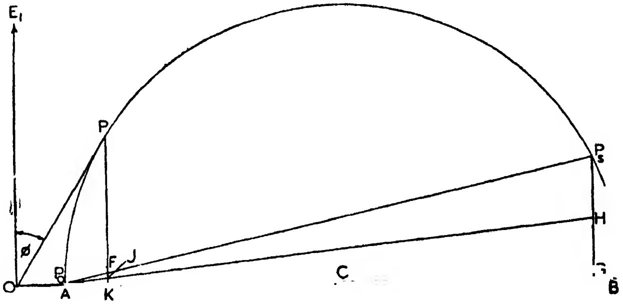


Fig. 51

OA represents the magnetizing current of 50 amperes and AP₀ perpendicular to OA represents the in-phase component of the no-load current. OP₀ is the no-load current.

P₀B is drawn parallel to OA and equal to $\dfrac{E_1}{X_1 + X_2^1}$ to the same scale as that chosen for the above current vectors. A convenient scale is 1 cm. = 25 amperes which is the scale of the diagram from which the following measurements were taken.

Then $$P_oB = \frac{3000}{\sqrt{3}} \times \frac{1}{3{\cdot}023} \text{ amperes,}$$

$$= 573{\cdot}1 \text{ amperes,}$$

$$= 22{\cdot}92 \text{ cm.}$$

P_oB is bisected at C and with C as centre the semicircle is drawn on the diameter P_oB. The point P_s which represents the short-circuit operating point is found by drawing P_oP_s at an angle $\varnothing_s$ to the vertical axis, where

$$\tan \varnothing_s = \frac{X_1 + X_2^1}{R_1 + R_2^1}$$

$$= \frac{3{\cdot}023}{0{\cdot}703} = 4{\cdot}3$$

Hence $$\varnothing_s = 76°55'$$

Then P_oP_s is the output line and if P_sG, the perpendicular on to P_oB, is divided at H in the ratio

$$\frac{P_sH}{HG} = \frac{R_2^1}{R_1} = \frac{0{\cdot}353}{0{\cdot}35}$$

Then P_oH is the torque line.

With the current scale chosen, lines perpendicular to P_oB represent power to the scale of

$$1 \text{ cm.} = \frac{\sqrt{3} \times 3000 \times 25}{746} \text{ h.p.}$$

$$= 174{\cdot}2 \text{ h.p.}$$

Rated output on full load $= 1000$ h.p.

$$= \frac{1000}{174{\cdot}2} \text{ cm. to the above power scale,}$$

$$= 5{\cdot}74 \text{ cm.}$$

The full-load operating point on the circle is P, obtained by making the vertical intercept PF between the circle and the output line equal to 5·74 cm. Then the required data may be found by measurement from the circle diagram as follows:

Full-load current $= OP$ $=$ **181 amperes.**

Full-load power factor $= \cos \angle \underline{POE_1} = \cos 31{\cdot}5° =$ **0·853**

Full-load efficiency $$= \frac{PF}{PK} = \frac{5{\cdot}74}{6{\cdot}15} \times 100 \text{ per cent}$$

$=$ **93·3 per cent.**

Full-load slip $$= \frac{FJ}{PJ} = \frac{0{\cdot}2}{5{\cdot}94} = \mathbf{0{\cdot}034}$$

$$\frac{\text{Starting torque}}{\text{Full-load torque}} = \frac{P_sH}{PJ} = \frac{2{\cdot}6}{5{\cdot}94} = 0{\cdot}438$$

i.e. the starting torque is 43·8 per cent of the full-load torque.

79. *Give a diagram representing the "equivalent circuit" of a 3-phase induction motor, and briefly discuss its validity.*

Estimate the stator current, equivalent rotor current, efficiency, output, and input power factor at a slip of 5 *per cent for a motor having the following data: Stator impedance,* $1{\cdot}0 + j3{\cdot}0$ *ohms: rotor standstill impedance,* $1{\cdot}0 + j2{\cdot}0$ *ohms; no-load shunt impedance,* $10 + j50$ *ohms; volts per phase,* 250.
(*I.E.E., Pt. II, Nov.*, 1942)

Fig. 52 shows the equivalent circuit per phase of the motor referred to the stator. It is assumed that the impedance figures given are phase values.

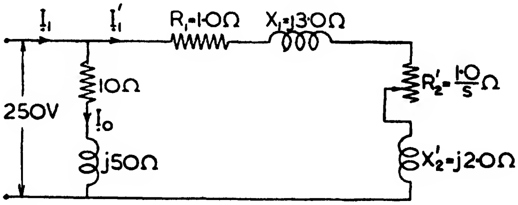


Fig. 52

When the slip is s the equivalent resistance of the rotor is $R_2' = \frac{R_2}{s} = \frac{1{\cdot}0}{s}$ ohms. In this case as $s = 0{\cdot}05$,

$$R_2' = \frac{1{\cdot}0}{0{\cdot}05} = 20 \text{ ohms.}$$

Effective impedance per phase $= (R_1 + R_2') + j\,(X_1 + X_2')$
$= 21 + j5$ ohms.

Stator load current $= I_1' = \frac{250}{(21 + j5)}$ amperes
$= \frac{250\,(21 - j5)}{(21^2 + 5^2)}$
$= 11{\cdot}27 - j2{\cdot}682$ amperes.

Stator no-load current $= I_o = \frac{250}{(10 + j50)}$ amperes
$= \frac{250\,(10 - j50)}{(10^2 + 50^2)}$
$= 0{\cdot}961 - j4{\cdot}807$ amperes.

Hence, **total stator current** $= I_1 = I_o + I_1'$
$= (0{\cdot}961 - j4{\cdot}807) + (11{\cdot}27 - j2{\cdot}682)$
$= 12{\cdot}23 - j7{\cdot}489$ amperes.
$I_1 =$ **14·34 amperes.**

Equivalent rotor current $= I_1' = 11{\cdot}27 - j2{\cdot}682$ amperes.
$I_1' =$ **11·58 amperes.**

Input phase angle $= \text{arc tan } \dfrac{7{\cdot}489}{12{\cdot}23} = 31^\circ 29'$

Input power factor $= \cos 31^\circ 29' =$ **$0{\cdot}853$**

Power input to rotor circuit $= (I_1')^2 R_2'$

$= 11{\cdot}58^2 \times 20 = 2684$ watts.

Mechanical power developed per phase $= (1 - s) \times$ power input to rotor

$= (1 - 0{\cdot}05) \times 2684$ watts,

$= 2550$ watts.

Hence, **output of motor** $= \dfrac{2550 \times 3}{746}$ h.p. allowing for 3 phases,

$=$ **10·26 h.p.**

Efficiency $= \dfrac{\text{Output}}{\text{Input}} \times 100$ per cent,

$$= \frac{2550 \times 3 \times 100}{3 \times 250 \times 14{\cdot}34 \times 0{\cdot}853}$$

$=$ **83·4 per cent.**

Note.—It has been assumed throughout this problem that there is a unity turns ratio between stator and rotor.

(iv) Double-cage motors.

80. *Describe briefly how a high starting torque is obtained with a double-cage induction motor.*

The resistance and reactance values of such a motor are as follows: stator resistance 0·25 *ohm, reactance* 3·5 *ohms; outer-cage resistance* 1·0 *ohm, reactance zero; inner-cage resistance* 0·15 *ohm, reactance* 3 *ohms.*

Find the starting torque if the phase voltage is 250 *volts and the synchronous speed is* 1000 *r.p.m.* (*I.E.E., Pt. II, May,* 1940)

The equivalent circuit of one phase of the machine is shown in Fig. 53.

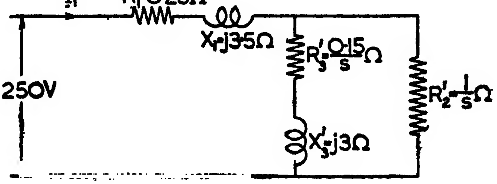


Fig. 53

At starting $s = 1$, therefore $R_2' = 1$ ohm, and $R_3' = 0{\cdot}15$ ohm.

Admittance per phase of the rotor circuit
$$= \frac{1}{R_2'} + \frac{1}{(R_3' + jX_3')}$$
$$= \frac{1}{1} + \frac{1}{(0{\cdot}15 + j3{\cdot}0)}$$
$$= 1{\cdot}0 + \frac{(0{\cdot}15 - j3{\cdot}0)}{(0{\cdot}15^2 + 3{\cdot}0^2)}$$
$$= 1{\cdot}0 + 0{\cdot}0166 - j0{\cdot}3325$$
$$= (1{\cdot}0166 - j0{\cdot}3325) \text{ mho.}$$

Impedance per phase of the rotor circuit
$$= \frac{1}{1{\cdot}0166 - j0{\cdot}3325}$$
$$= 0{\cdot}8872 + j0{\cdot}29 \text{ ohm.}$$

Hence, total impedance per phase
$$= (0{\cdot}25 + j3{\cdot}5) + (0{\cdot}8872 + j0{\cdot}29)$$
$$= (1{\cdot}137 + j3{\cdot}79) \text{ ohms.}$$

Input current per phase
$$I_1 = \frac{250}{1{\cdot}137 + j3{\cdot}79} \text{ amperes}$$
$$I_1 = 63{\cdot}19 \text{ amperes.}$$

Total copper loss per phase in rotor circuit
$$= I_1^2 r = 63{\cdot}19^2 \times 0{\cdot}8872 \text{ watts,}$$
$$= 3542 \text{ watts.}$$

Total copper loss in 3 phases $= 3 \times 3542 = 10626$ watts.

Power input to rotor $= \text{Copper loss in rotor} \times \dfrac{1}{s}$

At starting $s = 1$, therefore, power input to rotor circuit $=$ Copper loss in rotor $= 10626$ watts.
$$= \frac{10626}{746} \text{ h.p.}$$

Now if T_s is the starting torque and N_s is the synchronous speed,

power input to rotor circuit $= \dfrac{2\pi N_s T_s}{33000}$ h.p.

where T_s is in lb.-ft. and N_s is in r.p.m.
Therefore since $N_s = 1000$ r.p.m.,

$$\frac{2\pi \times 1000 \times T_s}{33000} = \frac{10626}{746}$$
$$T_s = \frac{10626 \times 33000}{(2\pi \times 1000 \times 746)}$$
$$= 74{\cdot}84 \text{ lb.-ft.}$$

i.e. **starting torque $= 74{\cdot}84$ lb.-ft.**

81. *Discuss the relative merits and disadvantages of single-cage and double-cage induction motors. If the standstill impedance of the outer cage of a double-cage machine is* $0{\cdot}3 + j0{\cdot}4$ *ohm, and of the inner cage is* $0{\cdot}1 + j1{\cdot}5$ *ohms, compare the relative currents and torques of the two cages (a) at standstill, (b) at a slip of* 5 *per cent.* (*I.E.E., Pt. II, Nov.*, 1938)

Fig. 54 shows the equivalent circuit diagram of the two cages.

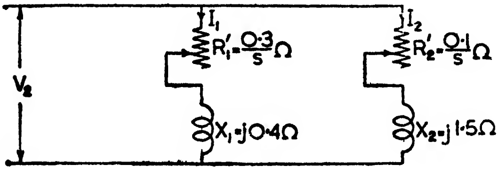


Fig. 54

(a) **At standstill,** $s = 1$, therefore the effective resistances of the two cages are:

Outer cage $R_1' = \frac{0{\cdot}3}{s} = 0{\cdot}3$ ohm,

Inner cage $R_2' = \frac{0{\cdot}1}{s} = 0{\cdot}1$ ohm.

Outer cage impedance $= (0{\cdot}3 + j0{\cdot}4) = 0{\cdot}5$ ohm.

Outer cage current $I_1 = \frac{V_2}{0{\cdot}5} = 2V_2$ amperes.

Inner cage impedance $= (0{\cdot}1 + j1{\cdot}5) = 1{\cdot}503$ ohms.

Inner cage current $I_2 = \frac{V_2}{1{\cdot}503} = 0{\cdot}665V_2$ amperes.

Hence, $\frac{I_1}{I_2} = \frac{2V_2}{0{\cdot}665V_2} = 3{\cdot}006$

i.e. **outer cage current** $=$ **3·006 × inner cage current.**

Power input to outer cage $= I_1'^2.R_1'$

$= (2V_2)^2.\ 0{\cdot}3 = 1{\cdot}2V_2^2$ watts,

Power input to inner cage $= I_2^2R_2'$

$= (0{\cdot}665V_2)^2 \times 0{\cdot}1 = 0{\cdot}0443V_2^2$

Now the torque developed by a cage is proportional to the power input to the cage, hence

$$\frac{\text{Torque of outer cage}}{\text{Torque of inner cage}} = \frac{1{\cdot}2V_2^2}{0{\cdot}0443V_2^2}$$

$= 27{\cdot}1$

Torque of outer cage $=$ **27·1 × torque of inner cage.**

(b) **At a slip of 5 per cent** s = 0·05, therefore the effective resistances are:

Outer cage $R_1' = \dfrac{0{\cdot}3}{0{\cdot}05} = 6$ ohms.

Inner cage $R_2' = \dfrac{0{\cdot}1}{0{\cdot}05} = 2$ ohms.

Outer cage impedance $= (6 + j0{\cdot}4) = 6{\cdot}013$ ohms.

Outer cage current $I_1 = \dfrac{V_2}{6{\cdot}013}$ amperes.

Impedance of inner cage $= (2 + j1{\cdot}5) = 2{\cdot}5$ ohms.

Inner cage current $I_2 = \dfrac{V_2}{2{\cdot}5}$ amperes.

Hence, $\dfrac{I_1}{I_2} = \dfrac{V_2}{6{\cdot}013} \times \dfrac{2{\cdot}5}{V_2} = 0{\cdot}416$

i.e. **outer cage current** $=$ **0·416 × inner cage current.**

Power input to outer cage $= I_1^2 R_1'$

$$= \left\{\frac{V_2}{6{\cdot}013}\right\}^2 \times 6 = \frac{V_2^2}{6{\cdot}03} \text{ watts.}$$

Power input to inner cage $= I_2^2 R_2'$

$$= \left\{\frac{V_2}{2{\cdot}5}\right\}^2 \times 2 = \frac{V_2^2}{3{\cdot}125} \text{ watts.}$$

$$\frac{\text{Torque of outer cage}}{\text{Torque of inner cage}} = \frac{V_2^2}{6{\cdot}03} \times \frac{3{\cdot}125}{V_2^2}$$

$$= 0{\cdot}52$$

i.e. **outer cage torque** $=$ **0·52 × inner cage torque.**

(v) Design calculations.

82. A 10-*h.p.*, 220-*volt*, 4-*pole*, 50-*cycle*, *star-connected*, 3-*phase induction motor with a bore of* 18 *cm. and a core-length of* 13·5 *cm. is to have an average air-gap flux density of about* 4000 *lines per sq. cm. Find particulars of a suitable stator winding, stating the number of slots, conductors per slot, coil-pitch, and flux per pole.* (*I.E.E., Pt. II, May,* 1940)

The length of the pole pitch is given by

$$Y = \frac{\pi . D}{p} = \pi \times \frac{18}{4} \text{ cm.} = 14{\cdot}14 \text{ cm.}$$

Flux per pole $=$ YL × Mean flux density

$= (14{\cdot}14 \times 13{\cdot}5)$ sq. cm. × 4000 lines per sq. cm.

$= 0{\cdot}764 \times 10^6$ lines.

The e.m.f. per phase will be approximately $\dfrac{220}{\sqrt{3}}$ volts, therefore

$$\frac{220}{\sqrt{3}} = 4{\cdot}44 k_m f \Phi T_s \cdot 10^{-8} \text{ volts.}$$

Taking $k_m = 0{\cdot}955$, i.e. assuming that the winding has a 60° phase spread,

$$T_s = \frac{(220 \times 10^8)}{(\sqrt{3} \times 4{\cdot}44 \times 50 \times 0{\cdot}955 \times 0{\cdot}764 \times 10^6)}$$
$$= 78{\cdot}54 \text{ turns, say } 78 \text{ turns.}$$

Hence, total number of turns on the whole stator $= 3 \times 78$
$= 234.$

For mechanical reasons the slot pitch should not be less than about 1 cm. On the other hand the number of slots per pole per phase should be at least three, otherwise the leakage reactance will be too high. If the number of slots per pole per phase is three,

total number of slots on the stator $= 3 \times 3 \times 4 = 36$

Slot pitch $= \pi \times \frac{18}{36}$ cm. $= 1{\cdot}57$ cm. which is satisfactory. Therefore make **the number of stator slots** $=$ **36.**

Conductors per slot $= 234 \times \frac{2}{36} =$ **13**

Thus each phase will consist of 6 coils, each of 13 turns.

Using the revised value of $T_s = 234$,

Flux per pole $$= \frac{220 \times 10^8}{(\sqrt{3} \times 4{\cdot}44 \times 0{\cdot}955 \times 50 \times 78)}$$
$$= \mathbf{0{\cdot}768 \text{ megalines.}}$$

The **coil span** will be **9 slots** (*see* Fig. 55). A suitable winding would be a "mush" type or constant span and the diagram shows a developed view of the winding and phase connections.

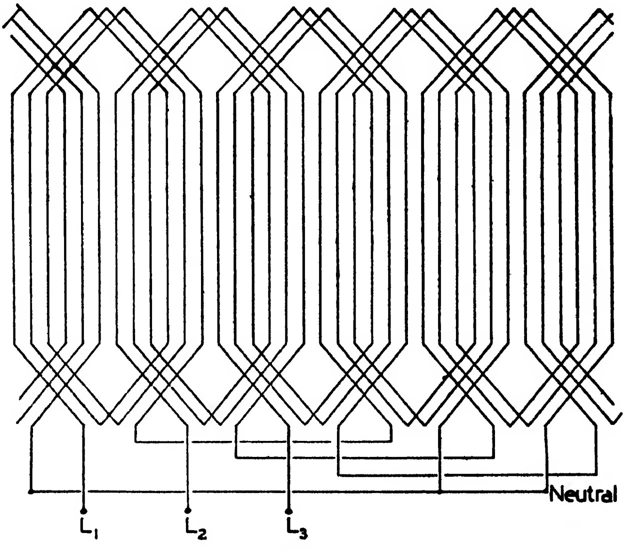


Fig. 55

Summarizing the winding details for this stator:

Total number of slots = 36
Conductors per slot = 13
Coils per phase = 6
Turns per phase = 78
Coil pitch = 9 slots.
Flux per pole = 0·768 megaline.

Winding: Lattice or mush type, single layer constant span.

CHAPTER V

SYNCHRONOUS MOTORS

(i) Input current, power factor, etc.

83. *A single-phase synchronous motor for use on a 500-volt circuit has a synchronous impedance of 3·2 ohms. The armature resistance is 0·2 ohm. To what voltage must the motor be excited so that it may develop 40 h.p. at unity power factor, the mechanical losses being 5 h.p.? What will be the armature current?* (*I.E.E., Pt. II, Nov.,* 1938)

Let I = the armature current when the motor is developing 40 h.p. The vector diagram is shown in Fig. 56.

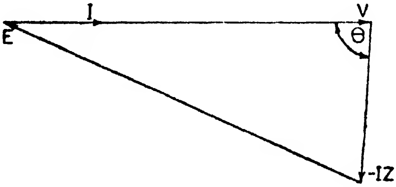


Fig. 56

Mechanical power developed including mechanical losses $= 40 + 5$ h.p. $= 45$ h.p.

$= 45 \times 746$ watts $= 33570$ watts.

Power input $= VI\cos\phi = 500I$ watts at unity power factor.

Copper losses $= I^2R = 0{\cdot}2I^2$

Mechanical power developed = Input power — copper losses.

Hence, $500I - 0{\cdot}2I^2 = 33570$

i.e. $I^2 - 2500I + 167850 = 0$

Solving for I, $I = 69$ **amperes = the armature current.**

Armature impedance drop $= IZ = 69 \times 3{\cdot}2$ volts $= 220{\cdot}8$ volts.

Internal phase angle $\theta = \text{arc cos}\,\dfrac{0{\cdot}2}{3{\cdot}2} = 86°25'$

If E is the induced e.m.f. in the armature, then from the vector diagram,

$$E^2 = V^2 + (IZ)^2 - 2V.IZ.\cos\theta$$
$$= 500^2 + 220{\cdot}8^2 - 2 \times 500 \times 220{\cdot}8 \times \cos 86°25'$$
$$= 250000 + 48760 - 13870 = 284890$$
$$E = \sqrt{284890} = 533{\cdot}8 \text{ volts.}$$

Hence, **motor must be excited to 533·8 volts.**

84. *A single-phase synchronous motor has a synchronous reactance of* X *ohms and negligible resistance. The terminal voltage is V* + *j*0 *and the induced e.m.f. E (j sin a — cos a). Deduce the current, power input and power factor.*

Determine the power factor and a *for an input of* 250 *kW, when* $V = 1000$, $X = 1{\cdot}2$ *ohms, and* $E = 1100$.

(*London B.Sc. Eng., July,* 1945)

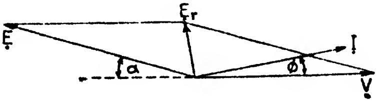


Fig. 57

The resultant voltage acting in the stator circuit is

$$\begin{aligned}\underset{\cdot}{E}_r &= \underset{\cdot}{V} + \underset{\cdot}{E}\\ &= V\,(1 + j0) + E\,(-\cos a + j \sin a)\\ &= (V - E \cos a) + jE \sin a\end{aligned}$$

The armature current under these conditions is

$$\begin{aligned}\underset{\cdot}{I} &= \frac{\underset{\cdot}{E}_r}{jX}\\ &= \frac{(V - E \cos a) + jE \sin a}{jX}\\ &= \frac{E}{X} \sin a - j\,\frac{V - E \cos a}{X}\end{aligned}$$

Hence
$$\begin{aligned}I &= \frac{1}{X}\sqrt{(E \sin a)^2 + (V - E \cos a)^2}\\ &= \frac{1}{X}\sqrt{V^2 + E^2 - 2VE \cos a}\end{aligned}$$

The phase angle $\varnothing$ is given by

$$\tan \varnothing = -\frac{V - E \cos a}{E \sin a}$$

Hence
$$\cos \varnothing = \frac{E \sin a}{\sqrt{V^2 + E^2 - 2VE \cos a}}$$

= **circuit power factor.**

$$\begin{aligned}\textbf{Power input} &= V\,I \cos \varnothing\\ &= V \times \frac{1}{X}\sqrt{V^2 + E^2 - 2VE \cos a} \times \frac{E \sin a}{\sqrt{V^2 + E^2 - 2VE \cos a}}\\ &= \frac{VE}{X} \sin a\end{aligned}$$

With the numerical values given,

$$250000 = \frac{1000 \times 1100}{1{\cdot}2} \sin a$$

$$\sin a = 0{\cdot}2727$$

Therefore $\mathbf{a = 15°49'}$

$$\textbf{Power factor} = \frac{1100 \sin 15°49'}{\sqrt{1000^2 + 1100^2 - 2 \times 1000 \times 1100 \times \cos 15°49'}}$$

$$= \mathbf{0{\cdot}979.}$$

85. *Explain with vector diagrams how the power factor of a synchronous motor working on a constant mechanical load depends on its excitation.*

A synchronous motor has an equivalent armature reactance of 3·3 *ohms. The exciting current is adjusted to such a value that the generated e.m.f. is* 950 *volts. Find the power factor at which the motor would operate when taking* 80 *kW from* 800-*volt supply mains.* (*C. and G. Final, Pt. I,* 1936)

In the vector diagram (Fig. 58),

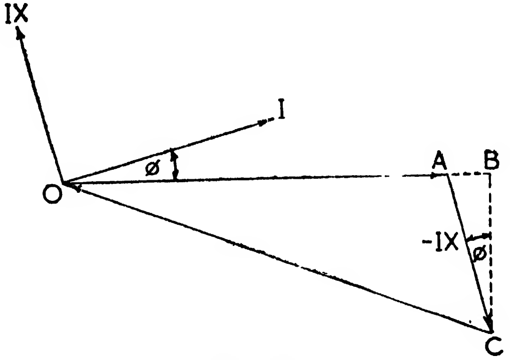


Fig. 58

OA = the input p.d. of 800 volts,
AC = the impedance drop in the armature
$= I \times X = 3{\cdot}3I$ volts.
CO = the generated e.m.f. of 950 volts.
Power input $= VI\cos\varnothing$
$= 800I\cos\varnothing = 80000$ watts.
Hence, $I\cos\varnothing = 100$ amperes.

If a perpendicular be dropped from C on OA produced then we have

$$OB^2 + BC^2 = CO^2$$
$$OB^2 + (AC.\cos\varnothing)^2 = 950^2$$
$$OB^2 + (3{\cdot}3I.\cos\varnothing)^2 = 950^2$$
$$OB^2 + 330^2 = 950^2$$

Whence $OB = 891$ volts.

Now $AB = OB - OA = 891 - 800 = 91$ volts.

$$\tan\varnothing = \frac{AB}{BC} = \frac{91}{330} = 0{\cdot}276$$
$$\varnothing = 15°26'$$

Therefore, **power factor of motor** = cos ø = **0·964 leading.**

86. *A* 2000-*volt,* 3-*phase, star-connected synchronous motor has an effective resistance and synchronous reactance of* 0·2 *ohm and* 2·2 *ohms respectively. The input is* 800 *kW at normal voltage and the generated line e.m.f. is* 2500 *volts. Calculate the line current and power factor.*

(*I.E.E., Pt. II, Nov.,* 1938)

$$\text{Impedance per phase} = \sqrt{0{\cdot}2^2 + 2{\cdot}2^2}$$
$$= 2{\cdot}21 \text{ ohms.}$$

$$\text{Applied p.d. per phase} = \frac{2000}{\sqrt{3}} = 1155 \text{ volts.}$$

$$\text{Induced e.m.f. per phase} = \frac{2500}{\sqrt{3}} = 1443 \text{ volts.}$$

$$\text{Internal phase angle of the motor } \theta = \text{arc tan } \frac{2{\cdot}2}{0{\cdot}2}$$
$$= 84^\circ 48'$$

From the vector diagram, Fig. 59

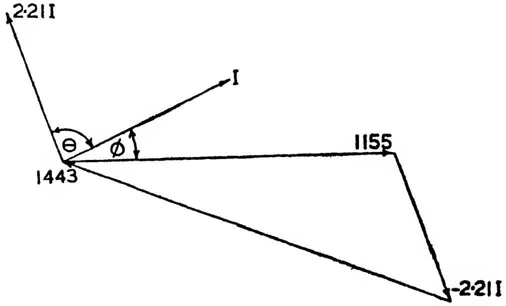


Fig. 59

$$1155^2 + (2{\cdot}21\text{I})^2 - (2 \times 1155 \times 2{\cdot}21\text{I} \times \cos(\theta + \varnothing)) = 1443^2$$

from which $\text{I}^2 - 1046\text{I}.\cos(\theta + \varnothing) - 153300 = 0$ (1)

Also, $\sqrt{3} \times 2000 \times \text{I} \times \cos\varnothing = 800000$ watts,

and $\text{I}\cos\varnothing = 231$ amperes.

Therefore $\cos\varnothing = \dfrac{231}{\text{I}}$, and $\sin\varnothing = \sqrt{1 - \left\{\dfrac{231}{\text{I}}\right\}^2}$ (2)

From (1), $\text{I}^2 - 1046\text{I}\,(\cos\theta.\cos\varnothing - \sin\theta.\sin\varnothing) - 153300 = 0$

From (2) $\text{I}^2 - 1046\text{I}\left\{\dfrac{0{\cdot}0906 \times 231}{\text{I}} - 0{\cdot}996.\dfrac{\sqrt{\text{I}^2 - 231^2}}{\text{I}}\right\} - 153300 = 0$

i.e. $\text{I}^2 - 21890 + 1042\sqrt{\text{I}^2 - 53360} - 153300 = 0$

$$\text{I}^2 - 175190 = -1042\sqrt{\text{I}^2 - 53360}$$

Squaring, $\text{I}^4 - 350380\text{I}^2 + 3{\cdot}069 \times 10^{10} = 1085000\text{I}^2 - 5{\cdot}792 \times 10^{10}$

Whence, $\text{I}^4 - 1435000\text{I}^2 + 8{\cdot}861 \times 10^{10} = 0$

Solving for I^2 $\quad \text{I}^2 = 65000$

$$\text{I} = \sqrt{65000} = 255 \text{ amperes}$$

$$\cos\varnothing = \frac{231}{\text{I}} = \frac{231}{255} = 0{\cdot}906$$

Therefore, **line current** = **255 amperes** and **power factor** = **0·906.**

87. *The input to an* 11000-*volt,* 3-*phase, star-connected synchronous motor is* 50 *amperes. The effective synchronous reactance and the resistance per phase are* 29 *ohms and* 0·95 *ohm respectively. Calculate the power supplied to the*

motor and the induced e.m.f. for a power factor of (*a*) 0·8 *lagging, and* (*b*) 0·8 *leading.* (*I.E.E., Pt. II, May,* 1938)

$$\text{Resistance drop per phase} = 50 \times 0{\cdot}95 = 47{\cdot}5 \text{ volts.}$$
$$\text{Reactance drop per phase} = 50 \times 29 = 1450 \text{ volts.}$$
$$\text{Input voltage per phase} = \frac{11000}{\sqrt{3}} = 6352 \text{ volts.}$$

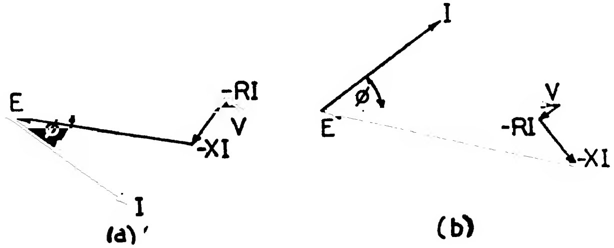


Fig. 60

(a) **Power factor 0·8 lagging.**

Power input $= \sqrt{3} \times 11000 \times 50 \times 0{\cdot}8 \times 10^{-3}$ kW
$= \mathbf{762{\cdot}1}$ **kW.**

From the vector diagram, Fig. 60 (a) the induced e.m.f. per phase E is given by

$$E^2 = (V - IR.\cos\varnothing - IX.\sin\varnothing)^2 + (IX.\cos\varnothing - IR.\sin\varnothing)^2$$
$$= (6352 - 47{\cdot}5 \times 0{\cdot}8 - 1450 \times 0{\cdot}6)^2 + (1450 \times 0{\cdot}8 - 47{\cdot}5 \times 0{\cdot}6)^2$$
$$= 5444^2 + 1131{\cdot}5^2$$

Whence, $E = 5562$ volts = induced e.m.f. per phase.

Therefore, **line induced e.m.f.** $= \sqrt{3} \times 5562 =$ **9632 volts.**

(b) **Power factor 0·8 leading.**

As in (a) the **power input** = **762·1 kW.**

Also IR = 47·5 volts, IX = 1450 volts, V = 6352 volts.

From the vector diagram Fig. 60(b) we see that

$$E^2 = (V - IR.\cos\varnothing + IX.\sin\varnothing)^2 + (IR.\sin\varnothing + IX.\cos\varnothing)^2$$
$$= (6352 - 38 + 870)^2 + (28{\cdot}5 + 1160)^2$$
$$= 7184^2 + 1188{\cdot}5^2$$
$$E = 7281 \text{ volts} = \text{the induced e.m.f. per phase.}$$

Hence, **line induced e.m.f.** $= \sqrt{3} \times 7281 =$ **12610 volts.**

88. *Construct curves showing the relation* (*a*) *between armature and field current;* (*b*) *between power factor and field current, for a star-connected synchronous motor with a synchronous reactance of* 8·25 *ohms per phase, and negligible resistance. The machine takes a constant power input of* 800 *kW at* 6.6 *kV. The open-circuit characteristic is as follows:*

Terminal voltage, kV	3	4	5	6	6·6	7	7·5	8
Field current, amperes	16	23	31	41	50	56	69	85

(*I.E.E., Pt. II, May,* 1942)

This problem is solved graphically as follows:

$$\text{Power input} = \sqrt{3} \times 6{\cdot}6 \times 10^3 \times I.\cos\varnothing = 800 \times 10^3 \text{ watts}$$

where I = the armature current

$\varnothing$ = the load phase angle

Hence, $I \times \cos\varnothing = 70$ amperes.

$$\text{Internal reactance drop} = \sqrt{3}.IX = \sqrt{3} \times \left\{\frac{70}{\cos\varnothing}\right\} \times 8{\cdot}25 \text{ volts}$$

$$= \frac{1000}{\cos\varnothing} \text{ volts.}$$

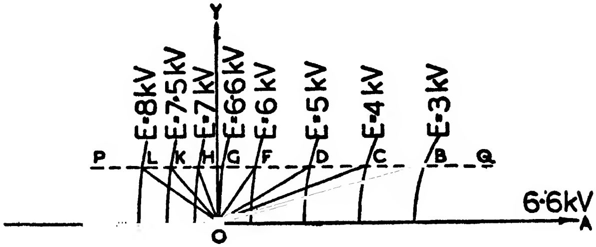


Fig. 61

Referring to Fig. 61, OA is drawn to represent the applied p.d. of 6·6 kV. Parallel to this line and at a perpendicular distance from it to represent 1000 volts another line PQ is drawn. This line is the locus of the end of the reactance drop vector since any line joining O to a point on PQ has a length representing $\frac{1000}{\cos\varnothing}$ volts where $\varnothing$ is the angle between the line and OY. $\varnothing$ is also the angle between OA and the perpendicular to the reactance drop vector, i.e. the current.

e.g. When $\varnothing = \angle YOB$, the reactance drop = OB. Then OI_B lagging OB by 90° represents the current and $\angle AOI_B = \varnothing$

With centre A and radii representing 3, 4, 5, . . . 8 kV respectively, circles are drawn to cut PQ at B, C, D, . . . L. These points are joined to O and OB, OC, OD, . . . OL represent the reactance drops for the conditions when the generated e.m.f.s are AB, AC, AD, . . . AL, i.e. 3, 4, 5, . . . 8 kV respectively.

$$\text{Then the armature current} = \left\{\frac{\text{Reactance drop}}{\sqrt{3}}\right\} \div 8{\cdot}25$$

$$= \frac{\text{Reactance drop}}{14{\cdot}29} \text{ amperes.}$$

The load phase angles for these generated e.m.f.s are the angles BOY, COY, DOY, . . . LOY respectively and the cosines of these angles are the corresponding power factors.

The following table may be constructed from the results of measurements taken from the diagram:

Field current, amperes	16	23	31	41	50	56	69	85
Open circuit e.m.f., kV.	3	4	5	6	6·6	7	7·5	8
Reactance drop, volts	3800	2880	1960	1180	1020	1040	1300	1660
Armature current, amperes	266	202	137	82·6	71·4	72·8	99·7	127
Phase angle, ∅	75°	69°30′	59°30′	32°	11°30′	16°	39°30′	53°
Power factor, cos ∅	0·26	0·35	0·51	0·85	0·98	0·96	0·77	0·60
	lagging					leading		

This table contains all the necessary data for the required curves to be plotted (Fig. 62).

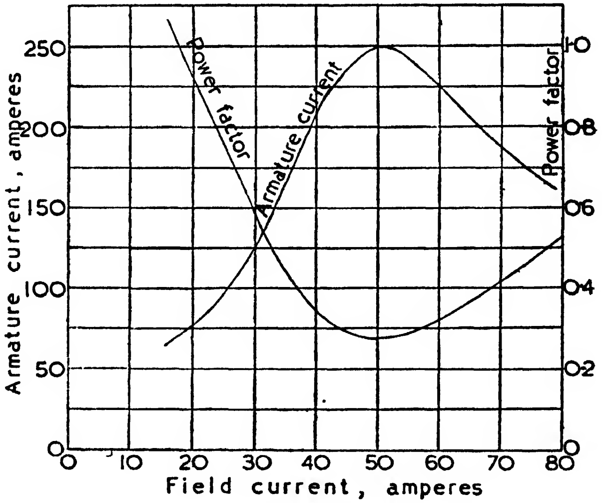


Fig. 62

89. *A star-connected, 3-phase synchronous motor taking on full-load 93 amperes at 500 volts and unity power factor has an armature resistance and reactance per phase of 0·03 and 0·3 ohm respectively. Calculate for full-load current and 0·8 power factor leading, the total mechanical power developed in kW and the e.m.f. generated by the motor. Assume an efficiency of 94 per cent.* (*I.E.E., Pt. II, May,* 1937)

At 0·8 power factor,

$$\text{Input current} = \frac{93}{0\cdot8} \text{ amperes} = 116\cdot25 \text{ amperes.}$$

$$\text{Power input} = \sqrt{3} \times 500 \times 93 \text{ watts} = 80540 \text{ watts}$$

$$= 80\cdot54 \text{ kW.}$$

$$\text{Total copper loss} = 3 \times (116\cdot25)^2 \times 0\cdot03 \times 10^{-3} \text{ kW}$$

$$= 1\cdot22 \text{ kW.}$$

Hence, **total mechanical power developed** (including core loss) = Input − copper losses

= (80·54 − 1·22) kW

= **79·32 kW.**

Net mechanical power developed = Input × efficiency,

= 80·54 × 0·94 kW

= **75·7 kW.**

Impedance per phase $= \sqrt{(0{\cdot}03)^2 + (0{\cdot}3)^2}$

= 0·3015 ohm.

Line impedance drop on full-load 0·8 power factor $= \sqrt{3} \times 116{\cdot}25 \times 0{\cdot}3015$

= 60·73 volts.

In the vector diagram (Fig. 63) OA represents the input p.d. of 500 volts, OI represents the input current leading OA by an angle ∅ = arc cos 0·8 = 36°52′, OB represents the line impedance drop of 60·73 volts, leading OI by an angle arc $\tan\frac{0{\cdot}3}{0{\cdot}03} = 84°18'$. Then AB represents the e.m.f. generated by the motor. Therefore

Fig. 63

$$\text{AB}^2 = \text{OA}^2 + \text{OB}^2 - 2.\text{OA}.\text{OB}.\cos \angle\text{AOB}$$
$$= 500^2 + 60{\cdot}73^2 - 2 \times 500 \times 60{\cdot}73 \times \cos(36°52' + 84°18')$$
$$= 500^2 + 60{\cdot}73^2 - 2 \times 500 \times 60{\cdot}73 \times (-\,0{\cdot}5175)$$
$$= 285118$$
$$\text{AB} = \sqrt{285118} = 533{\cdot}9 \text{ volts.}$$

i.e. **e.m.f. generated by the motor = 533·9 volts.**

(ii) The circle diagram, torque, excitation, etc.

90. *What are the relative merits of salient-pole synchronous motors and synchronous induction motors? Show how the rotor of the latter machine may be connected for d.c. excitation. Find the d.c. excitation needed to run a* 40-*h.p.*, 400-*volt*, 3-*phase induction motor on full-load at synchronous speed at a power factor of* 0·9 *leading, if its no-load current as an induction motor is* 16 *amperes at a power factor of* 0·2, *and its full-load current is* 60 *amper s at a power factor of* 0·85. *The ratio of stator to rotor turns per phase is* 2·0. *Find also the pull-out torque in synchronous kilowatts.*

(*I.E.E.*, *Pt. II*, *May*, 1943)

The circle diagram for this machine as an induction motor (Fig. 64(a)) is drawn as described in Chapter IV, except that the full-load current is given instead of the short-circuit current.

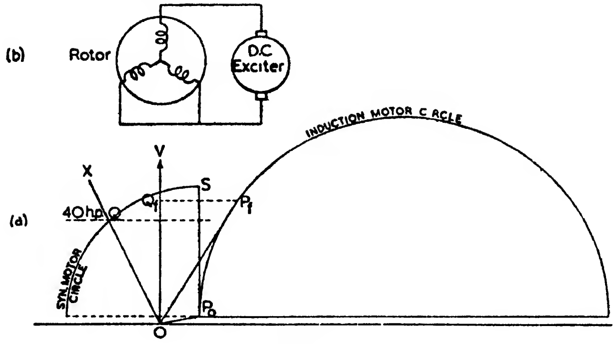


Fig. 64

In this diagram,

$OP_o = 16$ amperes (no-load) at a phase angle VOP_o = arc cos 0·2

$OP_f = 60$ amperes (full-load) at a phase angle VOP_f = arc cos 0·85

The current scale assumed for the diagram is 1 inch = 25 amperes.

Therefore the power scale is 1 inch $= \sqrt{3} \times 400 \times 60 \times \frac{0{\cdot}85}{2{\cdot}04}$ watts because P_fQ_f (2·04 inches) represents a power input of $(\sqrt{3} \times 400 \times 60 \times 0{\cdot}85)$ watts.

i.e. power scale is 1 inch = 17320 watts.

A horizontal line is drawn through P_o and above it at a vertical distance of 1·72 inches, to represent 40 h.p. (29840 watts) another horizontal line is drawn. This line meets OX (drawn through O at a leading phase angle of arc cos 0·9) at Q.

The synchronous motor circle is drawn with centre P_o and radius P_oQ

$P_oQ = 2{\cdot}3$ inches, representing 57·5 amperes. This is the a.c. rotor current equivalent to the d.c. exciting current. The d.c. excitation bears a fixed relation to this a.c. depending on the method of connecting the rotor. For the connections assumed (Fig. 64(b)) the equivalent d.c. excitation is

$$I_{DC} = \frac{\sqrt{2}I_{AC} \times \text{stator turns}}{\text{rotor turns}}$$
$$= \sqrt{2} \times 57{\cdot}5 \times 2 = 162{\cdot}6 \text{ amperes.}$$

i.e. **d.c. excitation required = 162·6 amperes.**

To obtain the pull-out torque a vertical line is drawn through P_o meeting the synchronous motor circle at S. P_oS is the pull-out torque to scale in synchronous watts. From the diagram,

$$P_oS = 2{\cdot}3 \text{ inches} = 2{\cdot}3 \times 17320 \text{ watts}$$
$$= 40 \text{ kW.}$$

Therefore, **pull-out torque = 40 synchronous kilowatts.**

91. *The following test figures relate to a* 500-*h.p.*, 2000-*volt*, 50-*cycle*, 8-*pole*, 3-*phase synchronous motor*, *driving its own exciter:*

As induction motor:
No-load: 2000 *volts,* 90 *amperes, power factor* 0·045.
Short-circuit: 500 *volts,* 180 *amperes, power factor* 0·14.
Ratio rotor to stator copper loss, 1 *to* 4.

As synchronous motor:
No-load: 2000 *volts,* 90 *amperes, power factor* 0·08.

Draw the current diagram and find (*a*) *the full-load current and power factor*; (*b*) *the synchronous overload capacity*; (*c*) *the greatest torque in lb.-ft. against which the motor is capable of starting from rest.*

(*I.E.E., Pt. II, Nov.,* 1937)

From the no-load and short-circuit data for the induction motor the circle diagram (Fig. 65) is drawn through P_0 (no-load) and P_s (short-circuit) as described in Chapter IV. H divides P_sG, the total stator copper loss on short-circuit in the ratio $P_sH : HG :: 1 : 4$. Then P_0P_s is the output line and P_0H is the torque line for the machine functioning as an induction motor.

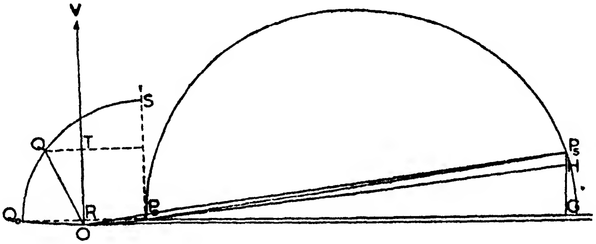


Fig. 65

The scales assumed for this diagram are:
Current: 1 inch = 100 amperes.
Power: 1 inch = $\sqrt{3} \times 2000 \times 100$ watts = 346400 watts.
Torque: 1 inch = 346400 synchronous watts.

As a synchronous motor OQ_0 represents the no-load condition where $OQ_0 = 90$ amperes and $\angle VOQ_0 = \text{arc cos } 0{\cdot}08$ (leading) $= 85°24'$. The current circle is drawn with centre P_0 and radius P_0Q_0. The horizontal line Q_0R is drawn and above it and parallel with it another line through T is drawn at a distance RT which represents 500 h.p. (full-load) to the power scale.

$$RT = 500 \times \frac{746}{346400} = 1{\cdot}08 \text{ inches.}$$

The horizontal line through T cuts the synchronous motor circle at Q. Then OQ represents full-load current in magnitude and phase.

By measurement from the diagram,

(a) **full-load current** OQ = **126 amperes,**
full-load power factor = cos VOQ = cos 24°30′ = **0·91 leading.**

(b) The pull-out load, i.e. the load beyond which the motor fails to run in synchronism, is represented by P_0S where S is the point where the synchronous motor circle cuts the vertical line through P_0.

By measurement from the diagram, $P_oS = 627$ kW $= 840$ h.p.

Synchronous over-load capacity $= \dfrac{840 - 500}{500} \times 100$ per cent

$=$ **68 per cent.**

(c) The greatest torque against which the motor is capable of starting from rest is found from the induction motor circle. It is represented by P_8H. By measurement from the diagram,

$$P_8H = 67000 \text{ synchronous watts.}$$

To convert this to lb.-ft., 1 synchronous watt $= \dfrac{33000}{(746 \times 2\pi N_s)}$ lb.-ft.

where N_s is the synchronous speed. For 8 poles and a frequency of 50 cycles per second, $N_s = 750$ r.p.m. Hence,

$$P_8H = \frac{67000 \times 33000}{(746 \times 2\pi \times 750)} = 629{\cdot}1 \text{ lb.-ft.}$$

i.e. **Motor will start from rest against a maximum torque of 629·1 lb.-ft.**

CHAPTER VI

INDUCTION REGULATORS

(i) Voltage variation and kVA rating.

92. *Describe the construction and action, and state the principal uses, of the induction regulator.*

A single-phase induction regulator has a pulsating flux of 1·5 *megalines. There are* 15 *coils each of* 2 *turns spread over* 120°. *Find the maximum and minimum voltages available if the regulator is used on a* 440*-volt supply.*
(*C. and G. Final, Pt. II*, 1943)

For a winding spread of 120° the distribution factor $k_m = 0{\cdot}827$. The number of turns in series per phase $= T_s = 30$ turns.

e.m.f. induced in the rotor $E_r = 4{\cdot}44 k_m.f\Phi T_s.10^{-8}$ volts,
$= 4{\cdot}44 \times 0{\cdot}827 \times 50 \times 1{\cdot}5 \times 10^6 \times 30 \times 10^{-8}$
$= 82{\cdot}62$ volts.

Maximum voltage available = Supply voltage + E_r
$= 440 + 82{\cdot}62$ volts
= **522·6 volts.**

Minimum voltage available = Supply voltage − E_r
$= 440 - 82{\cdot}62$ volts
= **357·4 volts.**

Note.—A supply frequency of 50 cycles per second has been assumed.

93. *It is desired to regulate the voltage on a 3-phase feeder dealing with* 1200 *kVA between the limits of* 10000 *and* 12000 *volts by means of an induction regulator. Calculate the rating of the regulator. How could this machine be made to function automatically?* (*I.E.E., Pt. II, Nov.,* 1939)

Total voltage variation = 12000 − 10000 volts
= 2000 volts
= ± 1000 volts about a mean value of 11000 volts.

Line current when line voltage is at its mean value of 11000 volts $= \dfrac{1200 \times 10^3}{\sqrt{3} \times 11 \times 10^3}$

$= \dfrac{1200}{11\sqrt{3}}$ amperes.

Rating of regulator $= \sqrt{3} \times$ line current $\times$ maximum line voltage variation up or down,

$$= \sqrt{3} \times \frac{1200}{11\sqrt{3}} \times 1000 \times 10^{-3} \text{ kVA}$$

$$= 109 \text{ kVA.}$$

i.e. **regulator will be rated at 109 kVA.**

94. *Discuss the relative merits of the induction regulator and the tapped transformer for voltage regulation.*

The voltage at the receiving end of a 3-phase feeder delivering 1000 *kVA varies between* 3000 *and* 3500 *volts. Determine the kVA rating of an induction regulator to maintain the voltage constant at* 3300 *volts.*

(*I.E.E., Pt. II, Nov.*, 1937)

$$\text{Full-load line current} = \frac{1000 \times 10^3}{(\sqrt{3} \times 3300)} \text{ amperes}$$

$$= 175 \text{ amperes.}$$

The regulator must increase the line voltage by 200 volts and reduce it by 300 volts. It will therefore be rated on the larger voltage variation of 300 volts.

kVA rating of regulator $= \sqrt{3} \times 175 \times 300 \times 10^{-3}$ kVA

$= \mathbf{90{\cdot}9\ kVA.}$

SYNCHRONOUS CONVERTORS

(i) a.c./d.c. voltage and current relations.

95. *Deduce the relation between the direct voltage and the slip-ring voltage in a 6-ring, diametrically connected synchronous convertor.*

Draw a connection diagram for a 3-wire direct-current supply with 440 *volts across outers, and a 3-phase high-voltage supply of* 11000 *volts. Mark in the several voltages and determine the transformer turn ratio.*

The transformer is mesh-connected on the high-voltage side.

(*I.E.E., Pt. II, May*, 1939)

Fig. 66 shows the diagram of connections for the 3-wire supply.

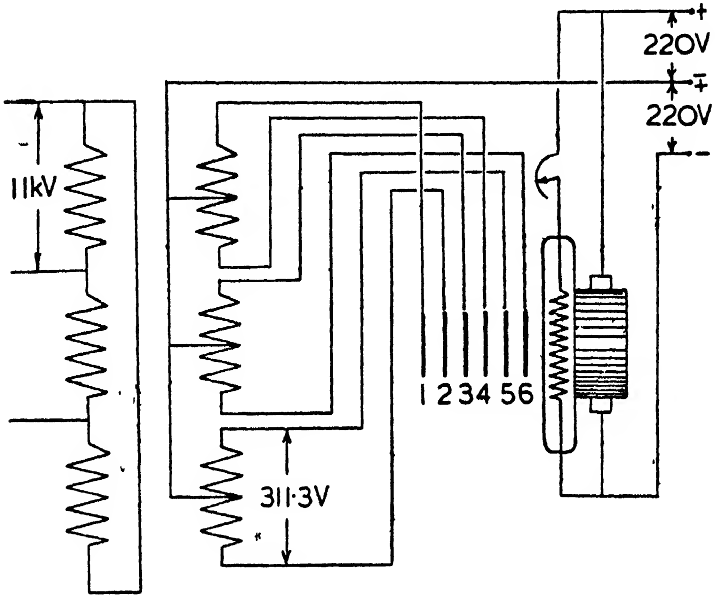


Fig. 66

The relation between the transformer input voltage to the convertor and the d.c. output voltage when the convertor is diametrically connected is, neglecting the voltage drop in the armature,

$$V_{AC} = \frac{V_{DC}}{\sqrt{2}}$$

where V_{AC} = input voltage across diametral slip-rings,
V_{DC} = output voltage across commutator brushes.

In this problem, V_{DC} = 440 volts, therefore

$$V_{AC} = \frac{440}{\sqrt{2}} = \mathbf{311{\cdot}3\ volts.}$$

This is the **transformer secondary phase voltage.**

Transformer primary phase voltage = 11000 volts.

Hence, **turns ratio per phase** $= \frac{11000}{311 \cdot 3} =$ **35·34 to 1.**

Note: If the transformer secondary is regarded as a 6-phase winding, the phase turns ratio is 70·68 to 1.

96. *Draw the essential connections for a self-synchronizing* 6-*ring synchronous convertor connected to a* 3-*wire direct-current network and supplied from* 3-*phase high-voltage mains.*

If the voltage between the outers of the d.c. mains is 500 *volts and that of the* 3-*phase system is* 6600 *volts, calculate the turn ratio of the transformer windings and the voltage between opposite and between adjacent slip-rings. The machine has diametral tappings and the transformer is delta-connected on the high-voltage side.* (*C. and G. Final, Pt. I,* 1936)

The diagram of connections is given in Fig. 66.

The transformer secondary phase voltage for diametral connection is

$$V_{AC} = \frac{V_{DC}}{\sqrt{2}} = \frac{500}{\sqrt{2}} = 353 \cdot 5 \text{ volts.}$$

The transformer primary phase voltage is 6600 volts.

Therefore, **turns ratio per phase** $= \frac{6600}{353 \cdot 5} =$ **18·66 to 1.**

Voltage between opposite rings $= V_{AC} =$ **353·5 volts.**

Voltage between adjacent rings $= \dfrac{V_{DC} \sin \frac{\pi}{N}}{\sqrt{2}}$

where N = number of rings,

$$= \frac{500 \sin \frac{\pi}{6}}{\sqrt{2}}$$

= **176·8 volts.**

97. *A* 6-*ring synchronous convertor is fed from* 33-*kV,* 3-*phase supply mains through a transformer connected star-double delta. The output of the convertor is* 250 *kW at* 500 *volts and its efficiency is* 90 *per cent. Assuming that the excitation is adjusted to give unity power factor on the a.c. side calculate the primary and secondary phase voltages and currents in the transformer. Neglect transformer losses and magnetizing current.*

Draw a diagram of the essential connections. (*H.N.C.,* 1943)

Fig. 67 is the diagram of connections for this problem.

In this method of connection there are two banks of secondaries each delta-connected, and each phase of the respective banks is connected to alternate slip-rings, e.g. Bank 1: 1—3, 3—5, 5—1; Bank 2: 6—4, 4—2, 2—6.

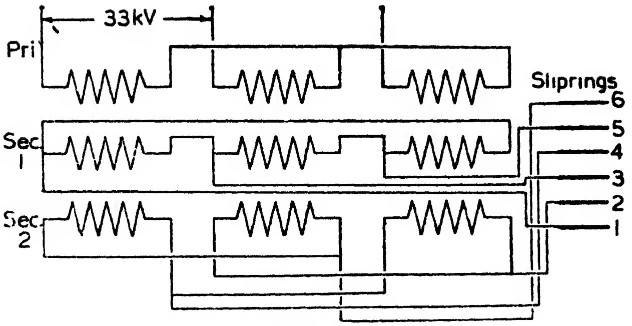


Fig. 67

The transformer secondary phase voltage may therefore be calculated by treating the convertor as a 3-ring one, thus

$$\text{transformer secondary phase voltage} = \frac{V_{DC}\sin\frac{\pi}{N}}{\sqrt{2}}$$

$$= \frac{500\sin\frac{\pi}{3}}{\sqrt{2}} = 306{\cdot}5 \text{ volts.}$$

The slip-ring current, which is the secondary **line** current, is calculated from the following expression :

$$\text{Slip-ring current} = I_{DC}\cdot\frac{2\sqrt{2}}{N}\cdot\frac{1}{\eta\cos\varnothing}$$

where N = total number of rings = 6
η = convertor efficiency = 0·9
$\cos\varnothing$ = convertor power factor = 1·0

$$\text{Hence, slip-ring current} = \left(\frac{250000}{500}\right)\times\left(\frac{2\sqrt{2}}{6}\right)\times\left(\frac{1}{0{\cdot}9}\right)$$

$$= 261{\cdot}8 \text{ amperes.}$$

$$\text{Therefore secondary phase current} = \frac{261{\cdot}8}{\sqrt{3}} \text{ amperes} = 151{\cdot}2 \text{ amperes}$$

$$\text{Transformer primary phase voltage} = \frac{33000}{\sqrt{3}} = 19000 \text{ volts.}$$

Line and phase current on primary side = Convertor input power ÷ $\sqrt{3} \times 33000 \times 1{\cdot}0$ assuming that the primary power factor is the same as that of the convertor, i.e. ignoring magnetizing current.

$$\text{Hence, primary phase current} = \left(\frac{250000}{0{\cdot}9}\right)\times\frac{1}{(\sqrt{3}\times 33000)}$$

$$= 4{\cdot}86 \text{ amperes.}$$

Summarizing,—**primary: 19000 volts, 4·86 amperes per phase,**
secondary: 306·5 volts, 151·2 amperes per phase.

98. *A* 1500-*kW*, 8-*pole*, 50-*cycle*, 6-*ring*, *diametrically connected synchronous convertor is required to supply a* 500-*volt direct-current network from an* 11000-*volt*, 3-*phase system. If the transformers between the a.c. lines and the slip-rings are mesh-connected on the high-voltage side, determine the number of turns on each winding and the cross-section of the conductors. Take the current density as* 3 *amperes per sq. mm., the total flux as* 8·5 × 10⁶ *lines, and neglect losses. Assume a power factor of* 0·9.

(*I.E.E., Pt. II, Nov.*, 1939)

Transformer secondary phase voltage = voltage between diametral slip-rings

$$= \frac{V_{DC}}{\sqrt{2}} \text{ volts,}$$

$$= \frac{500}{\sqrt{2}} = 353{\cdot}5 \text{ volts.}$$

Transformer primary phase voltage = a.c. line voltage = 11000 volts.

Now, for each winding of the transformer

$$\text{Induced e.m.f.} = 4{\cdot}44\ f\Phi T.\ 10^{-8} \text{ volts.}$$

Assuming that the induced e.m.f. is the same as the terminal voltage, i.e. neglecting resistance and leakage reactance in the windings, we have for the primary,

$$11000 = 4{\cdot}44 \times 50 \times 8{\cdot}5 \times 10^6 \times T_p \times 10^{-8}$$

$T_p = 582{\cdot}9$ or **583 primary turns.**

Similarly, for the secondary,

$$353{\cdot}5 = 4{\cdot}44 \times 50 \times 8{\cdot}5 \times 10^6 \times T_s \times 10^{-8}$$

whence, $T_s = 18{\cdot}74$ or **19 secondary turns.**

Neglecting losses in the transformers and convertor, each transformer supplies one-third of the total power input of the convertor, i.e.

Power supplied by each transformer = 500 kW.

Let I_{AC} = the slip-ring current, i.e. the secondary current of each transformer.

Then, $353{\cdot}5 \times I_{AC} \times 0{\cdot}9 = 500000$

$I_{AC} = 1571$ amperes.

Cross-section of secondary conductors $= \frac{1571}{3}$ sq. mm.

= **524 sq. mm.**

Neglecting the magnetizing current on the primary side the power factor will be the same as for the secondary.

$$\text{Primary line current} = \frac{1500000}{(\sqrt{3} \times 11000 \times 0{\cdot}9)}$$

$= 87{\cdot}5$ amperes.

$$\text{Primary phase current} = \frac{87{\cdot}5}{\sqrt{3}} = 50{\cdot}52 \text{ amperes.}$$

Cross-section of primary conductors $= \frac{50{\cdot}52}{3}$ sq. mm.

= **16·84 sq. mm.**

99. *A* 6-*ring*, 12-*pole, lap-wound synchronous convertor with diametral tappings runs at* 600 *r.p.m. and supplies a direct-current of* 2000 *amperes. There are* 1200 *armature conductors and the flux per pole is* 5 *megalines. The transformer is delta-connected to a* 6600-*volt*, 3-*phase supply. Assuming unity power factor and an armature efficiency of* 96 *per cent, calculate* (*a*) *the turn ratio of the transformer and the voltage between adjacent rings*, (*b*) *the current per ring, and* (*c*) *the current in the armature connection between a slip-ring and a tapping point. Draw a diagram of the essential connections.*

(*C. and G. Final, Pt. I*, 1939)

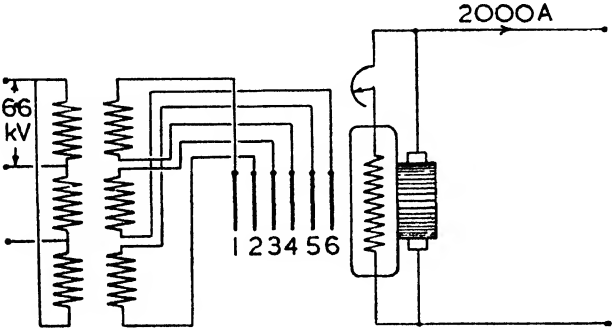


Fig. 68

The d.c. output voltage of the convertor is calculated from the data given, treating the machine as a d.c. generator.

$$\text{d.c. voltage} = \frac{p.N.Z.\Phi.10^{-8}}{60\,a}\ \text{volts,}$$

where p = 12 poles, N = 600 r.p.m., Z = 1200 conductors, $\Phi = 5 \times 10^6$ lines per pole, a = 12 parallel paths between the brushes.

$$\text{d.c. voltage} = 12 \times 600 \times 1200 \times 5 \times 10^6 \times \frac{10^{-8}}{60 \times 12}$$

$$= 600\ \text{volts.}$$

$$\text{Hence, transformer phase secondary voltage} = \frac{600}{\sqrt{2}}\ \text{volts}$$

$$= 424{\cdot}3\ \text{volts.}$$

$$\text{Transformer primary phase voltage} = 6600\ \text{volts.}$$

(a) Therefore, **turns ratio per phase** $= \dfrac{6600}{424{\cdot}3}$

$= \mathbf{15{\cdot}55\ to\ 1.}$

Voltage between adjacent rings $= \dfrac{V_{DC} \sin \frac{\pi}{N}}{\sqrt{2}}$

$= 424{\cdot}3 \sin \frac{\pi}{6}$

$= \mathbf{212{\cdot}1\ volts.}$

(b) **Slip-ring current** $= I_{DC} \times \left(\dfrac{2\sqrt{2}}{N}\right) \times \left(\dfrac{1}{\eta.\cos\varnothing}\right)$

where $I_{DC} = 2000$ amperes, $N = 6$ rings, $\eta = 0{\cdot}96$ efficiency, $\cos \varnothing$ = unity,

$$= 2000 \times \left(\frac{2\sqrt{2}}{6}\right) \times \left(\frac{1}{0{\cdot}96}\right)$$

$$= \mathbf{982\ amperes.}$$

(c) In a lap-wound machine each slip-ring is connected to as many tapping points on the armature as there are pairs of poles, i.e. six in this case. Therefore the current between a slip-ring and a tapping point is one-sixth of the total slip-ring current.

Current between a slip-ring and a tapping point $= \frac{982}{6}$ amperes

$$= \mathbf{163{\cdot}7\ amperes}$$

(ii) **Armature current waveform, heating.**

100. *A single-phase synchronous convertor has a wavewound armature and operates at unity power factor. Assuming that its efficiency is* 100 *per cent, plot curves showing the waveform of the resultant armature current*

(*a*) *in the coil midway between the tapping points,*

(*b*) *in the coil which is tapped.*

Also in (*a*) *and* (*b*) *calculate the ratio between the copper loss due to the direct current alone and that due to the resultant current.*

As the armature has a wave winding there are two paths in parallel between the brushes. Let the direct current in each path be I_D, corresponding to a commutator brush current of $2I_D$. Then

$$\text{slip-ring current} = (2I_D) \times \frac{2\sqrt{2}}{N} \times \frac{1}{\eta.\cos\varnothing}$$

$$= (2I_D) \times \left(\frac{2\sqrt{2}}{2}\right) = 2\sqrt{2}I_D$$

for 2 rings (single-phase), unity power factor, and 100 per cent efficiency.

With the armature tapped for single-phase current the current in each conductor is one-half of the slip-ring current. Calling this I_A

$$I_A = \sqrt{2}I_D$$

This is the R.M.S. value of the alternating current in the armature conductors so its maximum value will be $2I_D$.

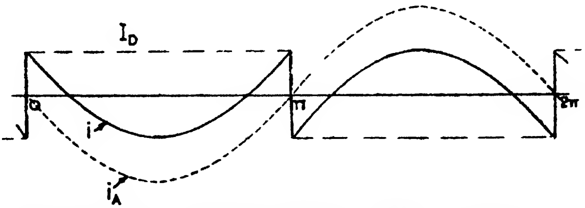
Regarding I_D as an alternating current of rectangular waveform and I_A as of sine waveform the resultant current in an armature conductor at any instant is the algebraic sum of the two.

(a) **In the coil midway between the tapping points,** I_D reverses at the instant when the coil passes a commutator brush. Also at the same instant I_A is zero and reverses because one-half of the phase is under one pole and the other half is under the other pole. Hence the slip-ring voltage and current are zero at this instant (provided the power factor is unity). The resultant current is therefore the algebraic sum of two waveforms having the same fundamental frequency but 180° out of phase. The 180° phase displacement arises because the convertor is motoring from the a.c. side and generating on the d.c. side.

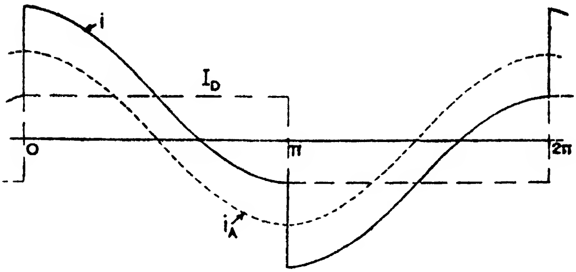
Fig. 69(a) shows the resultant current for this coil.

(b) **In the coil which is tapped,** when I_D reverses, i.e. as the coil passes a brush, the alternating component in the coil is at its maximum value

because all the conductors in the phase are lying beneath the same pole. Therefore I_A does not reverse until 90° later and the two waveforms are 90° out of phase instead of 180°. Fig. 69(b) shows the waveform of the resultant current in this case.



(a) Conductor midway between tapping points



(b) Conductor at a tapping point

Fig. 69

Copper loss ratios.

(a) The equation of the resultant current over the first half-cycle of the waveform is

$$i = I_D - 2I_D \sin\theta$$

Hence,
$$\begin{aligned} i^2 &= I_D^2\,(1 - 2\sin\theta)^2 \\ &= I_D^2\,(1 - 4\sin\theta + 4\sin^2\theta) \\ &= I_D^2\,(3 - 4\sin\theta - 2\cos2\theta) \end{aligned}$$

$$\text{Mean value of } i^2 = \frac{\int_0^\pi I_D^2\,(3 - 4\sin\theta - 2\cos2\theta)\,d\theta}{\pi}$$

$$= \frac{I_D^2\,(3\theta + 4\cos\theta - \sin2\theta)_0^\pi}{\pi}$$

$$= \frac{I_D^2\,(3\pi - 8)}{\pi} = 0{\cdot}454 I_D^2$$

If I is the R.M.S. value of the resultant current, $I^2 = 0{\cdot}454 I_D^2$

i.e. $$\frac{\textbf{Copper loss due to resultant current}}{\textbf{Copper loss due to d.c. alone}} = \frac{I^2}{I_D^2} = 0{\cdot}454$$

(b) The equation for the resultant current is

$$\begin{aligned} i &= I_D + 2I_D\cos\theta \\ &= I_D(1 + 2\cos\theta) \\ i^2 &= I_D^2(1 + 4\cos\theta + 4\cos^2\theta) \\ &= I_D^2(3 + 4\cos\theta + 2\cos2\theta) \end{aligned}$$

$$\text{Mean value of } i^2 = \frac{\int_0^\pi I_D^2(3 + 4\cos\theta + 2\cos2\theta)\,d\theta}{\pi}$$

$$= \frac{I_D^2(3\theta + 4\sin\theta + \sin 2\theta)_0^\pi}{\pi}$$

$$= \frac{I_D^2(3\pi)}{\pi} = 3I_D^2$$

Hence, $I^2 = 3I_D^2$

i.e. $\dfrac{\textbf{Copper loss due to resultant current}}{\textbf{Copper loss due to d.c. alone}} = \dfrac{I^2}{I_D^2} = \mathbf{3{\cdot}0}$

101. *If a 3-phase, 6-ring synchronous convertor has an efficiency of 92 per cent at full-load and the power factor is adjusted to unity, compare the heating of an armature conductor adjacent to one of the slip-ring tappings with the heating of the same conductor when the machine is run as a dynamo with the same output. Sketch the wave shape of the current in the two cases.*

(*C. and G. Final, Pt. II,* 1945)

In a conductor adjacent to a slip-ring tapping point the alternating current component reverses 30° after the direct current component. The resultant current in the conductor is the algebraic sum of the two components instant by instant and has the waveform shown in Fig. 70. When the machine is being run as a dynamo the alternating component is absent and the current in the conductor is of rectangular waveform.

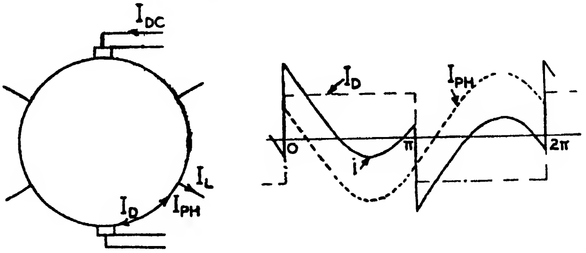


Fig. 70

Let I_L = the R.M.S. slip-ring current.
I_{PH} = the alternating component of the current in the conductor.
I_{DC} = the direct current output, i.e. the brush current.
I_D = the direct component of the current in the conductor
= 0·5 I_{DC}, assuming the armature is wave-wound.
N = number of slip-rings.

η = the efficiency of the convertor.

$\cos \varnothing$ = the power factor of the convertor.

Then $$\mathrm{I_L} = \mathrm{I_{DC}} \frac{2\sqrt{2}}{\mathrm{N}.\eta.\cos \varnothing}$$

Peak value of $$\mathrm{I_L} = \mathrm{I_{DC}} \frac{2\sqrt{2} \times \sqrt{2}}{\mathrm{N}.\eta.}$$ since the power factor is unity,

$$= \mathrm{I_{DC}} \frac{4}{6 \times 0{\cdot}92}$$

$$= 0{\cdot}725\ \mathrm{I_{DC}}$$

Now $$\mathrm{I_{PH}} = \frac{\mathrm{I_L}}{2 \sin \frac{\pi}{\mathrm{N}}}$$

$$= \frac{\mathrm{I_L}}{2 \sin \frac{\pi}{6}}$$

$$= \mathrm{I_L}$$

Therefore,

$$\text{peak value of } \mathrm{I_{PH}} = \text{peak value of } \mathrm{I_L}$$

$$= 0{\cdot}725\ \mathrm{I_{DC}}$$

$$= 1{\cdot}45\ \mathrm{I_D}$$

Hence the equation for the waveform of the alternating current component in the conductor is

$$\mathrm{i_{PH}} = -\,1{\cdot}45\ \mathrm{I_D} \sin \left\{\theta - \frac{\pi}{6}\right\}$$

where $\theta = 0$ at the instant the direct current reverses and becomes positive.

The resultant current in the conductor is now given by

$$\mathrm{i} = \mathrm{I_D} - 1{\cdot}45\ \mathrm{I_D} \sin\ \theta - \frac{\pi}{6}$$

$$\mathrm{i}^2 = \mathrm{I_D^2} \left\{1 - 1{\cdot}45 \sin \left(\theta - \frac{\pi}{6}\right)\right\}^2$$

$$= \mathrm{I_D^2} \left\{1 - 2{\cdot}9 \sin \left(\theta - \frac{\pi}{6}\right) + 2{\cdot}11 \sin^2 \left(\theta - \frac{\pi}{6}\right)\right\}$$

$$= \mathrm{I_D^2} \left\{1 - 2{\cdot}9 \sin \left(\theta - \frac{\pi}{6}\right) + 1{\cdot}055 \left[1 - \cos 2 \left(\theta - \frac{\pi}{6}\right)\right]\right\}$$

$$= \mathrm{I_D^2} \left\{2{\cdot}055 - 2{\cdot}9 \sin \left(\theta - \frac{\pi}{6}\right) - 1{\cdot}055 \cos 2 \left(\theta - \frac{\pi}{6}\right)\right\}$$

$$\int \mathrm{i}^2\, d\theta = \mathrm{I_D^2} \left\{2{\cdot}055\theta + 2{\cdot}9 \cos \left(\theta - \frac{\pi}{6}\right) - 0{\cdot}5275 \sin 2 \left(\theta - \frac{\pi}{6}\right)\right\}$$

$$\begin{aligned}\int_0^\pi i^2 d\theta &= I_D^2 \left\{\left(2{\cdot}055\pi + 2{\cdot}9\cos\frac{5\pi}{6} - 0{\cdot}5275\sin\frac{5\pi}{3}\right)\right.\\ &\quad \left. - \left[0 + 2{\cdot}9\cos\left(-\frac{\pi}{6}\right) - 0{\cdot}5275\sin\left(-\frac{\pi}{3}\right)\right]\right\}\\ &= I_D^2 \left\{(6{\cdot}45 - 2{\cdot}51 + 0{\cdot}458) - (2{\cdot}51 + 0{\cdot}458)\right\}\\ &= 1{\cdot}43\ I_D^2\end{aligned}$$

Mean value of i^2 during one-half of the cycle

$$= \frac{1{\cdot}43\ I_D^2}{\pi}$$

$$= 0{\cdot}455\ I_D^2$$

Since the heating of the conductor is proportional to the (R.M.S. current)^2, i.e. to the mean value of i^2 and the heating when run as a dynamo with the same output is proportional to $I_D{}^2$, therefore the heating of the conductor when the machine is running as a convertor will be **0·455 times** the heating when the machine is running as a dynamo.

(iii) d.c. voltage control.

102. *A reactor with a reactance of* 1 *ohm and negligible resistance is included in each line of the supply to a* 3*-ring synchronous convertor taking* 50 *amperes at a leading power factor of* 0·8. *If the voltage of supply is maintained constant at* 500 *volts, calculate the voltage at the commutator brushes, neglecting all losses.* (*C. and G. Final, Pt. II*, 1943)

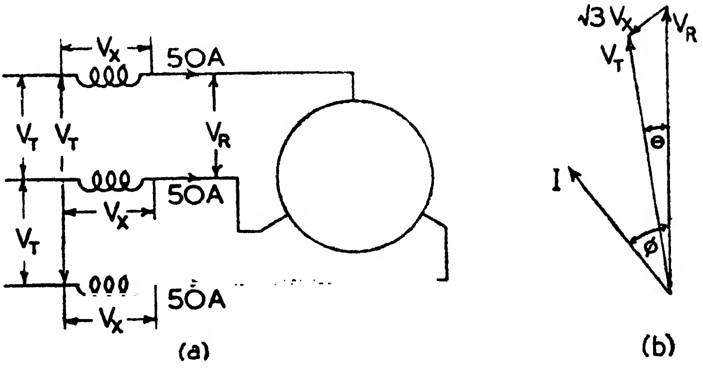


Fig. 71

Figs. 71(a) and (b) show the connection and vector diagrams respectively.

Voltage drop across each reactance = IX = 50 volts = V_x

Total voltage drop between the supply terminals and a pair of slip-rings = $\sqrt{3}V_x$ = 86·6 volts.

This voltage subtracted vectorially from the supply voltage V_T gives the slip-ring voltage V_R. This is shown in the vector diagram. Note that the reactance drop is in quadrature with the current.

Although it is not specifically stated, the power factor of 0·8 leading is assumed to be at the convertor slip-rings, not at the transformer terminals, i.e. angle ϕ between V_R and $I = \text{arc cos } 0{\cdot}8 = 36°52'$.

From the vector diagram,

$V_T \cos\theta = V_R - \sqrt{3}V_x \sin\phi$, where θ is the angle between V_T and V_R

$$500 \cos\theta = V_R - 86{\cdot}6 \times 0{\cdot}6 = V_R - 52 \text{ volts} \quad (1)$$

Also, $V_T \sin\theta = \sqrt{3}V_x \cos\phi$

$$500 \sin\theta = 86{\cdot}6 \times 0{\cdot}8 = 69{\cdot}28 \text{ volts} \quad (2)$$

Squaring (1) and (2) and adding,

$$500^2 = (V_R - 52)^2 + 69{\cdot}28^2$$
$$(V_R - 52)^2 = 250000 - 4800 = 245200$$
$$V_R - 52 = \sqrt{245200} = 495{\cdot}2 \text{ volts,}$$
$$V_R = 547{\cdot}2 \text{ volts} = \text{the slip-ring voltage.}$$

Now, slip-ring voltage $= \dfrac{V_{DC} \sin\frac{\pi}{N}}{\sqrt{2}}$

i.e. $547{\cdot}2 = \dfrac{V_{DC} \sin\frac{\pi}{3}}{\sqrt{2}}$

$$V_{DC} = \frac{547{\cdot}2 \times \sqrt{2}}{0{\cdot}866} = 893{\cdot}5 \text{ volts.}$$

Therefore, **voltage at commutator brushes = 893·5 volts.**

103. *Explain with a vector diagram the reactance method of voltage control of a synchronous convertor.*

A 6-ring convertor with diametral tappings is supplied from a 33-kV, 3-phase supply. The transformer is delta-connected on the high-voltage side with a turn ratio of 77 and an equivalent secondary reactance of 0·2 ohm. If the direct-current load remains constant at 500 amperes, find the change in commutator voltage when the power factor at the slip-rings increases from 0·8 leading to unity. Assume the efficiency to remain unaltered at 0·9.

(*C. and G. Final, Pt. I,* 1940)

Transformer secondary voltage $= \dfrac{33000}{77} = 428{\cdot}6$ volts.

Slip-ring current $I_{AC} = I_{DC} \times \dfrac{2\sqrt{2}}{N} \times \dfrac{1}{\eta . \cos\phi}$ amperes.

At 0·8 power factor, $I_{AC} = 500 \times \dfrac{2\sqrt{2}}{6} \times \dfrac{1}{0{\cdot}9 \times 0{\cdot}8}$

$= 327$ amperes.

At unity power factor $I_{AC} = 500 \times \dfrac{2\sqrt{2}}{6} \times \dfrac{1}{0{\cdot}9}$

$= 261{\cdot}6$ amperes.

Reactance drop at 0·8 power factor $= 327 \times 0{\cdot}2 = 65{\cdot}4$ **volts.**

Reactance drop at unity power factor $= 261{\cdot}6 \times 0{\cdot}2 = 52{\cdot}3$ **volts.**

The vector diagrams for the two conditions are shown in Fig. 72(a) and (b).

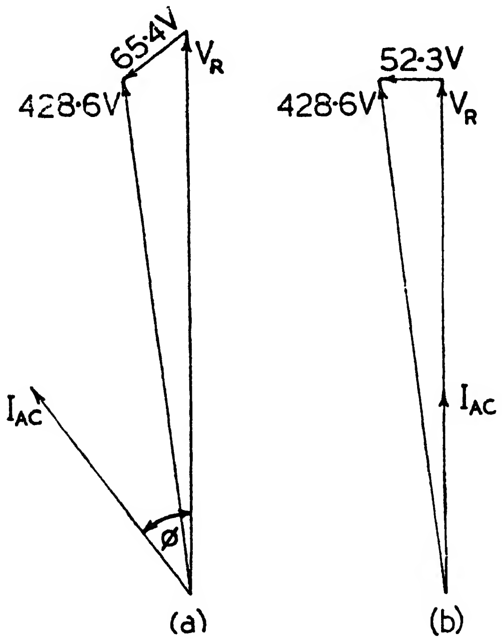


Fig. 72

In each case the slip-ring voltage is the vector difference between the transformer voltage and the reactance drop.

At 0·8 power factor, the phase angle between the slip-ring voltage V_R and the reactance drop is $(90° - \varnothing)$ where $\varnothing$ is the phase angle between V_R and the slip-ring current. Therefore

$$\cos(90 - \varnothing) = \frac{(V_R^2 + 65{\cdot}4^2 - 428{\cdot}6^2)}{(2 \times V_R \times 65{\cdot}4)}$$

$$V_R^2 + 65{\cdot}4^2 - 428{\cdot}6^2 = 2 \times V_R \times 65{\cdot}4 \times 0{\cdot}6$$

$$V_R^2 - 78{\cdot}48V_R - 179423 = 0, \text{ and solving this quadratic}$$

$$V_R = 464{\cdot}7 \text{ volts.}$$

Hence, $V_{DC} = \sqrt{2} \times 464{\cdot}7 = 657$ volts.

At unity power factor, $V_R = \sqrt{(428{\cdot}6^2 - 52{\cdot}3^2)}$

$$= 425{\cdot}5 \text{ volts.}$$

$$V_{DC} = \sqrt{2} \times 425{\cdot}5 = 602 \text{ volts.}$$

Therefore, **the change in commutator voltage is — 55 volts.**

104. *A* 6600/370-*volt,* 600-*kVA, star/diametral transformer with a* 5 *per cent reactance supplies a* 6-*ring,* 500-*kW, synchronous convertor. Estimate the additional reactance necessary in* each *slip-ring lead in order that the convertor may develop* 550 *volts on full-load with the excitation adjusted to give a power factor of* 0·8 *leading at the slip-rings. Neglect losses and assume a constant normal primary transformer voltage.* (*I.E.E., Pt. II May,* 1938)

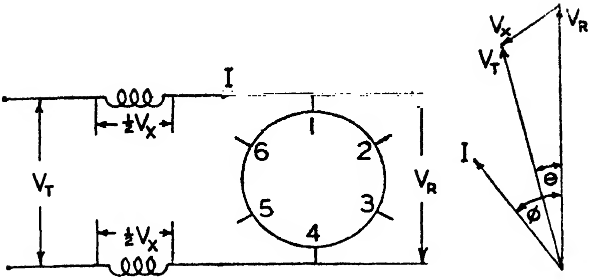


Fig. 73

Let V_R = the voltage across diametral slip-rings,
V_T = the secondary phase voltage of the transformer,
V_x = the total reactance drop per phase on full-load.

Then, $V_R = \dfrac{V_{DC}}{\sqrt{2}} = \dfrac{550}{\sqrt{2}} = 389$ volts.

Also, $V_T = 370$ volts, and the phase angle between V_R and the slip-ring current I is $\phi = \text{arc}\cos 0{\cdot}8$ (Fig. 73).

From the vector diagram,

$$V_T \sin\theta = V_x \cos\phi$$

i.e. $370 \sin\theta = 0{\cdot}8\ V_x$ (1)

Also, $V_T \cos\theta = V_R - V_x \sin\phi$

i.e. $370 \cos\theta = (389 - 0{\cdot}6\ V_x)$ (2)

Squaring (1) and (2) and adding,

$$370^2 = (0{\cdot}8\ V_x)^2 + (389 - 0{\cdot}6\ V_x)^2$$

$$136900 = V_x^2 - 466{\cdot}8\ V_x + 151300$$

Solving, $V_x = 33{\cdot}4$ volts.

Each secondary winding supplies one-third of the total load, i.e. $\dfrac{500}{3}$ kW.

Therefore, at 0·8 power factor,

full-load secondary current $= \dfrac{500000}{370 \times 3 \times 0{\cdot}8}$ amperes
$= 563{\cdot}1$ amperes.

Total reactance needed in each secondary phase $= \dfrac{33{\cdot}4}{563{\cdot}1}$ ohm $= 0{\cdot}0593$ ohm.

Transformer secondary reactance per phase $= \dfrac{0{\cdot}05 \times 370^2}{\text{kVA} \times 10^3}$ ohm.

$= \dfrac{0{\cdot}05 \times 370^2}{200 \times 10^3} = 0{\cdot}0342$ ohm

Added reactance necessary $= 0{\cdot}0593 - 0{\cdot}0342$
$= 0{\cdot}0251$ ohm.

Reactance necessary in each slip-ring lead $= \dfrac{0{\cdot}0251}{2}$ **= 0·0126 ohm.**

105. *Explain with vector diagrams the reactance method of voltage control of a synchronous convertor.*

A 6-*ring synchronous convertor with diametral tappings is supplied from a transformer giving a constant voltage of* 350 *volts on each secondary phase. Calculate the no-load d.c. voltage.*

If a reactor of 0·15 *ohm is included in each slip-ring lead, calculate the d.c. voltage for power factors of* 0·8 *leading and* 0·7 *lagging at the transformer secondary terminals, the d.c. output remaining unchanged at* 300 *kW. Neglect losses.*

(*C. and G. Final, Pt. I,* 1937)

No-load d.c. voltage = Diametral slip-ring voltage × $\sqrt{2}$
= $350 \times \sqrt{2} =$ **495 volts.**

Let V_{AC} and I_{AC} be the secondary voltage and current per phase of the transformer and $\varnothing$ the phase angle between them. Then since each phase supplies one-third of the total power, i.e. 100 kW,

$$V_{AC} \, . \, I_{AC} \, . \cos \varnothing = 100000 \text{ watts.}$$

Hence, when $\cos \varnothing = 0{\cdot}8$,

$$I_{AC} = \frac{100000}{(350 \times 0{\cdot}8)} = 357 \text{ amperes.}$$

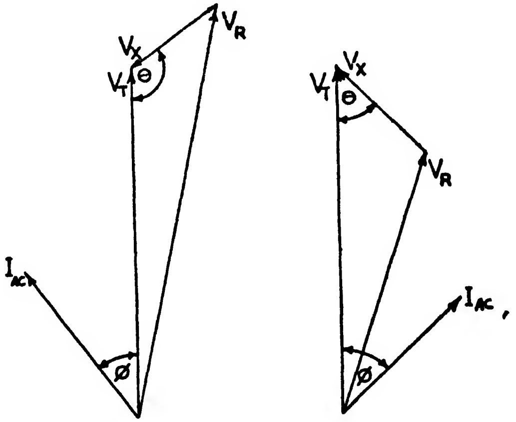
When $\cos \varnothing = 0{\cdot}7$,

$$I_{AC} = \frac{100000}{(350 \times 0{\cdot}7)} = 408 \text{ amperes.}$$

Therefore, at 0·8 power factor, reactance drop per phase $= 2 \times 357 \times 0{\cdot}15 = 107{\cdot}1$ volts.

At 0·7 power factor, reactance drop per phase $= 2 \times 408 \times 0{\cdot}15 = 122{\cdot}4$ volts.

The vector diagrams for the two power factor conditions are shown in Fig. 74.



0·8 p.f.   0·7 p.f.

Fig. 74

Referring to these diagrams,

at 0·8 power factor, $\cos \phi = 0{\cdot}8$ and $\phi = 36^\circ52'$

$\theta = 90^\circ + \phi = 126^\circ52'$

$$\cos 126^\circ52' = \frac{(V_T^2 + V_x^2 - V_R^2)}{(2V_T\,V_x)}$$

$$-0{\cdot}6 = \frac{(350^2 + 107{\cdot}1^2 - V_R^2)}{(2 \times 350 \times 107{\cdot}1)}$$

Whence, $V_R = 423$ volts.

d.c. voltage $= 423\sqrt{2} =$ **598 volts.**

At 0·7 power factor, $\cos \phi = 0{\cdot}7$ and $\phi = 45^\circ36'$

$\theta = 90 - \phi = 44^\circ24'$

$$\cos 44^\circ24' = \frac{(350^2 + 122{\cdot}4^2 - V_R^2)}{(2 \times 350 \times 122{\cdot}4)}$$

Whence, $V_R = 276$ volts.

d.c. voltage $= 276\sqrt{2} =$ **390 volts.**

Note.—The reactance drop **per phase** is twice that across each reactor because there is a reactor in **each** slip-ring lead.

CHAPTER VIII

MERCURY ARC RECTIFIERS

(i) a.c./d.c. voltage and current relations.

106. *The turn ratio of a* 3*-phase delta/star transformer connected to* 11000*-volt mains and supplying a* 3*-anode mercury arc rectifier is* 65/3. *Find the mean value of the d.c. voltage, neglecting arc drop. Deduce any formula used.*

(*C. and G. Final, Pt. I,* 1938)

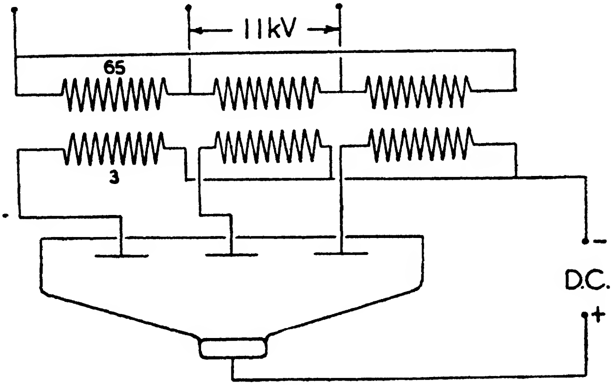


Fig. 75

Consider an N-anode rectifier giving a constant direct current output of I amperes and supplied by a transformer whose phase e.m.f. is E volts (R.M.S). The anodes then operate one at a time in sequence, and each anode supplies the load current for a period $\frac{2\pi}{N}$.

The d.c. voltage of the rectifier, including the drop in the arc, has the fluctuating waveform shown in Fig. 76. At any instant its value is equal to the voltage between the anode which is conducting at that instant and transformer neutral. The mean value of the d.c. voltage is found by integrating the voltage wave over the period $\frac{2\pi}{N}$ using the point of maximum value as the origin.

Let e_D be the instantaneous value of the d.c. voltage, and E_D be the mean value of the d.c. voltage.

Then, $$E_D = \frac{\int_{-\pi/N}^{+\pi/N} e_D d\theta}{\frac{2\pi}{N}}$$

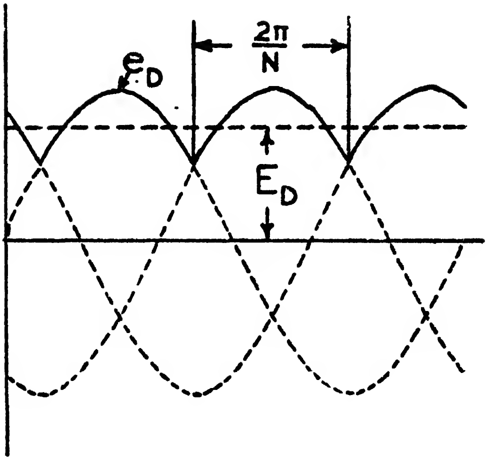


Fig. 76

Now, with the point of maximum value as the origin,

$$e_D = E\sqrt{2}\cos\theta \qquad \text{where } \theta = \omega t.$$

Hence,
$$E_D = \frac{\int_{-\pi/N}^{\pi/N} E\sqrt{2}\cos\theta \, d\theta}{\frac{2\pi}{N}}$$

$$= \frac{(E\sqrt{2}\sin\theta)_{-\pi/N}^{+\pi/N}}{\frac{2\pi}{N}}$$

$$= \frac{E\sqrt{2}\sin\frac{\pi}{N}}{\frac{\pi}{N}}$$

In this problem, the transformer secondary phase e.m.f. is

$$E = \frac{11000}{21\frac{2}{3}}$$
$$= 507{\cdot}6 \text{ volts.}$$

Mean value of d.c. voltage $= \dfrac{507{\cdot}6\sqrt{2}\sin\frac{\pi}{3}}{\frac{\pi}{3}}$ volts

$$= \frac{507{\cdot}6 \times \sqrt{2} \times \sqrt{3} \times 3}{2 \times \pi} \text{ volts}$$

$= \mathbf{594}$ **volts.**

107. *Deduce an expression for the relation between the d.c. (mean) voltage and the a.c. (R.M.S) voltage in an N-anode mercury arc rectifier.*

A d.c. load is supplied at 500 *(mean) volts from a* 6*-anode mercury arc rectifier. The high-voltage supply is* 6600 *volts and the transformer is star-connected on the primary side. Calculate the turn ratio of the transformer. Allow an arc drop of* 30 *volts for each electrode.*

(*C. and G. Final, Pt. I,* 1940)

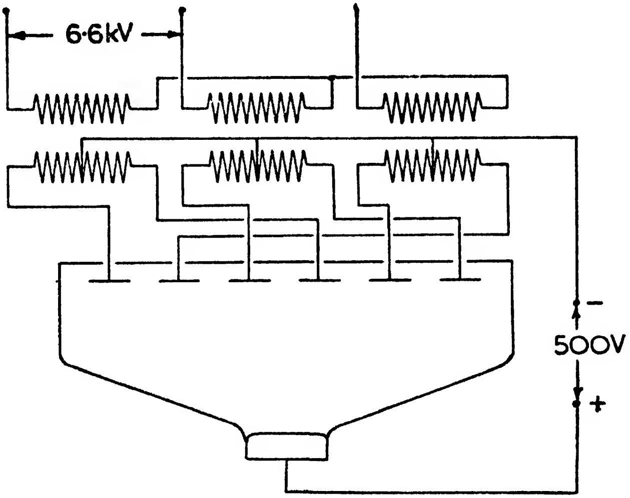


Fig. 77

Mean d.c. voltage including arc drop = 500 + 30 = 530 volts.

Using the formula given in Problem 106,

R.M.S value of a.c. voltage between each anode and the neutral

$$= \frac{530 \times \frac{\pi}{6}}{\sqrt{2} \times \sin\frac{\pi}{6}} \text{ volts}$$

$$= \frac{530 \times \pi}{\sqrt{2} \times 0{\cdot}5 \times 6} \text{ volts}$$

$$= 392{\cdot}6 \text{ volts.}$$

Hence, secondary phase voltage of transformer $= 392{\cdot}6 \times 2$

$= 785{\cdot}2$ volts.

Primary phase voltage of transformer $= \frac{6600}{\sqrt{3}} = 3811$ volts.

Therefore, **turns ratio per phase** $= \frac{3811}{785{\cdot}2}$

$$= \frac{\mathbf{4{\cdot}852}}{\mathbf{1}}$$

108. *Deduce the relation between the average d.c. voltage of a mercury arc rectifier and the phase voltage of each secondary winding of the star-connected transformer.*

A 3-*anode rectifier supplies a load of* 10 *kW at* 220 *volts, the arc drop being* 20 *volts. Find the kVA rating of the transformer.*

(*C. and G. Final, Pt. I,* 1943)

Mean value of d.c. voltage including arc drop $= 220 + 20 = 240$ volts.

Secondary phase voltage $V_{AC} = \dfrac{240 \times \frac{\pi}{3}}{\sqrt{2} \times \sin\frac{\pi}{3}}$ volts

$= \dfrac{240 \times \pi \times 2}{\sqrt{2} \times 3 \times \sqrt{3}}$ volts

$= 205{\cdot}2$ volts.

Output current $I_{DC} = \dfrac{10000 \text{ watts}}{220 \text{ volts}}$

$= 45{\cdot}45$ amperes.

R.M.S value of secondary phase current $= \dfrac{I_{DC}}{\sqrt{N}}$

i.e. $I_{AC} = \dfrac{45{\cdot}45}{\sqrt{3}}$ amperes $= 26{\cdot}25$ amperes

Transformer rating per phase $= V_{AC} \,.\, I_{AC}$

$= 205{\cdot}2 \times 26{\cdot}25 \times 10^{-3}$ kVA.

Overall transformer rating $= 3 \times 205{\cdot}2 \times 26{\cdot}25 \times 10^{-3}$ kVA

$=$ **16·15 kVA.**

(ii) Voltage regulation, anode current overlap, grid control.

109. *Give an account of the operation of a mercury arc rectifier. Explain the terms "transition load" and "angle of overlap."*

A 3-*anode mercury arc rectifier has an anode current overlap of* 30°. *What is the percentage regulation, neglecting the arc drop?*

(*C. and G. Final, Pt. I,* 1944)

Let $E =$ the R.M.S value of the transformer secondary phase e.m.f.

$$\text{No-load d.c. voltage} = \frac{E\sqrt{2}\sin\frac{\pi}{3}}{\frac{\pi}{3}}$$

$= 1{\cdot}169E$ volts.

The anode current overlap is due to reactance in the anode circuit. If the reactance per phase is X ohms, then on a load current of I amperes the overlap angle is given by

$$\cos a = 1 - \frac{IX}{E\sqrt{2}\sin\frac{\pi}{N}}$$

i.e.

$$IX = E\sqrt{2}\sin\frac{\pi}{N}(1 - \cos a)$$

$$= E\sqrt{2}\sin\frac{\pi}{3}\left(1 - \cos\frac{\pi}{6}\right)$$

$$= \frac{E\sqrt{2} \times \sqrt{3}}{2}\left(1 - \frac{\sqrt{3}}{2}\right)$$

$$= 0{\cdot}164 \text{ E volts.}$$

The average drop in d.c. voltage on this load $= \frac{IX}{2\pi/N}$

$$= \frac{0{\cdot}164 \text{ E} \times 3}{2\pi} \text{ volts}$$

$$= 0{\cdot}0783 \text{ E volts.}$$

Hence, d.c. voltage on load $= 1{\cdot}169 \text{ E} - 0{\cdot}0783 \text{ E}$

$$= 1{\cdot}0907 \text{ E volts.}$$

Percentage regulation $= \dfrac{\text{d.c. voltage on no-load} - \text{d.c. voltage on load}}{\text{d.c. voltage on load}} \times 100$ per cent

$$= \frac{0{\cdot}0783 \text{ E}}{1{\cdot}0907 \text{ E}} \times 100 \text{ per cent,}$$

= **7·18 per cent.**

110. *What is meant by the "overlap angle" of a mercury arc rectifier and how does it affect the d.c. voltage output?*

A 6-anode mercury arc rectifier is fed from a transformer giving a phase voltage of 400 volts at 50 cycles per second, and the output current may be considered constant at 200 amperes. Calculate the effective leakage inductance per phase of the transformer if the overlap angle on this load is to be limited to 15°.

(*H.N.C.*, 1943)

The reactance per phase of the transformer is found from the expression,

$$\cos a = 1 - \frac{IX}{E\sqrt{2}\sin\frac{\pi}{N}}$$

In this problem $a = 15°$, $I = 200$ amperes, $E = 400$ volts, and $N = 6$ anodes.

Hence,

$$\cos 15° = 1 - \frac{200 \text{ X}}{400\sqrt{2}\sin\frac{\pi}{6}}$$

$$0{\cdot}9659 = 1 - \frac{200 \text{ X}}{400\sqrt{2} \times 0{\cdot}5} = 1 - \frac{X}{\sqrt{2}}$$

$$\frac{X}{\sqrt{2}} = 0{\cdot}0341$$

$$X = 0{\cdot}0482 \text{ ohm.}$$

$$\text{Therefore, } \textbf{inductance per phase} = \frac{X}{2\pi f}$$

$$= \frac{0{\cdot}0482}{100\pi} \text{ henry,}$$

$$= \mathbf{0{\cdot}1535 \text{ millihenry.}}$$

111. *Explain and differentiate between power factor and distortion factor in the supply to a mercury arc rectifier. Give a diagram of connections for the grid control of a* 6*-anode rectifier. Calculate the average value of the output voltage of a* 6*-anode rectifier with grids biassed to give a delay of* 30°; *the transformer has a* 3*-phase mesh-connected primary supplied at* 3000 *volts and a* 6*-phase star secondary, the phase turn ratio being* 150/11. *Assume an arc drop of* 25 *volts and neglect the effect of overlap.*

(*I.E.E., Pt. II, May,* 1938)

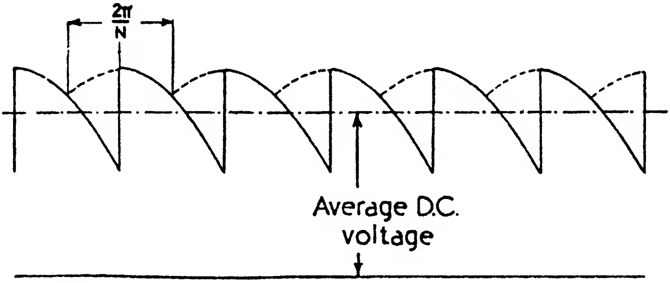


Fig. 78

The effect of grid control on a mercury arc rectifier is to delay the point at which each anode takes over the load from the preceding anode. The waveform of the output voltage, including the arc drop, becomes that shown in heavy outline in Fig. 78. Taking the point of maximum voltage as the origin the equation for the d.c. voltage is

$e_D = E\sqrt{2} \cos \theta$, where E is the R.M.S phase voltage.

Without grid control each anode would conduct over a period from $\theta = -\frac{\pi}{N}$ to $\theta = +\frac{\pi}{N}$. With grid control which delays the firing point of each anode by an angle β, the conduction period becomes from $\theta = -\frac{\pi}{N} + \beta$ to $+\frac{\pi}{N} + \beta$. The average value of the d.c. voltage is found by integrating the expression for the waveform over this range of θ.

Let E_D be the average d.c. voltage including arc drop.

$$E_D = \frac{\int_{-\pi/N+\beta}^{\pi/N+\beta} E\sqrt{2}\cos\theta\, d\theta}{\frac{2\pi}{N}}$$

$$= \int_{-\pi/N+\beta}^{\pi/N+\beta} \cos\theta\, d\theta \times \frac{E\sqrt{2N}}{2\pi}$$

$$= \frac{E\sqrt{2N}}{2\pi}\Big[\sin\theta\Big]_{-\pi/N+\beta}^{\pi/N+\beta}$$

$$= \frac{E\sqrt{2N}}{2\pi} \sin\left[\left(\beta+\frac{\pi}{N}\right) - \sin\left(\beta-\frac{\pi}{N}\right)\right]$$

$$= \frac{E\sqrt{2N}}{2\pi}\left[2\cos\beta .\ \sin\frac{\pi}{N}\right]$$

$$= \frac{E\sqrt{2}\sin\frac{\pi}{N}}{\frac{\pi}{N}}\cos\beta$$

$= E_{Do}\cos\beta$ where E_{Do} is the average d.c. voltage when there is no grid control.

This expression holds only until $\beta = \frac{\pi}{2} - \frac{\pi}{N}$, i.e. until $\beta = 60°$ for a 6-anode rectifier.

In this problem,
secondary voltage per phase $= \frac{3000 \times 11}{150} = 220$ volts.

Without grid control, mean value of d.c. voltage $= \frac{220\sqrt{2}\sin\frac{\pi}{6}}{\frac{\pi}{6}}$

$= 297$ volts.

With a delay angle of 30° imposed by grid control,
mean value of d.c. voltage $= 297\cos 30° = 257$ volts.

Allowing an arc drop of 25 volts,
mean value of output voltage $= 257 - 25 =$ **232 volts.**

112. *A* 6-*anode mercury-arc rectifier giving* 1200 *kW at* 630 *volts on the d.c. side is supplied from a transformer with its secondary windings connected in double-star with an interphase transformer joining the two star points.*

Give a diagram of the connections of transformers and rectifier. Explain the action of the interphase transformer and calculate its rating.

(*London B.Sc.Eng.*, 1945)

The diagram of connections for the transformers and rectifier is shown in Fig. 79.

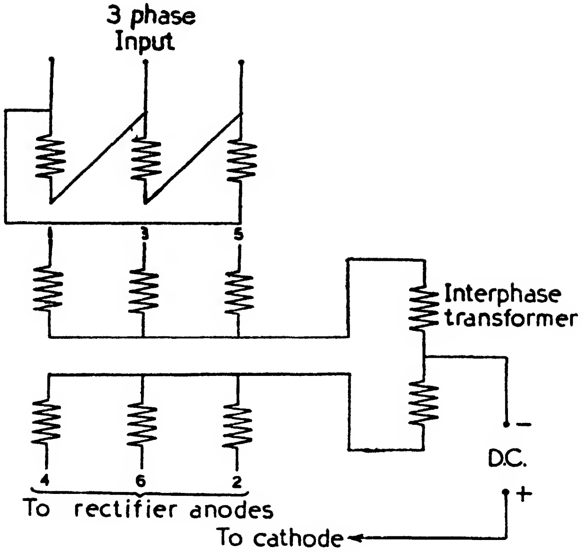


Fig. 79

The number of phases for which a rectifier is to be designed is governed by the following considerations:

(a) the rectifier supply transformer should be utilized to the best advantage,

(b) the power factor of the system should be as high as possible,

(c) the voltage regulation should be low,

(d) the harmonic generation should be low.

To meet requirements (a), (b) and (c) the number of phases should be low while for (d) it should be high. The use of the interphase transformer with a 6-anode rectifier combines the low harmonic percentage of a 6-phase system with the better transformer utilization factor, power factor and voltage regulation of a 3-phase system.

The function of the interphase transformer is to equalize the potentials of two anodes at a time so that at any given instant the load is being shared by those two anodes which are effectively working in parallel. Thus although the voltage waveform has the characteristics of a 6-phase rectifier with its smaller harmonic content of twice the supply frequency, the load is actually being shared by two 3-phase systems which operate in parallel at a terminal voltage which is the mean of the terminal voltages of the phases operating together. Each phase of the transformer operates for one-third of the cycle instead of one-sixth and the mean output voltage is the same as that of a rectifier with three anodes instead of six.

The principle on which the interphase transformer operates may be explained by examining the conditions which prevail at an instant on the voltage waveforms in Fig. 80 when anodes 1 and 2 are conducting. Anode 1 is at the highest potential and as the current to this anode increases, flowing through one winding of the interphase transformer, it sets up a magnetic field in the core of this transformer. The effect of the magnetic field is to induce e.m.f.s in the two windings of the transformer such that the potential of anode 1 is reduced and that of anode 2 is increased until the potentials of both are equalized at a value which is the mean of their respective phase potentials. Hence at any one time there are two anodes, one from each group, in operation in parallel, e.g. 1 and 2, 2 and 3, 3 and 4, 4 and 5, 5 and 6, 6 and 1. Phase 1 carries one half of the load current until its voltage wave is intersected by that of phase 3 to which it then transfers, and each group operates as a 3-phase rectifier.

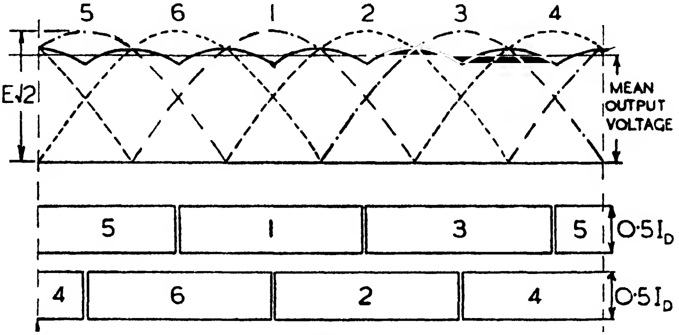


Fig. 80

The direct currents in the two windings of the interphase transformer flow in opposite directions and therefore there is no magnetization of the core. The core is, however, magnetized by the third-harmonic current which produces a third-harmonic flux.

Calculation of interphase transformer rating.

Each winding of the transformer carries one half of the d.c. load. The current rating is therefore one half of the load current.

The voltage across each winding is of triplen frequency and may be shown to have practically triangular waveform with an R.M.S. value of 0·204 E where E is the R.M.S. phase voltage of the supply transformer, or 0·24 E_D where E_D is the mean value of the output voltage.

The interphase transformer must be designed for third-harmonic flux and compared with what it would need to be at supply frequency for the same iron losses the flux density will be very much less, actually it is halved. Its rating is calculated as a two-winding transformer working at supply

frequency and on this basis the voltage, which is proportional to the product of frequency and flux density is reduced in the ratio 2 : 3.

i.e. Equivalent voltage rating of transformer at supply frequency = $\frac{2}{3} \times 0{\cdot}24\ E_D = 0{\cdot}16\ E_D$.

In the problem given, $E_D = 630$ volts,

$$I_D = \frac{1200000 \text{ watts}}{630 \text{ volts}}$$

$$= 1905 \text{ amperes.}$$

Current rating of interphase transformer = $0{\cdot}5\ I_D$ = **953 amperes.**

Voltage rating of transformer = $0{\cdot}16\ E_D$ = **100 volts.**

kVA rating of transformer $= \frac{100 \times 953}{1000}$ = **95·3 kVA.**

CHAPTER IX

DIRECT CURRENT MACHINES

(i) Losses, speed control, starting.

113. *Enumerate the losses occurring in a d.c. machine. Calculate the brush friction and brush-contact-resistance losses in a 4-pole,* 500-*V*, 100-*kW*, 800-*r.p.m. shunt generator with the following particulars:*

Commutator diameter = 24 *cm.*
Commutator length = 16 *cm.*
Brush pressure = 150 *gm. per cm.*^{2}
Coefficient of friction = 0·24
Brush current density = 7·5 *amperes per cm.*^{2} *(approximately).*
Brush contact drop = 1 *volt per brush set.*

(*C. and G. Final, Pt. II*, 1942)

Output current of generator $= \dfrac{100000}{500}$
$= 200$ amperes.

As the current density is approximately 7·5 amperes per sq. cm.
Contact area of positive or negative brushes $= \dfrac{200}{7{\cdot}5}$
$= 26{\cdot}67$ sq. cm.

Total area of brush contact at positive and negative brushes $= 26{\cdot}67 \times 2$
$= 53{\cdot}34$ sq. cm.

Total pressure over contact surface $= 53{\cdot}34 \times 150$
$= 8000$ gm.

Frictional resistance to be overcome $= 0{\cdot}24 \times 8000$
$= 1920$ gm.

Peripheral speed of commutator $= \pi \times 24 \times \dfrac{800}{60}$
$= 1000$ cm. per sec.

Therefore, work done in overcoming friction $= 1920 \times 1000$ gm.-cm. per sec.
$= 1{\cdot}92 \times 10^6$ gm.-cm. per sec.

Now, 1 gm.-cm. = 981 ergs
and 10^7 ergs per sec. = 1 watt

Therefore, **power loss in overcoming friction** $= \dfrac{1{\cdot}92 \times 10^6 \times 981}{10^7}$
= 188 watts.

Suppose that there are N brush sets, i.e. $\dfrac{N}{2}$ positive brushes and $\dfrac{N}{2}$ negative brushes.

$$\text{Current per set} = \frac{200}{\frac{N}{2}}$$

$$= \frac{400}{N} \text{ amperes.}$$

$$\text{Contact resistance loss per set} = 1 \text{ volt} \times \frac{400}{N} \text{ amperes}$$

$$= \frac{400}{N} \text{ watts.}$$

Therefore, **total contact resistance loss for N sets** $= \frac{400}{N} \times N$

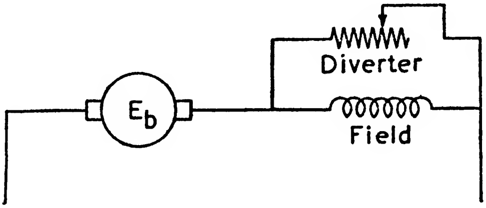
$=$ **400 watts.**

114. *Describe briefly the methods of speed-control available for d.c. motors.*

A series motor is required to produce constant full-load torque whilst its speed is raised 25 per cent above the full-load value by means of a diverter. Calculate the diverter resistance required as a percentage of the field resistance. Neglect losses in the motor. The relation between the field current and flux is as follows:

Current	50	75	100	*per cent of full-load value.*
Flux	66	86	100	*per cent of full-load value.*

(*C. and G. Final, Pt. II,* 1945)



Fg. 81

The relation between the back e.m.f., flux and speed of a d.c. motor if the torque is constant is

$$E_b = k\ \Phi\ N \text{ where k is a constant.}$$

If the losses in the motor are neglected the back e.m.f. will remain constant at all speeds and be equal to the supply voltage. Then ΦN will be a constant.

Let Φ_1 and N_1 be the flux and speed at full-load and Φ_2 and N_2 be the flux and speed at 125 per cent of full-load speed.

Then $N_2 = 1{\cdot}25\ N_1$

Therefore, $\Phi_2 \times 1{\cdot}25\ N_1 = \Phi_1 \times N_1$

i.e. $\Phi_2 = 0.8\ \Phi_1$

Therefore the diverter resistance must be of such a value that the flux is reduced to 80 per cent of its full-load value. If the current-flux graph is plotted from the data given it will be found that when the flux is 80 per cent of the full-load value the current is 67 per cent of its full-load value.

Hence, of the total current flowing into the motor the diverter resistance must allow only 67 per cent of full-load current to flow through the field.

Now the torque of a d.c. motor is proportional to the product of flux and armature current, i.e. Φ I is a constant if the torque is a constant.

At 125 per cent of full-load speed it has been shown that the flux is 80 per cent of full-load flux. Hence in order that the product of flux and armature current may be constant the armature current at 125 per cent full-load speed must be 125 per cent of its full-load value.

Hence, total current flowing through field and diverter resistance = 125 per cent of full-load current,

Current flowing through the field = 67 per cent of full-load current,

Current flowing through the diverter — 58 per cent of full-load current,

Hence, field current : diverter current : : 67 : 58
and field resistance : diverter resistance : : 58 : 67

$$\text{Therefore, diverter resistance} = \frac{67}{58} \times \text{field resistance}$$
$$= 1{\cdot}155 \times \text{field resistance.}$$

i.e. **the diverter resistance required is 115·5 per cent of the field resistance.**

115. *The speed of a* 500-*volt series motor coupled to a fan is reduced to one-half of full speed by a series resistance. At full speed the current is* 100 *amperes and the load torque is proportional to the square of the speed. Show that the power input is proportional to the cube of the speed, and calculate the resistance required assuming that the field is unsaturated and neglecting change of motor losses.* (*C. and G. Final, Pt. II,* 1944)

Let T represent the motor torque,
N represent the motor speed,
Φ represent the flux per pole,
I represent the armature current (which is also the input current),
E_b represent the back e.m.f. of the motor.

Then from the data given,

$$T = kN^2 \text{ where k is a constant}$$

But $T = k_1\Phi\, I$ in any motor, where k_1 is another constant.

Therefore $k_1\, \Phi\, I = kN^2$

If the field is unsaturated,

$$\Phi = k_2\, I \text{ where } k_2 \text{ is a third constant}$$

Hence, $k_1\, \Phi\, I = k_1\, k_2\, I^2$

and $$I^2 = \frac{k}{k_1 k_2} N^2$$

Now, $$E_b = \frac{p\,N\,Z\,\Phi}{60\,a} \times 10^{-8} \text{ volts}$$

i.e. $$E_b = k_3\,N\,\Phi \text{ where } k_3 \text{ is a constant}$$

Power input to armature
$$\begin{aligned} &= E_b\,I \\ &= k_3\,N\,\Phi\,I \\ &= k_3\,N\,k_2\,I\,I \\ &= k_2 k_3\,N\,I^2 \\ &= k_2 k_3\,N \frac{k}{k_1 k_2} N^2 \\ &= \frac{k \cdot k_3}{k_1} N^3 \end{aligned}$$

Therefore the power input is proportional to the cube of the speed.

At full-load speed I = 100 amperes.

From the above theory the current is proportional to the speed, hence if the speed is halved by the inclusion of series resistance the current will also be halved.

Therefore, current at half full-load speed = 50 amperes.

At full speed E_b is approximately 500 volts, ignoring the motor losses,

i.e. $$E_b\,I = 500 \times 100 \text{ watts} = 50000 \text{ watts.}$$

Since the power input is proportional to the cube of the speed, when the speed is halved the power input is reduced to one-eighth.

Therefore at half-speed,

$$E_b\,I = \frac{50000}{8} \text{ watts.}$$

Therefore $$E_b = \frac{50000}{8 \times 50} = 125 \text{ volts.}$$

Drop in series resistance $$= 500 - 125 = 375 \text{ volts.}$$

As the motor current is 50 amperes,

series resistance required $$= \frac{375}{50} = \mathbf{7{\cdot}5 \text{ ohms.}}$$

116. *A d.c. series motor drives a load the torque of which varies as the square of the speed. The motor current is* 20 *amperes when the speed is* 500 *r.p.m. Calculate the speed and current when the motor field winding is shunted by a resistor of the same resistance as the field winding. Neglect all motor losses and assume that the magnetic circuit is unsaturated.*

(*C. and G. Final, Pt. II,* 1942)

Neglecting all losses the back e.m.f. will be equal to the applied e.m.f. in each case.

Therefore, $$E_b = k\,N_1\,\Phi_1 = k\,N_2\,\Phi_2$$

where the symbols have the same meaning as in the previous problem and the suffixes represent the two sets of conditions.

Hence, $$\frac{N_2}{N_1} = \frac{\Phi_1}{\Phi_2} \qquad (1)$$

The torque ratio is $$\frac{T_1}{T_2} = \frac{\Phi_1 I_1}{\Phi_2 I_2} = \frac{2I_1^2}{I_2^2} = \frac{N_1^2}{N_2^2}$$

Therefore, $$\frac{N_2}{N_1} = \frac{I_2}{I_1\sqrt{2}} \qquad (2)$$

The motor field circuit is unsaturated and as the field winding is shunted by a resistance equal to itself the field current is one-half the input current.

Hence, $$\frac{\Phi_1}{\Phi_2} = \frac{I_1}{0{\cdot}5I_2}$$
$$= 2\frac{I_1}{I_2} \qquad (3)$$

From (1), (2) and (3),
$$\left(\frac{N_2}{N_1}\right)^2 = \frac{\Phi_1}{\Phi_2} \times \frac{I_2}{I_1}$$

i.e. $$\left(\frac{N_2}{N_1}\right)^2 = 2 \times \frac{I_1}{I_2} \times \frac{I_2}{I_1\sqrt{2}}$$
$$= \sqrt{2}$$
$$\left(\frac{N_2}{500}\right)^2 = \sqrt{2}$$
$$N_2 = 500\sqrt[4]{2} = \mathbf{595\ r.p.m.}$$

From (2), $$\frac{I_2}{\surd 220} = \frac{N_2}{N_1}$$
$$= \sqrt[4]{2}$$

Hence, $$I_2 = \mathbf{33{\cdot}6\ amperes.}$$

117. *The speed of a* 500-*volt shunt motor is raised from* 700 *r.p.m. to* 1000 *r.p.m. by field weakening, the total torque remaining unchanged. The armature and field resistances are* 0·8 *ohm and* 750 *ohms respectively, and the current at the lower speed is* 12 *amperes. Calculate the additional shunt field resistance required, assuming the magnetic circuit to be unsaturated and neglecting all losses.* (*C. and G. Final, Pt. II*, 1943)

It is assumed that at the lower speed the field circuit resistance is that of the field winding alone, i.e. 750 ohms.

Field current at 700 r.p.m. $= \frac{500}{750}$
$= 0{\cdot}67$ amperes.

Armature current at this speed $= 12 - 0{\cdot}67$
$= 11{\cdot}33$ amperes.

Voltage drop in the armature $= 11{\cdot}33 \times 0{\cdot}8$
$= 9{\cdot}06$ volts.

Back e.m.f. in the armature $= 500 - 9{\cdot}06$
$= 490{\cdot}94$ volts.

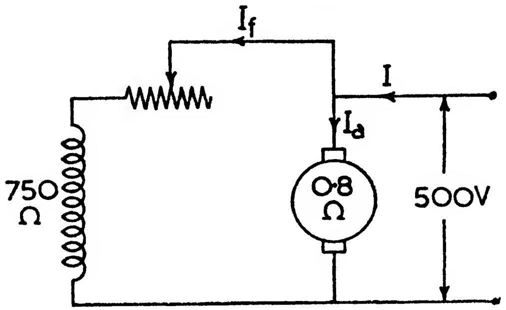


Fig. 82

The back e.m.f. is proportional to the product of flux and speed if the torque is constant, hence

At 1000 r.p.m. back e.m.f. $= 490{\cdot}94 \times \frac{1000}{700} \times \frac{\Phi_2}{\Phi_1}$

$= 701{\cdot}4 \times \frac{\Phi_2}{\Phi_1}$ (1)

Since the torque remains constant the product of flux and armature current is also constant.

i.e. $\Phi_1 I_{a_1} = \Phi_2 I_{a_2}$ (2)

Also the flux is proportional to the field current as the magnetic circuit is unsaturated.

Therefore from (2) $I_{f_1} I_{a_1} = I_{f_2} I_{a_2}$

Hence $0{\cdot}67 \times 11{\cdot}33 = I_{f_2} \times I_{a_2}$

$I_{a_2} = \frac{7{\cdot}56}{I_{f_2}}$ (3)

Back e.m.f. at 1000 r.p.m. $= 500 - 0{\cdot}8\, I_{a_2}$

$= 500 - 0{\cdot}8 \times \frac{7{\cdot}56}{I_{f_2}}$

$= 500 - \frac{6{\cdot}05}{I_{f_2}}$ (4)

But from (1), back e.m.f. at 1000 r.p.m. $= 701{\cdot}4 \times \frac{\Phi_2}{\Phi_1}$

$= 701{\cdot}4 \times \frac{I_{f_2}}{I_{f_1}}$

$= 701{\cdot}4 \times \frac{I_{f_2}}{0{\cdot}67}$

$= 1052{\cdot}1\, I_{f_2}$ (5)

From (4) and (5), $1052{\cdot}1\, I_{f_2} = 500 - \frac{6{\cdot}05}{I_{f_2}}$

i.e. $1052{\cdot}1\, I_{f_2}^2 - 500\, I_{f_2} + 6{\cdot}05 = 0$

Solving this equation for I_{f_2} gives the field current at 1000 r.p.m. as 0·463 ampere.

Therefore, at 1000 r.p.m., field circuit

$$\text{resistance} = \frac{500}{0{\cdot}463} \text{ ohms}$$
$$= 1080 \text{ ohms}$$

Additional resistance required $= 1080 - 750$

$= \mathbf{330\ ohms.}$

118. *Rheostatic braking is used to limit the speed of a d.c. crane motor to* 600 *r.p.m. when the supply is cut off. The motor has a resistance of* 1 *ohm and the descending load exerts a torque of* 200 *lb.-ft. Calculate the approximate value of the resistance to be connected across the motor terminals.*

Points on the magnetization curve at 600 *r.p.m. are:*

Current, amperes	42	43	44	45
e.m.f., volts	383	390	397	403

(*C. and G. Final, Pt. II,* 1944)

The descending load drives the motor as a d.c. generator which supplies power to the resistance connected across the terminals.

$$\text{Power input to the machine by the load} = \frac{2\pi \text{ N T}}{33000} \text{ h.p.}$$
$$= \frac{2\pi \times 600 \times 200}{33000} \text{ h.p.}$$
$$= 22{\cdot}85 \text{ h.p.}$$

Assuming an efficiency of 85 per cent for the machine acting as a generator,

$$\text{power output into the resistance} = 22{\cdot}85 \times 0{\cdot}85 \times 746$$
$$= 14500 \text{ watts.}$$

This is the product of the terminal p.d. and the current delivered to the resistance. If V is the terminal p.d., E the generated e.m.f., and I the current, then

$$\text{V I} = 14500$$
$$\text{V} = \frac{14500}{\text{I}} \qquad (1)$$

But V is the e.m.f. generated less the voltage drop in the motor resistance, i.e.

$$\text{V} = \text{E} - \text{I} \times 1$$
$$= \text{E} - \text{I} \qquad (2)$$

In Fig. 83 equation (1) is plotted over the range of current given in the question. Equation (2) is also plotted on the same axes from the data given. Where these two graphs intersect gives the terminal p.d. developed by the machine.

From the diagram, V = 342·8 volts

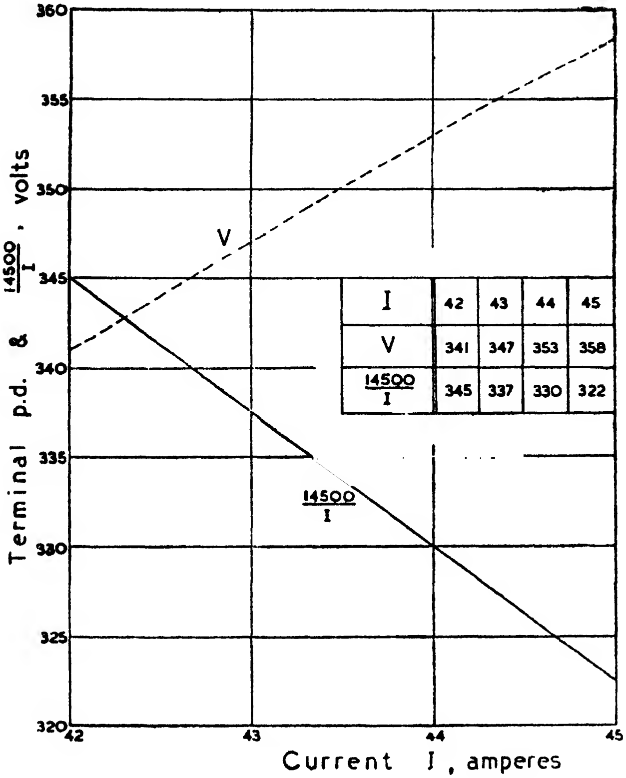
I = 42·3 amperes.

Now the ratio of V to I is the value of the resistance connected across the motor terminals.

Hence, **value of external resistance** $= \dfrac{342{\cdot}8}{42{\cdot}3}$ ohms

$= \mathbf{8{\cdot}12\ ohms,}$ approximately.

Note.—The value is only approximate owing to exact data regarding the motor efficiency not being given.



I	42	43	44	45
V	341	347	353	358
$\frac{14500}{I}$	345	337	330	322

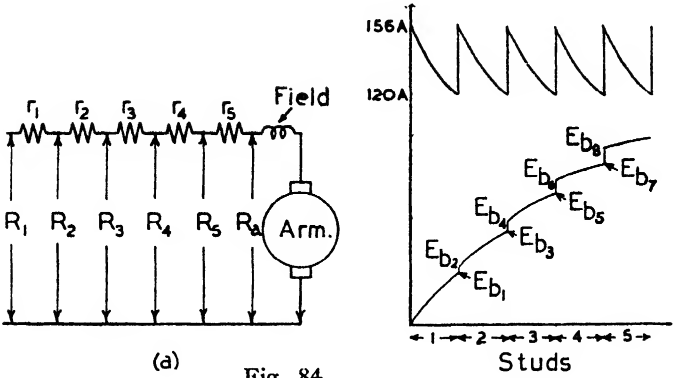
Fig. 83

119. *Explain in terms of the speed-current and torque-current characteristics the advantages of the series motor for haulage purposes.*

The startor for a 460-*volt d.c. series haulage motor has* 5 *resistance sections and the current limits during starting are* 120 *amperes and* 156 *amperes. The resistance of the machine is* 0·19 *ohm, and between these current limits the flux changes by* 10 *per cent. Find the resistance of each section.*

(*C. and G. Final, Pt. II,* 1941)

Referring to Fig. 84(a), on making contact with stud 1 the total resistance in circuit can be calculated by assuming that the back e.m.f. is momentarily zero and that the current is limited by the resistance R_1 to 156 amperes with 460 volts applied.



(a)

Fig. 84

(b)

Hence, $156 = \frac{460}{R_1}$

$R_1 = 2{\cdot}95$ ohms.

When the contact arm is about to be moved from stud 1 the current has fallen to 120 amperes due to the back e.m.f. generated when the speed is N_1. Calling the back e.m.f. E_{b1}

$$120 = \frac{460 - E_{b1}}{2{\cdot}95}$$

$E_{b1} = 106$ volts.

When stud 2 is contacted the speed is momentarily the same at N_1, but due to cutting out the section r_1 the current rises again to 156 amperes, thus increasing the flux by 10 per cent. This increases the back e.m.f. by 10 per cent at the same speed to E_{b2} where $E_{b2} = 1{\cdot}1 \times E_{b1} = 116{\cdot}6$ volts.

Hence, $156 = \frac{460 - 116{\cdot}6}{R_2}$

and $R_2 = 2{\cdot}2$ ohms.

When the contact arm is about to be moved from stud 2 the speed has risen to N_2 and the back e.m.f. to E_{b3}, the current has fallen to 120 amperes and the resistance in circuit is still R_2.

Therefore, $120 = \frac{460 - E_{b3}}{2{\cdot}2}$

$E_{b3} = 196$ volts.

When stud 3 is contacted the current rises to 156 amperes increasing the back e.m.f. momentarily to $E_{b4} = 1{\cdot}1 \times E_{b3}$ while the speed is still N_2 and the resistance in circuit is R_3.

Hence, $156 = \frac{460 - 1{\cdot}1 \times 196}{R_3}$

and $R_3 = 1{\cdot}57$ ohms.

Similarly the circuit resistance measured on the other studs is evaluated as follows:

On leaving stud 3, $120 = \frac{460 - E_{b5}}{1{\cdot}57}$

$E_{b5} = 272$ volts.

On making stud 4,
$$156 = \frac{460 - 1{\cdot}1 \times E_{b6}}{R_4} = \frac{460 - 1{\cdot}1 \times 272}{R_4}$$
$$R_4 = 1{\cdot}03 \text{ ohms.}$$

On leaving stud 4,
$$120 = \frac{460 - E_{b7}}{1{\cdot}03}$$
$$E_{b7} = 336{\cdot}4 \text{ volts.}$$

On making stud 5,
$$156 = \frac{460 - E_{b8}}{R_5} = \frac{460 - 1{\cdot}1 \times 336{\cdot}4}{R_5}$$
$$R_5 = 0{\cdot}58 \text{ ohm.}$$

Hence **the resistances of the various startor sections are:**

$r_1 = R_1 - R_2 =$ **0·75 ohm.**
$r_2 = R_2 - R_3 =$ **0·63 ohm.**
$r_3 = R_3 - R_4 =$ **0·54 ohm.**
$r_4 = R_4 - R_5 =$ **0·45 ohm.**
$r_5 = R_5 - R_a =$ **0·39 ohm.**

(ii.) Design calculations.

120. *A field coil has an internal diameter of* 30 *cm. and an external diameter of* 40 *cm.; it is* 17·5 *cm. long. The outside surface can dissipate* 0·1 *watt per cm.*^{2} *and the cooling effect of the other surfaces may be neglected. The p.d. is* 50 *volts. Calculate the ampere-turns of the coil. Assume the space factor to be* 0·6 *and the resistivity of copper wire to be* 2 *microhms per cm.-cube.*
(*C. and G. Final, Pt. II,* 1944)

Area of the outside curved surface $= \pi \times 40 \times 17{\cdot}5$ sq. cm.
$= 2200$ sq. cm.

Power which can be dissipated $= 2200 \times 0{\cdot}1$ watts
$= 220$ watts.

Field current at 50 volts p.d. $= \frac{220}{50}$ amperes
$= 4{\cdot}4$ amperes.

Resistance of the coil $= \frac{50}{4{\cdot}4}$ ohms
$= 11{\cdot}37$ ohms.

Gross cross section of the coil $= 5 \times 17{\cdot}5$ sq. cm.
$= 87{\cdot}5$ sq. cm.

Nett cross section of copper $= 0{\cdot}6 \times 87{\cdot}5$ sq. cm.
$= 52{\cdot}5$ sq. cm.

Let T = number of turns,
a_w = cross section of copper per conductor.

Then $$T \,.\, a_w = 52{\cdot}5 \text{ sq. cm.} \qquad (1)$$

Mean diameter per turn $= 35$ cm.

Mean length of wire used per turn $= \pi \times 35$ cm.
$= 110$ cm.

Hence, $$\frac{2 \times 10^{-6} \times 110\, T}{a_w} = 11{\cdot}37$$

i.e. $$\frac{T}{a_w} = 51700 \qquad (2)$$

Multiplying (1) and (2),

$$T^2 = 52{\cdot}5 \times 51700$$
$$T = 1640 \text{ turns.}$$

Therefore, **ampere-turns of the coil** $= 4{\cdot}4 \times 1640$
$= \mathbf{7200}$

121. *A shunt coil has to develop* 9,000 *ampere-turns. The volt drop in the coil is* 40, *and the resistivity of the round wire used is* 2·1 *microhms per cm. cube. The winding depth is* 3·5 *cm. approximately and the mean length of turn* 140 *cm. Design a coil so that the power wasted is* 7 *watts per sq. decimetre of the* total *surface of the coil* (*i.e., inner, outer and end surfaces*). (*Take the diameter of the insulated wire to be* 0·4 *mm. greater than the bare wire.*)
(*London B.Sc.Eng., July,* 1945)

Let L_o = the length of an outside turn,
L_i = the length of an inner turn,
L_m = the mean length of turn,
a_w = the copper cross section of the wire,
L = the winding length,
D = the winding depth = 3·5 cm.
ρ = the resistivity of the wire,
T = the number of turns,
V = the voltage drop in the coil,
I = the field current.

Ampere-turn excitation per pole $= TI$
$$= \frac{TV}{R}$$

Coil resistance $$R = \frac{TL_m\rho}{a_w}$$

Hence, $$TI = \frac{TV\, a_w}{TL_m\rho} = \frac{V\, a_w}{L_m\rho}$$

Therefore, $$a_w = \frac{(TI)\, L_m\rho}{V} = \frac{9000 \times 140 \times 2{\cdot}1 \times 10^{-6}}{40} = 0{\cdot}06613 \text{ sq. cm.}$$

Diameter of copper $$= \sqrt{\frac{4 \times 0{\cdot}06613}{\pi}}$$

$= 0.29$ cm.

Diameter over insulation $= 0.29 + 0.04$ cm.

$= 0.33$ cm.

Surface area of coil $= L_oL + L_iL + 2DL_m$

$= 2L_mL + 2DL_m$

$= 2L_m\,(L + D)$

$= 280\,(L + 3.5)$ sq. cm.

Permissible power loss $= 0.07 \times 280\,(L + D)$

$= 19.6\,(L + 3.5) = VI$

Hence $$I = \frac{19.6\,(L + 3.5)}{40}$$

$$= 0.49\,(L + 3.5) \qquad (1)$$

$$\frac{\text{Copper cross section}}{\text{Gross cross section}} = \frac{0.29^2}{0.33^2}$$

$= 0.77$

Therefore, $T\,a_w = 0.77 \times L \times D$

$$T = \frac{0.77 \times L \times 3.5}{0.06613}$$

$$= 41.0\,L \qquad (2)$$

Multiply (1) and (2), $TI = 41.0\,L \times 0.49\,(L + 3.5)$

i.e. $9000 = 20.1\,L^2 + 70.4\,L$

$20.1\,L^2 + 70.4\,L - 9000 = 0$

Solving this quadratic for L,

$L = 19.4$ cm. = winding length.

Number of turns $T = 41.0 \times 19.4$

$= 795$

Number of layers $$= \frac{3.5}{0.33}$$

$= 10.6$ say 11 layers.

Number of turns per layer $$= \frac{795}{11}$$

$= 72.3$ say 72

Hence the coil will consist of 11 layers of 72 turns per layer, the wire diameter being 0·33 cm. over the insulation.

Note.—The nearest standard wire gauge to the above calculated wire diameter is No. 11 S.W.G., of which the uninsulated diameter is 0·295 cm.

Check on the above design.

Resistance of the coil $$= \frac{792 \times 140 \times 2.1 \times 10^{-6}}{0.06613}$$

$= 3.52$ ohms.

Field current $$= \frac{40 \text{ volts}}{3.52 \text{ ohms}}$$

$= 11.35$ amperes.

Power dissipated in the coil $= 40$ volts $\times\ 11.35$ amperes.

$= 454$ watts.

Surface area of coil $= 2 \times 19.4 \times 140 + 2 \times 140 \times 3.5$ sq. cm.

$= 6410$ sq. cm.

$= 64.1$ sq. dm.

$$\text{Power dissipated per unit area} = \frac{454 \text{ watts}}{64{\cdot}1 \text{ sq. dm.}}$$
$$= 7{\cdot}08 \text{ watts per sq. dm.}$$

122. *Find suitable values for the number of poles and the diameter and length of the armature core of a* 400-*kW*, 550-*volt*, 180-*r.p.m. direct current generator. Assume an average flux-density over the whole armature periphery of about* 6,000 *lines per square centimetre and the ampere-conductors per centimetre periphery to be about* 350. (*C. and G. Final, Pt. II*, 1936)

Assuming that the generator has an efficiency of 92 per cent,

$$\text{armature output} = \frac{400}{0{\cdot}92} \text{ kW}$$
$$= 435 \text{ kW.}$$
$$\text{Output in watts per r.p.m.} = \frac{435{,}000}{180}$$
$$= 2420$$

The relation between the output in watts per r.p.m. and the dimensions of the armature core is given by

watts per r.p.m. = GD^2L, where
D = the core diameter,
L = the core length,
G = the output coefficient, which is a constant for the machine.

The value of the output coefficient depends upon the mean flux density, the ampere-conductors per unit length of armature periphery and the ratio of pole arc to pole pitch. If the armature dimensions are given in centimetre units the expression for the output coefficient is

$$G = \frac{\pi^2 \delta \bar{B} \text{ ac}}{60 \times 10^8}$$

In this problem $\bar{B} = 6000$, ac = 350 and δ may be assumed to have a value of 0·7. Then

$$G = \frac{\pi^2 \times 0{\cdot}7 \times 6000 \times 350}{60 \times 10^8}$$
$$= 0{\cdot}002418$$
$$\text{Therefore} \qquad D^2L = \frac{2420}{0{\cdot}002418}$$
$$= 10^6$$

At this stage it is necessary to assume a suitable value for either D or L in order to be able to calculate the other.

For a machine of this output a suitable core length would be about 30 cm. and based on this value for L,

$$D = \sqrt{\frac{10^6}{30}}$$
$$= 183 \text{ cm.}$$

The minimum number of poles which must be provided is set by the armature reaction ampere-turns. For the value of core diameter obtained above and the ampere-conductors per cm. given the total armature ampere-turns can be calculated.

$$\text{Ampere-turns per cm. of periphery} = \frac{\text{Ampere-conductors per cm.}}{2}$$
$$= 175$$

$$\text{Total ampere-turns around whole periphery} = \pi D \times 175$$
$$= \pi \times 183 \times 175$$
$$= 101000$$

The magnetic effect of armature reaction in modern generators is equivalent to a number of ampere-turns on the field windings of the order of 5000 to 9000 ampere-turns per pole. Taking an average value of about 7000

$$\text{Number of poles} = \frac{101000}{7000}$$
$$= 14{\cdot}4 \text{ say 14 poles.}$$

As an alternative method the number of poles may be calculated by making use of the knowledge that the core length is usually about 70 per cent of the pole pitch. Hence if the diameter is 183 cm. and the core length 30 cm.,

$$\text{pole pitch} = \frac{30 \times 100}{70} \text{ cm.}$$
$$= 42{\cdot}8 \text{ cm.}$$
$$\text{Number of poles} = \frac{\pi \times 183}{42{\cdot}8}$$
$$= 13{\cdot}4 \text{ or, say, } 14$$

which checks with the number obtained above.

Therefore this machine could be satisfactorily designed for

Core length = 30 cm.
Core diameter = 183 cm.
Number of poles = 14

123. *Find suitable values for the numbers of armature conductors, Z; commutator sectors, C; slots, S; and all winding pitches, for a symmetrical, 6-pole, 2-circuit, wave-wound armature where Z lies between* 295 *and* 305, *S between* 35 *and* 45, *and C between* 145 *and* 153.

(*C. and G. Final, Pt. II*, 1940)

For a 6-pole, 2-circuit wave-winding neither the number of slots, coils, coil sides per slot, nor commutator sectors may be a multiple of 3. This restricts the values of S and C within the limits given in the question to the following possibilities:

S may be 35, 37, 38, 40, 41 or 43.
C may be 145, 146, 148, 149, 151, 152.

The number of coils is the same as C and hence C must be exactly divisible by S in order that the number of coil sides per slot may be an integer. Moreover this integer must not be a multiple of 3.

These conditions are satisfied by making **S = 37 slots** and **C = 148 sectors,** giving 148 coils and 4 coil sides per slot.

Using single-turn coils there will be twice as many conductors as coil sides, i.e.

$$Z = 148 \times 2$$
$$= \mathbf{296} \text{ conductors}$$
which is within the limits specified.

Slot span $= \dfrac{S}{p}$ where p = number of poles

$$= \frac{37}{6}$$

$= 6\frac{1}{6}$, say 6, i.e., one side of a coil will lie at the top of slot 1 and its other side will be at the bottom of slot 7, and so on.

Commutator span $y_c = \dfrac{C \pm 1}{\frac{1}{2}p}$

$$= \frac{148 \pm 1}{3}$$

$$= \mathbf{49}$$

The winding table may now be drawn up as follows with the numbering referring to coil sides and commutator sectors. From this table a section of the winding is constructed in Fig. 85:

Back end connections		Front end connections		Commutator
1 to 50	and	50 to 99	at	50
99 to 148	and	148 to 197	at	99
197 to 246	and	246 to 295	at	148
295 to 48	and	48 to 97	at	49
201 to 250	and	250 to 3	at	2
3 to 52	and	52 to 101	at	51
101 to 150	and	150 to 199	at	100
199 to 248	and	248 to 1	at	1

close of winding.

124. *Determine suitable main armature dimensions and a winding for a* 10 *h.p.*, 400-*volt, d.c. shunt motor to run at* 1000 *r.p.m. Assume suitable values for loading constants.* (*C. and G. Final, Pt. II*, 1941)

Assume that the motor has a full-load efficiency of 85 per cent.

Full-load input to motor $= \dfrac{10 \times 746 \times 100}{400 \times 85}$ amperes

$= 21{\cdot}9$ amperes.

Of this total input current the field current may be about 0·9 ampere leaving 21 amperes as the armature current.

Total full-load losses $= 400 \times 21{\cdot}9 - 7460$ watts,

$= 1300$ watts.

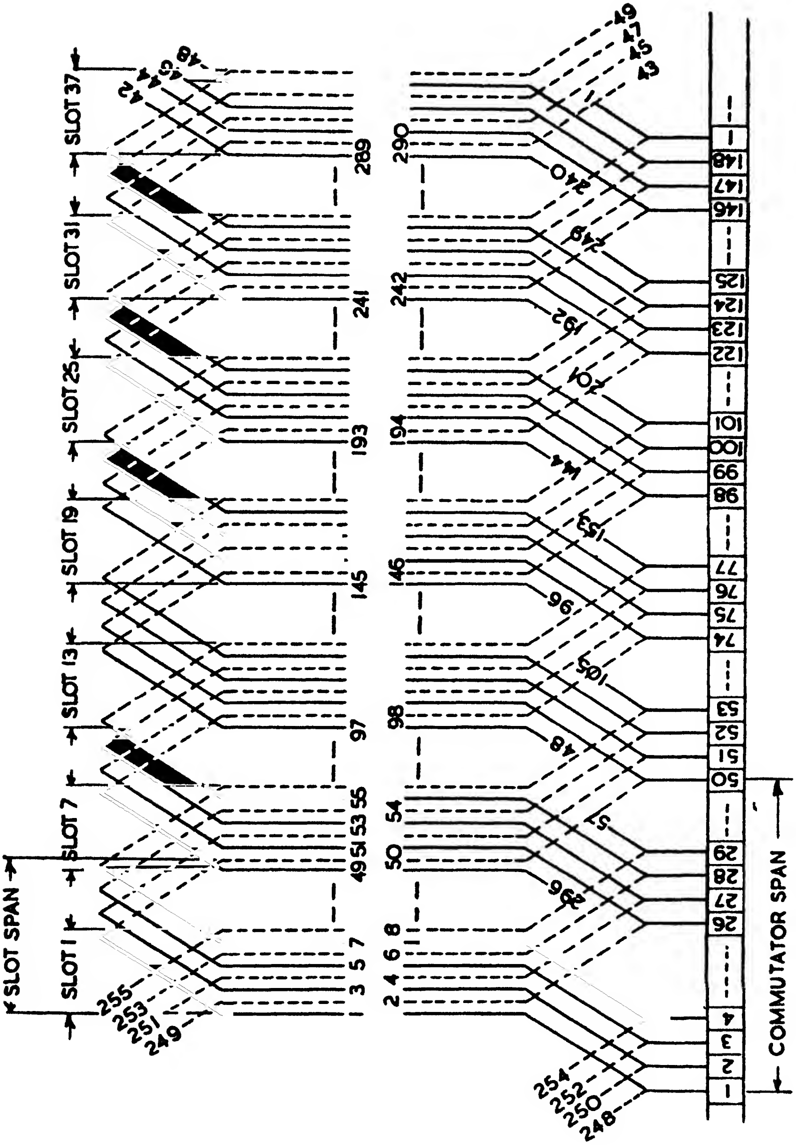

SLOT SPAN
SLOT 1
SLOT 7
SLOT 13
SLOT 19
SLOT 25
SLOT 31
SLOT 37
COMMUTATOR SPAN
255
253
251
249
254
252
250
248
3 5 7
2 4 6 8
49 51 53 55
50 54
97
98
145
146
193
194
241
242
289
290
42
44
46
48
43
45
47
49
1
2
3
4
26
27
28
29
50
51
52
53
74
75
76
77
98
99
100
101
122
123
124
125
146
147
148
1

Fig. 85

If the motor is designed to have maximum efficiency on full-load, one-half of the losses on full-load will occur in the armature.

i.e. Full-load armature loss $= 650$ watts.

Hence, voltage drop in armature $= \dfrac{650}{21}$ volts

$= 31$ volts.

Full-load armature back e.m.f. $= 400 - 31$

$= 369$ volts.

Full-load armature output $= \dfrac{369 \times 21}{1000}$ watts per r.p.m.

$= 7{\cdot}75$ watts per r.p.m.

Output coefficient $G = \dfrac{\pi^2 \delta \bar{B}\, ac}{60 \times 10^8}$

where δ = ratio of pole arc to pole pitch, assumed to be 0·7,

$\bar{B}$ = average flux density, assumed to be 4500 gauss,

ac = specific electric loading, assumed to be 200 ampere-conductors per cm.

Then $G = \dfrac{\pi^2 \times 0{\cdot}7 \times 4500 \times 200}{60 \times 10^8}$

$= 0{\cdot}00104$

and $D^2L = \dfrac{7{\cdot}75}{0{\cdot}00104}$

$= 7450$

Assuming an armature core length of 10 cm.

$D = \sqrt{\dfrac{7450}{10}}$

$= 27$ cm.

Hence the main armature dimensions may be: **core length 10 cm.**
core diameter 27 cm.

Winding. Since the armature current is quite small a simple wave winding will be suitable. The approximate number of conductors required may be calculated from the specific electric loading and the core diameter.

$$\text{Number of conductors} = \frac{\pi D \times ac}{I_a}$$

$$= \frac{\pi \times 27 \times 200}{21} = 810$$

The choice of the number of slots is largely a compromise and for machines up to 50 cm. diameter the number chosen usually works out at about 8—10 slots per pole. A motor of the size, output and speed under consideration would almost certainly have 4 poles, which would fix the number of slots as between 32 and 40. Also, the number of slots when

multiplied by the number of conductors per slot (an even number) must give a total number of conductors in the region of 810.

These conditions are approached fairly closely by having **33 slots** with **24 conductors per slot,** i.e. a total of 792 conductors.

In a machine of this size it would be impossible to have single turn coils as the number of commutator sectors required (396) would make the width of sector too small. Therefore multi-turn coils are necessary and 3 turns per coil would reduce the number of coils and commutator sectors to 132, which is reasonable.

The number of coils and sectors, however, for a 4-pole machine must be odd in order that the expression for the commutator span

$$y_c = \frac{C \pm 1}{\frac{1}{2}p}$$

may be an integer. If C is made 131

$$y_c = \frac{131 \pm 1}{2}$$

$$= 65 \text{ or } 66, \text{ say } 65.$$

Thus there will be 131 commutator sectors, a commutator span of 65, and an **active** number of coils of 131. In order to fill up all the slots completely there must be 132 coils, of which one will therefore be a **dummy coil.** This coil need not form part of the winding, but it is usual to connect it in such a way that it is merely in parallel with one of the other coils. The winding is then effectively a 131-coil one.

With 3 turns per coil there will be 4 coil sides per slot.

$$\text{Slot span} = \frac{33}{4}$$

$$= 8, \text{ say.}$$

Fig. 86 shows a section of the winding drawn from the winding table below. The numbering of the diagram and the table refers to coil sides and it should be remembered that each coil contains 3 turns, with the start and finish of the **coil** connected to commutator sectors. For the sake of clarity the diagram is drawn for single-turn coils.

Back end connections				**Front end connections**				**Commutator**
1	to	66	and	66	to	131	at	66
131	to	196	and	196	to	261/263	at	131
261/263	to	62/64	and	62/64	to	129	at	65
129	to	194	and	194	to	259	at	130
3	to	68	and	68	to	133	at	67
133	to	198	and	198	to	1	at	1

winding closes.

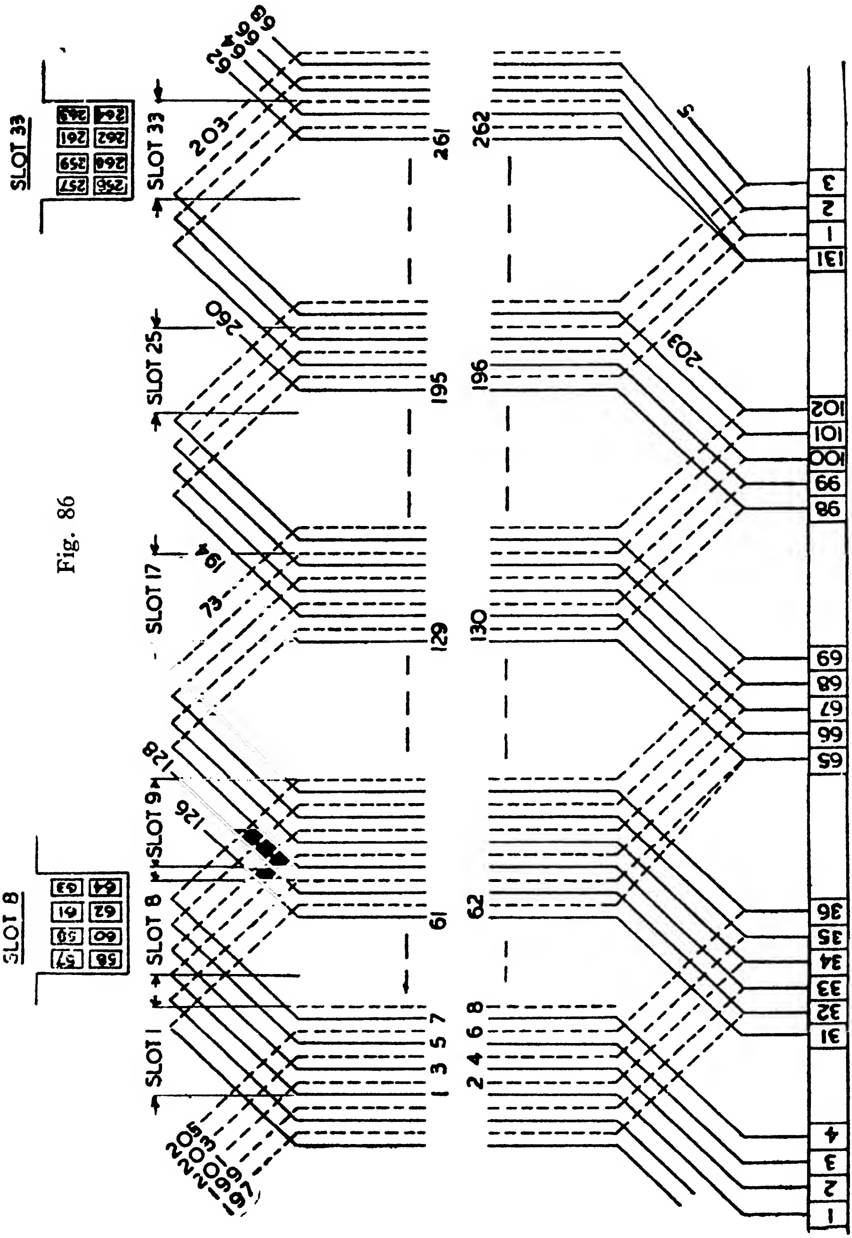

SLOT 33
256
257
259
260
261
262
263
264
SLOT 8
57
58
59
60
61
62
63
64
SLOT 1
SLOT 8
SLOT 9
SLOT 17
SLOT 25
SLOT 33
1 3 5 7
2 4 6 8
61
62
129
130
195
196
261
262
69
68
66
64
203
260
194
73
128
126
205
203
199
197
S
203
1
2
3
4
31
32
33
34
35
36
65
66
67
68
69
98
99
100
101
102
131
1
2
3

Fig. 86

SECTION II

THE GENERATION AND TRANSMISSION OF ELECTRICAL ENERGY

CHAPTER X

GENERATING STATIONS

(i) Thermal efficiency, fuel consumption, etc.

125. *Explain the reasons for the present tendency towards the use of higher steam pressures and temperatures in modern generating stations.*

The relation between the water evaporated (W *lb.*), *coal consumed* (C *lb.*) *and kWh generated per 8-hour shift for a generating station is as follows:* $W = 60000 + 10{\cdot}5$ *kWh,* $C = 11000 + 1{\cdot}75$ *kWh. To what limiting value does the water evaporated per lb. of coal consumed approach as the station output increases? How much coal per hour would be required to keep the station running at no-load (i.e. zero kWh output)?*

(*I.E.E., Pt. II, May*, 1942)

For an 8-hour shift,

$\mathrm{W} = 60000 + 10{\cdot}5$ kWh = weight of water evaporated (lb.)

$\mathrm{C} = 11000 + 1{\cdot}75$ kWh = weight of coal consumed (lb.)

Hence the weight of water evaporated per lb. of coal consumed is

$$\frac{\mathrm{W}}{\mathrm{C}} = \frac{60000 + 10{\cdot}5 \text{ kWh}}{11000 + 1{\cdot}75 \text{ kWh}} = 6 - \frac{6000}{11000 + 1{\cdot}75 \text{ kWh}}.$$

The limiting value of this expression as the number of kWh increases towards infinity is 6.

Therefore **the weight of water evaporated per lb. of coal consumed approaches a limiting value of 6 lb. as the kWh output increases.**

On no-load, in 8 hours,

$$\text{coal consumed} = 11000 + 1{\cdot}75 \times 0$$
$$= 11000 \text{ lb.}$$

$$\textbf{Coal consumed per hour} = \frac{11000}{8}$$
$$= \mathbf{1375\ lb.}$$

126. *Give a brief account of the routine measurements in a central station to determine the efficiency under working conditions.*

The output of a generating station per 8-hour shift is related to the total weight of coal burnt and water evaporated by the equations $C = 9000 + 1{\cdot}25$ *kWh and* $W = 64800 + 10$ *kWh, where* C *and* W *are the pounds of coal burnt and water evaporated respectively.*

If the coal used has a calorific value of 13000 *British Thermal Units per pound, find the percentage overall efficiency of the station if the output during the shift is* 200×10^3 *kWh.*

Deduce the relation between the weight of coal burnt and the water evaporated per shift and hence find the weight of coal per shift which corresponds to boiler-room and to engine-room losses. (*C. and G. Final, Pt. II,* 1939)

Weight of coal used per shift $= 9000 + 1{\cdot}25 \times 200 \times 10^3$ lb.

$= 259000$ lb.

Calorific value of this fuel $= 259000 \times 13000$ B.Th.U.

$= 3{\cdot}367 \times 10^9$ B.Th.U.

Now, 1 kWh $= 3412$ B.Th.U.

Therefore if the efficiency of the station were 100 per cent, the number of kWh generated would be

$$= \frac{3{\cdot}367 \times 10^9}{3412}$$

$= 986500$ kWh.

The actual output per shift, however, is 200000 kWh.

Hence, **overall efficiency** $= \frac{200000}{986500} \times 100$ per cent

$=$ **20·3 per cent.**

The relation between the weight of coal burnt per shift and the weight of water evaporated is obtained by eliminating the kWh from the equations for W and C, thus

$$W = 64800 + 10 \text{ kWh}$$

$$\text{hence kWh} = \frac{W - 64800}{10}$$

$$C = 9000 + 1{\cdot}25 \text{ kWh}$$

$$= 9000 + 1{\cdot}25\left(\frac{W - 64800}{10}\right)$$

$$= 9000 + 0{\cdot}125W - 8100$$

Therefore $\mathbf{C = 900 + 0{\cdot}125W}$

which is the relation required.

If there were no kWh output per shift,

weight of coal burnt $= 9000$ lb., which corresponds to the combined boiler-room and engine-room losses. Also, if there were no water evaporated per shift, i.e. $W = 0$,

weight of coal burnt $= 900$ lb., from the relation between C and W deduced above. This corresponds to the boiler-room losses.

i.e. **Boiler-room losses are equivalent to 900 lb. of coal per shift.** Hence, **Engine-room losses are equivalent to 9000 — 900 lb.,** i.e. **8100 lb. of coal per shift.**

127. *Discuss the considerations which govern the type and size of turbine to be installed in a hydro-electric power station. What conditions are favourable for the use of* (*a*) *propellor-type turbines,* (*b*) *reaction-type turbines? Sketch the layout of a station for* one *of these types.*

A hydro-electric station operates with a mean head of 180 *ft. and is supplied from a reservoir lake which drains a catchment area of* 200 *square miles over which the average rainfall is* 120 *in. per annum. If* 60 *per cent of this rainfall can be utilized, calculate the power in kW for which the station should be*

designed. Assume that 5 *per cent of the head is lost in pipes, penstocks, etc.; the mechanical efficiency of the turbines is* 85 *per cent; and the efficiency of the generators is* 95 *per cent.* (*I.E.E., Pt. II, November,* 1943)

Volume of water drained per annum $= 200 \times 5280^2 \times \frac{120}{12}$ cu. ft.
$= 55{\cdot}76 \times 10^9$ cu. ft.

Taking the density of water as 62·5 lb. per cu. ft.,

Effective weight of water used per annum $= 0{\cdot}6 \times 55{\cdot}76 \times 10^9 \times 62{\cdot}5$ lb.
$= 2{\cdot}091 \times 10^{12}$ lb.

Effective head of water $= 0{\cdot}95 \times 180$ ft.
$= 171$ ft.

Weight of water falling through this height per minute $= \frac{2{\cdot}091 \times 10^{12}}{365 \times 24 \times 60}$ lb.
$= 3{\cdot}978 \times 10^6$ lb.

Work done on the turbines per minute $= 171 \times 3{\cdot}978 \times 10^6$ ft. lb.
$= 6{\cdot}803 \times 10^8$ ft. lb.

With a turbine efficiency of 85 per cent, mechanical output of the turbines $= \frac{0{\cdot}85 \times 6{\cdot}803 \times 10^8}{33000}$ h.p.
$= 17520$ h.p.

With a generator efficiency of 95 per cent,
electrical output of the generators $= 0{\cdot}95 \times 17520 \times 0{\cdot}746$ kW
= 12420 kW.

128. *Discuss the chief factors which affect the thermal efficiency of a power station equipped with steam-driven turbo-alternators.*

The maximum demand on a station is 80 *MW, the annual load factor is* 45 *per cent, and the thermal efficiency is* 23 *per cent. Calculate the average daily coal consumption if the calorific value of the coal is* 11000 *B.Th.U. per lb. Assume* 1 *kWh to be equivalent to* 3412 *B.Th.U.*

What factors under the control of the station require careful consideration if a high thermal efficiency is to be obtained?

(*I.E.E., Pt. II, November,* 1944)

Average daily kWh produced $= 80000 \times 24 \times 0{\cdot}45$ kWh
$= 8{\cdot}64 \times 10^5$ kWh

Number of B.Th.U. required per day $= 8{\cdot}64 \times 10^5 \times 3412 \times \frac{100}{23}$
$= 1{\cdot}282 \times 10^{10}$ B.Th.U.

With the coal having a calorific value of 11000 B.Th.U. per lb.,
average daily coal consumption $= \frac{1{\cdot}282 \times 10^{10}}{11000 \times 2240}$ tons,
= 519·5 tons.

129. *Enumerate the principal auxiliary plant required for the boilers and turbines of a coal-fired generating station, and state the function of each item enumerated. Sketch the layout of the condensing plant, showing the relative positions of turbine, condenser, pumps, etc.*

A boiler-feed pump, driven by a 3*-phase,* 3000*-volt induction motor, delivers* 500,000 *lb. of water per hour at a pressure of* 350 *lb./in.*^{2}. *If the efficiencies of the pump and motor are* 85 *per cent and* 92 *per cent respectively, and the power factor of the motor is* 0·93, *calculate the current input. Assume the weight of* 1 *cubic foot of hot water to be* 59 *lb.*

(*I.E.E., Sect. B, October,* 1945)

Work done by the pump per lb. of water delivered $= \dfrac{p}{w}$,

where p = the pressure in lb./ft.^{2}
w = the density of water in lb./ft.^{3}

$$= \frac{350 \times 144}{59} \text{ ft. lb.}$$

$$= 855 \text{ ft. lb.}$$

Therefore, work done by the pump per minute $= 855 \text{ ft. lb./lb.} \times \dfrac{500000}{60} \text{ lb.}$

$$= 7{\cdot}125 \times 10^6 \text{ ft. lb.}$$

Horsepower output of the pump $= \dfrac{7125000}{33000}$

$$= 216 \text{ h.p.}$$

Input to the pump $= \dfrac{216}{0{\cdot}85}$ h.p.

Input to the motor $= \dfrac{216}{0{\cdot}85 \times 0{\cdot}92}$ h.p.

$$= \frac{216 \times 746}{0{\cdot}85 \times 0{\cdot}92} \text{ watts,}$$

$$= 206100 \text{ watts.}$$

If I is the motor current in amperes,

$$\sqrt{3} \times 3000 \times I \times 0{\cdot}93 = 206100 \text{ watts,}$$

Whence $I =$ **42·7 amperes**

CHAPTER XI

SHORT-CIRCUIT CALCULATIONS ON GENERATING PLANT AND CURRENT LIMITING REACTORS

(i) Short-circuit kVA.

130. *Each of the three generators in a central station has a short-circuit reactance of* 20 *per cent based upon the respective ratings of* 75 *MVA*, 90 *MVA*, *and* 110 *MVA. Each machine is connected to its own sectional bus-bar and each bus-bar is connected to a tie-bar through a reactor of* 10 *per cent reactance based upon the rating of the alternator connected to it. Calculate the MVA fed into a short-circuit occurring between the bars of the section to which the* 110 *MVA machine is connected.* (*C. and G. Final, Pt. II*, 1942)

The circuit diagram is shown in Fig. 87, A, B and C being the generators and D, E and F the reactors.

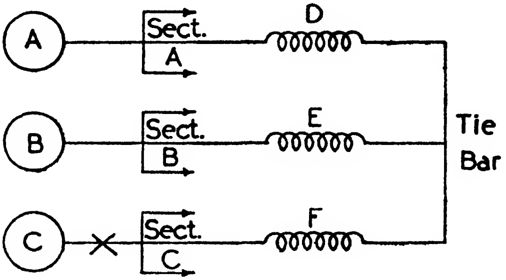


Fig. 87

A 75 MVA, 20 per cent reactance.
B 90 MVA, 20 per cent reactance.
C 110 MVA, 20 per cent reactance.
D 10 per cent reactance on 75 MVA.
E 10 per cent reactance on 90 MVA.
F 10 per cent reactance on 110 MVA.

The first step is to calculate the various reactances on a basis of a common MVA rating, which may be that of either of the machines or alternatively some convenient arbitrary value of base MVA may be chosen. This is necessary because 20 per cent reactance on a 75 MVA rating has a different ohmic value from 20 per cent reactance on 90 or 110 MVA. Before percentage reactances can be combined in series or parallel by the methods used with ohmic reactances, they must all be expressed on a common MVA base. Thus,

Reactance of A on 110 MVA base $= \frac{110}{75} \times 20 = 29{\cdot}35$ per cent.

Reactance of B on 110 MVA base $= \frac{110}{90} \times 20 = 24{\cdot}44$ per cent

Reactance of C on 110 MVA base $= 20$ per cent.

Reactance of D on 110 MVA base $= \frac{110}{75} \times 10 = 14{\cdot}68$ per cent.

Reactance of E on 110 MVA base $= \frac{110}{90} \times 10 = 12{\cdot}22$ per cent.

Reactance of F $= 10$ per cent.

The circuit diagram may now be reduced to the form shown in Fig. 88.

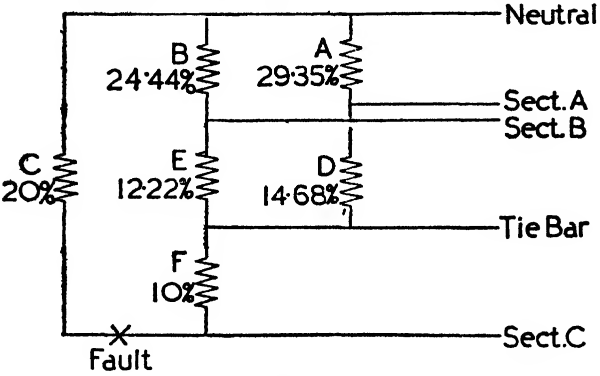


Fig. 88

B and E are in series and at the same time in parallel with A and D (which are themselves in series(. Therefore the diagram may be still further simplified to Fig. 89.

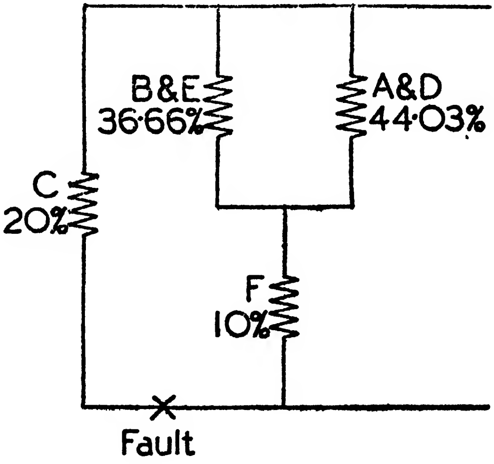


Fig. 89

The equivalent reactance of the parallel groups is

$$\frac{36{\cdot}66 \times 44{\cdot}03}{36{\cdot}66 + 44{\cdot}03} = 20 \text{ per cent.}$$

Still further simplification of the reactances is possible as indicated in Fig. 90.

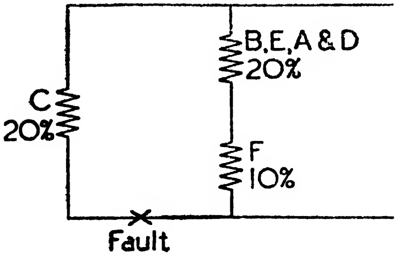


Fig. 90

The total reactance therefore reduces to

$$\frac{20 \times 30}{20 + 30} = 12 \text{ per cent.}$$

$$\text{Short circuit MVA fed into the fault} = 110 \times \frac{100}{12}$$
$$= 917 \text{ MVA.}$$

131. *Give an account of the use of reactance for the control of large powers in generating stations and interconnected systems. Explain precisely what is meant by the percentage rating of a reactor.*

The estimated short-circuit kVA at the bus-bars of a generating station is 1 million kVA, and of another station 666000 *kVA. The generated voltage of each station is* 11000 *volts. Calculate the possible short-circuit kVA at each station when they are linked by an interconnector cable having a reactance of* 0·4 *ohm.* (*C. and G. Final, Pt. II,* 1936)

Let the base kVA be 100000 then, referred to this base,

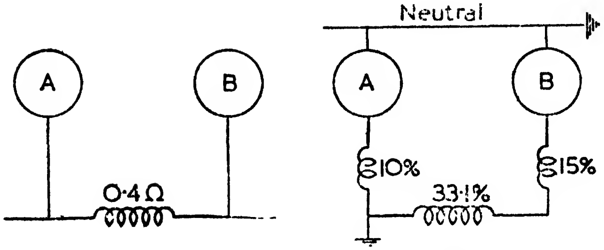
$$\text{reactance of A} = \frac{\text{Base kVA} \times 100}{\text{Short-circuit kVA}} \text{ per cent}$$
$$= \frac{100000 \times 100}{1000000}$$
$$= 10 \text{ per cent.}$$

$$\text{Reactance of B} = \frac{100000 \times 100}{666000}$$
$$= 15 \text{ per cent.}$$

$$\text{Reactance of interconnector} = \frac{\text{Ohmic reactance} \times \text{kVA base}}{10 \times (\text{kV})^2}$$
$$= \frac{0{\cdot}4 \times 100000}{10 \times (11)^2} \text{ per cent}$$
$$= 33{\cdot}1 \text{ per cent.}$$

Hence the equivalent circuit may be drawn as in Fig. 91 (b).

If a fault occurs at station A the net reactance from the fault to the neutral will be that due to 10 per cent in parallel with 48·1 per cent. If it occurs at B the net reactance from the fault to the neutral will be that of 15 per cent in parallel with 43·1 per cent.



(a) (b)

Fig. 91

Net short-circuit reactance at station A $= \dfrac{10 \times 48{\cdot}1}{10 + 48{\cdot}1}$

$= 8{\cdot}28$ per cent.

Short-circuit kVA at A $= \dfrac{100000 \times 100}{8{\cdot}28}$

$= \mathbf{1{\cdot}21 \times 10^6}$ **kVA.**

Net short-circuit reactance at B $= \dfrac{15 \times 43{\cdot}1}{15 + 43{\cdot}1}$

$= 11{\cdot}13$ per cent.

Short-circuit kVA at B $= \dfrac{100000 \times 100}{11{\cdot}13} = \mathbf{0{\cdot}9 \times 10^6}$ **kVA.**

132. *A 3-phase transmission line, operating at* 33 *kV and having a resistance and reactance of* 6 *ohms and* 21 *ohms respectively, is connected to the generating station bus-bars through a* 3000-*kVA step-up transformer which has a reactance of* 5 *per cent. Connected to the bus-bars are two similar* 10000-*kVA alternators, each with a reactance of* 15 *per cent, and one* 5000-*kVA alternator, having a reactance of* 10 *per cent. Calculate the kVA at a short-circuit fault between phases occurring* (*a*) *at the high-voltage terminals of the transformer,* (*b*) *at the load end of the transmission line.*

(*C. and G. Final, Pt. II,* 1944)

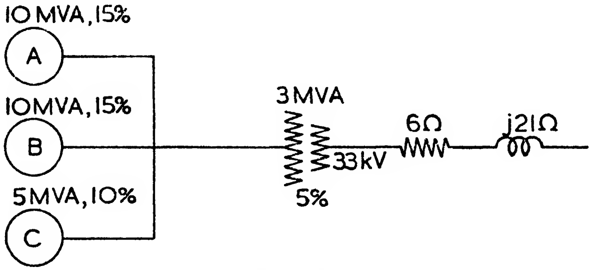


Fig. 92

(a) **Fault at the high-voltage terminals of the transformer.**

Total plant capacity $= 10000 + 10000 + 5000$
$= 25000$ kVA.

The reactances will now be based on the total kVA.

Reactance of A based on

$$25000 \text{ kVA} = \frac{25000}{10000} \times 15$$
$$= 37{\cdot}5 \text{ per cent.}$$

Reactance of B to same base $= 37{\cdot}5$ per cent.

Reactance of C to same base $= \dfrac{25000}{5000} \times 10$
$= 50$ per cent.

The reactances of these three machines are in parallel, hence referred to the total plant capacity of 25000 kVA, the effective reactance is

$$\frac{1}{\frac{1}{37{\cdot}5} + \frac{1}{37{\cdot}5} + \frac{1}{50}} = \frac{150}{11}$$
$$= 13{\cdot}64 \text{ per cent.}$$

Short-circuit kVA $= \dfrac{25000 \times 100}{13{\cdot}64}$
$=$ **183300 kVA.**

(b) **Fault at far end of the transmission line.**

Reactance of transformer referred

$$\text{to } 25000 \text{ kVA base} = \frac{25000}{3000} \times 5$$
$$= 41{\cdot}67 \text{ per cent.}$$

The percentage resistance and reactance of the transmission line must be found, calculated on the basis of 25000 kVA supplied to the line.

The line current corresponding to

$$25000 \text{ kVA at } 33 \text{ kV} = \frac{25000 \times 10^3}{\sqrt{3} \times 33 \times 10^3}$$
$$= 437{\cdot}4 \text{ amperes} = I$$

Percentage resistance drop on

$$25000 \text{ kVA base} = \frac{\sqrt{3} \times IR}{V} \times 100 \text{ per cent}$$

where R is the resistance of each conductor.

$$= \frac{\sqrt{3} \times 437{\cdot}4 \times 6 \times 100}{33 \times 10^3}$$
$$= 13{\cdot}77 \text{ per cent.}$$

Similarly, percentage reactance

$$\text{on this base} = \frac{\sqrt{3} \times 437{\cdot}4 \times 21 \times 100}{33 \times 10^3}$$
$$= 48{\cdot}2 \text{ per cent.}$$

Therefore for the whole plant, and referred to the same kVA base,

Percentage resistance $= 13{\cdot}77$

Percentage reactance $= 13{\cdot}64 + 41{\cdot}67 + 48{\cdot}2$

$= 103{\cdot}5$ per cent

Percentage impedance $= \sqrt{13{\cdot}77^2 + 103{\cdot}5^2}$

$= 104{\cdot}4$ per cent.

Short-circuit kVA $= 25000 \times \dfrac{100}{104{\cdot}4}$

$=$ **23940 kVA.**

133. *Explain what is meant by the percentage rating of a current-limiting reactor.*

A transformer rated at 30000 *kVA and having a short-circuit reactance of* 5 *per cent is connected to the bus-bars of a transformer station which is supplied through two* 33-*kV feeder cables each having an impedance of* $1 + j2$ *ohms. One of the feeders is connected to a generating station with plant rated at* 60000 *kVA connected to its bus-bars having a short-circuit reactance of* 10 *per cent and the other feeder to a station with* 80000 *kVA of generating plant with a reactance of* 15 *per cent. Calculate the kVA supplied to the fault in the event of a short-circuit occurring between the secondary terminals of the transformer.* (*C. and G. Final, Pt. II,* 1941)

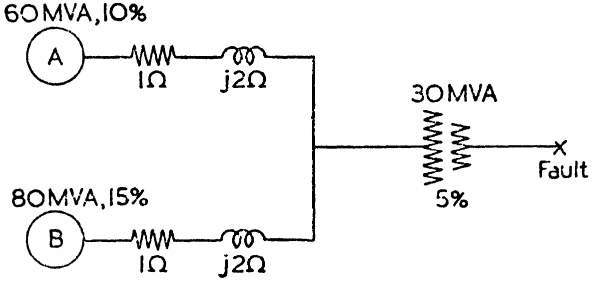


Fig. 93

All the resistances and reactances must be converted to a common kVA base, which may be quite arbitrarily chosen. The value of the base kVA makes no difference to the ultimate result. A convenient value in this case will be 120000 kVA.

Referred to this base,

Reactance of A $= \dfrac{120000}{60000} \times 10$

$= 20$ per cent.

Reactance of B $= \dfrac{120000}{80000} \times 15$

$= 22{\cdot}5$ per cent.

Reactance of transformer $= \dfrac{120000}{30000} \times 5$

$= 20$ per cent.

To convert the feeder impedance into a percentage impedance based on 120000 kVA the method used in Problem 132 may be used, or alternatively it may be found from the expression:

$$\left.\begin{array}{l}\text{Percentage resistance (or re-}\\ \text{actance) at given kVA base}\\ \text{for ohmic value R (or X)}\end{array}\right\} = \frac{100000 \times \text{kVA base} \times \text{R (or X)}}{(\text{Line voltage})^2}$$

In this case R = 1, X = 2, voltage = 33 kV, hence

Percentage feeder resistance on

$$120000\text{ kVA} = \frac{100000 \times 120000 \times 1}{(33 \times 10^3)^2}$$

$$= 11 \text{ per cent.}$$

Percentage feeder reactance on

$$120000\text{ kVA} = \frac{100000 \times 120000 \times 2}{(33 \times 10^3)^2}$$

$$= 22 \text{ per cent.}$$

Total impedance of A and feeder $= j20 + 11 + j22$

$= 11 + j42$ per cent.

Total impedance of B and feeder $= j22{\cdot}5 + 11 + j22$

$= 11 + j44{\cdot}5$ per cent.

These impedances are in parallel, therefore total impedance up to the transformer primary

$$= \frac{(11 + j42)\,(11 + j44{\cdot}5)}{11 + j42 + 11 + j44{\cdot}5}$$

$$= 5{\cdot}505 + j21{\cdot}6$$

This impedance is in series with the transformer reactance, therefore total impedance between

supply and fault $= 5{\cdot}505 + j21{\cdot}6 + j20$

$= 5{\cdot}505 + j41{\cdot}6$

$= 41{\cdot}96$ per cent.

Short-circuit kVA $= \dfrac{\text{Base kVA} \times 100}{\text{Percentage impedance to fault}}$

$$= \frac{120000 \times 100}{41{\cdot}96}$$

= **286000 kVA.**

134. *In a large central station the generating plant connected to each of N sectional bus-bars is rated at K kilovoltamperes with a short-circuit reactance of A per cent. Each sectional bus-bar is connected to a tie-bar through a reactor rated at B per cent. Obtain an expression for the total kVA supplied to a complete short-circuit between the bus-bars of one of the sections.*

Find this value if there are two sectional bus-bars each with two 10000-*kVA alternators, each alternator having a short-circuit reactance of* 20 *per cent and the section reactors each a reactance of* 30 *per cent.*

(*C. and G. Final, Pt. II,* 1940)

The equivalent circuit diagram for this problem is shown in Fig. 94:

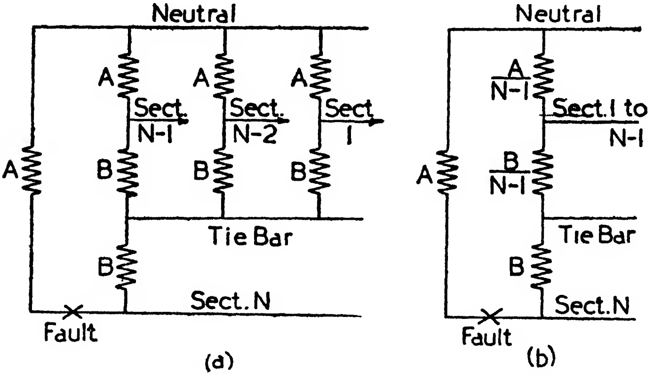


Fig. 94

If the Nth. section is short-circuited there will be (N — 1) paths in parallel between the neutral and the tie-bar, each path having a total reactance of (A + B) per cent. Therefore,

$$\text{total reactance between neutral and tie-bar} = \frac{A}{N-1} + \frac{B}{N-1}$$

$$\text{Total reactance between the neutral and fault via tie-bar} = \frac{A+B}{N-1} + B$$
$$= \frac{A+BN}{N-1}$$

This reactance is in parallel with the reactance A of the short-circuited machine.

$$\text{Total reactance between neutral and fault} = \frac{A \times \dfrac{A+BN}{N-1}}{A + \dfrac{A+BN}{N-1}}$$
$$= \frac{A\ (A+BN)}{N\ (A+B)} \text{ per cent}$$

$$\textbf{Total kVA supplied to fault} = \frac{K \times 100}{\dfrac{A\ (A+BN)}{N\ (A+B)}}$$
$$= \frac{\mathbf{KN\ (A+B) \times 100}}{\mathbf{A\ (A+BN)}}\ \textbf{kVA.}$$

For the values given,

$$\textbf{total short-circuit kVA} = \frac{20000 \times 2\ (20+30) \times 100}{20\ (20 + 30 \times 2)}$$
$$= \mathbf{125{,}000\ kVA.}$$

(ii) **Bus-bar reactors.**

135. *The main bus-bars in a generating station are divided into three sections, each section being connected to a tie-bar by a similar reactor. One*

20000 kVA, 3-phase, 50-frequency, 11000-volt generator, having a short-circuit reactance of 15 per cent, is connected to each section bus-bar. When a short-circuit takes place between the phases of one of the section bus-bars, the voltage on the remaining sections falls to 60 per cent of the normal value. Calculate the reactance in ohms of each reactor.

(*C. and G. Final, Pt. II,* 1943)

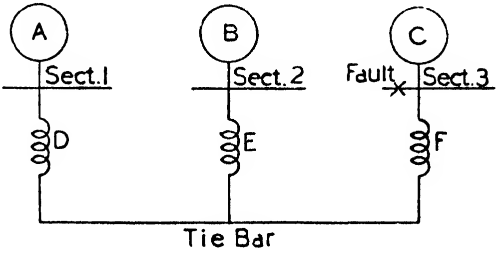


Fig. 95

A, B and C (Fig. 95) are the three generators, each 20000 kVA and of 15 per cent reactance.

D, E and F are the three reactors and suppose each has a reactance of X per cent based on 20000 kVA.

The circuit diagram may be redrawn in the manner described in Problem 130, assuming that the fault occurs on section 3.

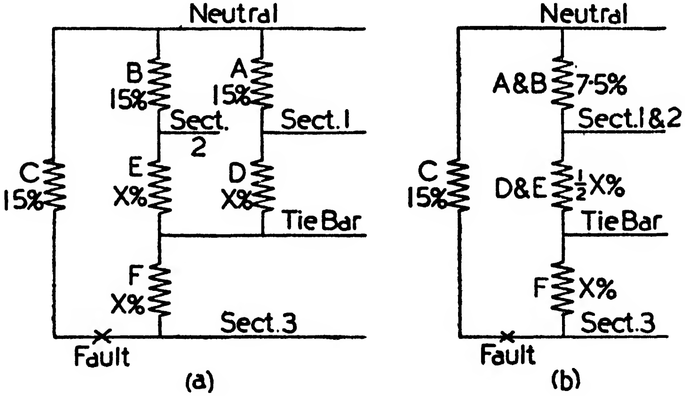


Fig. 96

A and D (in series) are in parallel with B and E (in series).

Reactance of A, B, D and E

$$\text{between tie-bar and neutral} = \frac{X + 15}{2}$$

$$= 0{\cdot}5X + 7{\cdot}5 \text{ per cent}$$

Reactance from neutral to fault

$$\text{via tie-bar} = 0{\cdot}5X + 7{\cdot}5 + X \text{ per cent}$$

$$= 1{\cdot}5X + 7{\cdot}5 \text{ per cent.}$$

The voltage on sections 1 and 2 drops to 60 per cent of the normal value, therefore 40 per cent of the normal voltage is dropped in the reactance of A and B, i.e. the reactance of A and B, which is 7·5 per cent, is 40 per cent of the total reactance from the neutral to the fault. Therefore

$$\begin{aligned} 7{\cdot}5 &= 0{\cdot}4\ (1{\cdot}5X + 7{\cdot}5) \\ &= 0{\cdot}6X + 3 \\ 0{\cdot}6X &= 4{\cdot}5 \\ X &= 7{\cdot}5 \text{ per cent.} \end{aligned}$$

A percentage reactance may be converted into an ohmic value by using the expression

$$X\ (\text{ohms}) = \frac{X\ (\text{per cent}) \times 10\ (\text{kV})^2}{\text{kVA base}}$$

where X per cent = the reactance on the base kVA
and kV = the line voltage.

Hence, **reactance of each reactor** $= \dfrac{7{\cdot}5 \times 10\ (11)^2}{20000}$

$= \mathbf{0{\cdot}454\ ohm.}$

(iii) **Interconnectors.**

136. *Two 3-phase generating stations A and B are linked together through a 33-kV interconnector having a resistance and a reactance of* 0·8 *and* 4·0 *ohms respectively per phase. The load on the generators at station A is* 80 *MW at a power factor of* 0·8 *lagging and the local load taken by consumers connected directly to the A bus-bars is* 50 *MW at a power factor of* 0·707 *lagging. Calculate the load in kW received from station A by station B, its power factor and the phase difference between the voltages of A and B.*

(*C. and G. Final, Pt. II,* 1941)

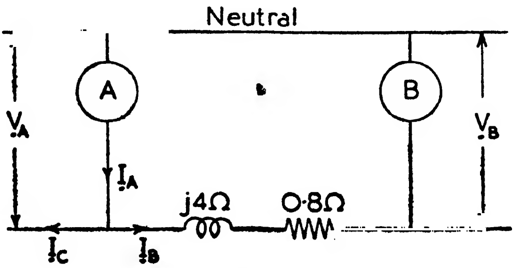


Fig. 97

Load current of machine A $= \dfrac{80 \times 10^6}{\sqrt{3} \times 33 \times 10^3 \times 0{\cdot}8}$

$= 1750$ amperes.

Local load current on A $= \dfrac{50 \times 10^6}{\sqrt{3} \times 33 \times 10^3 \times 0{\cdot}707}$

$= 1237$ amperes.

Phase voltage of A $= \dfrac{33000}{\sqrt{3}}$

$= 19050$ volts.

Let $\underset{\cdot}{V}_A = 19050\ (1 + j0)$

$= 19050\angle\underline{0^\circ}$ volts

i.e. $\underset{\cdot}{V}_A$ is taken as the reference vector.

Then $\underset{\cdot}{I}_A = 1750\ (0{\cdot}8 - j0{\cdot}6)$ since the power factor of $\underset{\cdot}{I}_A$ is 0·8 lagging

$= 1400 - j1050$ amperes

Also $\underset{\cdot}{I}_C = 1237\ (0{\cdot}707 - j0{\cdot}707)$ since the power factor of $\underset{\cdot}{I}_C$ is 0·707 lagging

$\underset{\cdot}{I}_C = 875 - j875$ amperes

Now $\underset{\cdot}{I}_B = \underset{\cdot}{I}_A - \underset{\cdot}{I}_C$

$= 525 - j175$

$= 553{\cdot}4\angle\underline{-18^\circ\ 26'}$ amperes.

Voltage drop in interconnector $= \underset{\cdot}{I}_B\ (0{\cdot}8 + j4)$

$= (525 - j175)\ (0{\cdot}8 + j4)$

$= 1120 + j1960$ volts

$\underset{\cdot}{V}_B = \underset{\cdot}{V}_A -$ drop in interconnector

$= 19050 - (1120 + j1960)$

$= 17930 - j1960$

$= 18040\angle\underline{-6^\circ\ 14'}$ volts

= phase voltage of B

Phase difference between $\underset{\cdot}{V}_B$ and $\underset{\cdot}{I}_B$ $= 18^\circ\ 26' - 6^\circ\ 14'$

$= 12^\circ\ 12'$ with $\underset{\cdot}{I}_B$ lagging

Power factor of load received by B $= \cos 12^\circ\ 12'$

$=$ **0·977 lagging.**

Power received by station B $= 3\ V_B\ I_B \cos 12^\circ\ 12'$

$$= \frac{3 \times 18040 \times 553{\cdot}4 \times 0{\cdot}977}{1000}\ \text{kW}$$

$=$ **29260 kW.**

Phase angle between voltages of A and B $=$ **6° 14′** with **B lagging.**

137. *Two generating stations, each having a bus-bar voltage of* 11000 *volts, are linked through an interconnector cable and a reactor which have a combined reactance of* 3 *ohms per phase and negligible resistance. A load of* 8000 *kW at a lagging power factor of* 0·87 *is taken by the consumers in one station area and the corresponding load in the other area is* 12000 *kW at* 0·71 *power factor. The generator loads on the two stations are equalized by the transference of power through the interconnector. Calculate the power factor at each station and the angle of phase difference between their bus-bar voltages.*

Give a vector diagram to illustrate these conditions and explain how the distribution of (*a*) *the power,* (*b*) *the reactive kVA between the interconnected stations can be controlled.* (*C. and G. Final, Pt. II,* 1937)

The phase voltage of each generator $= \dfrac{11000}{\sqrt{3}}$

$= 6352$ volts.

Let $\underset{\cdot}{V}_A = 6352\ (1 + j0)$ volts

which is taken throughout as the reference vector.

Also let $\underset{\cdot}{I}_A = I_A\ (\alpha - j\beta)$

If I_1 is the load current of the consumers in station A area, I_1 has a power factor of 0·87 referred to $\underset{\cdot}{V}_A$.

Therefore $I_1 = \dfrac{8000 \times 10^3}{\sqrt{3} \times 11000 \times 0{\cdot}87}$

$= 482{\cdot}7$ amperes.

Similarly if $\underset{\cdot}{I}_2$ is the load current taken by the consumers in station B area, $\underset{\cdot}{I}_2$ has a power factor of 0·71 **referred to $\underset{\cdot}{V}_B$**

Therefore $I_2 = \dfrac{12000 \times 10^3}{\sqrt{3} \times 11000 \times 0{\cdot}71}$

$= 887{\cdot}2$ amperes.

Now $\cos \varnothing_1 = 0{\cdot}87$ and $\sin \varnothing_1 = 0{\cdot}4932$, hence

$$\underset{\cdot}{I}_1 = 482{\cdot}7\ (0{\cdot}87 - j0{\cdot}4932)$$
$$= 420 - j238{\cdot}1$$

Since the power output of station A is 10000 kW

$$\sqrt{3} \times 11000 \times I_A\alpha = 10000000$$
$$I_A\alpha = 524{\cdot}9 \text{ amperes} \qquad (1)$$

Now $\underset{\cdot}{I}_3 = \underset{\cdot}{I}_A - \underset{\cdot}{I}_1$

where $\underset{\cdot}{I}_3$ is the current in the interconnector

$$= I_A\ (\alpha - j\beta) - (420 - j238{\cdot}1)$$
$$= 104{\cdot}9 - j\ (I_A\beta - 238{\cdot}1) \qquad (2)$$

$$\underset{\cdot}{V}_B = \underset{\cdot}{V}_A - \text{voltage drop in interconnector}$$

i.e. $\underset{\cdot}{V}_B = 6352\ (1 + j0) - j3\ (104{\cdot}9 - j\ (I_A\beta - 238{\cdot}1)\,)$

$$= 6352 - j314{\cdot}7 - 3I_A\beta + 714{\cdot}3$$
$$= (7066 - 3I_A\beta) - j314{\cdot}7 \qquad (3)$$

The bus-bar voltages are equal, i.e. $V_A = V_B = 6352$ volts

Let $\underset{\cdot}{V}_B = 6352\ (x - jy) \qquad (4)$

From (3) and (4)

$$6352\ y = 314{\cdot}7$$
$$y = 0{\cdot}04956, \text{ and since } x^2 + y^2 = 1,$$
$$x = 0{\cdot}9988$$

$\underset{\cdot}{V}_B$ lags $\underset{\cdot}{V}_A$ by arc tan $\dfrac{y}{x} = 2° \ 51'$

Hence, **angle between bus-bar voltages is 2° 51′** .

From (3) and (4),

$$6352x = 7066 - 3I_A\beta$$
$$6352 \times 0{\cdot}9988 = 7066 - 3I_A\beta$$
$$3I_A\beta = 7066 - 6343$$
$$= 723$$
$$I_A\beta = 241 \text{ amperes.}$$

From (1) $I_A\alpha = 524{\cdot}9$ amperes.

Hence, $\dfrac{\beta}{\alpha} = 0{\cdot}4592$

$= \tan \varnothing_A$ where $\varnothing_A$ is the angle between $\underset{\cdot}{V}_A$ and $\underset{\cdot}{I}_A$.

Therefore $\varnothing_A = 24^\circ\ 40'$

$\cos \varnothing_A = \alpha =$ **0·9087 = power factor of station A.**

Now $\underset{\cdot}{V}_B$ lags $\underset{\cdot}{V}_A$ by $2^\circ\ 51'$, and also $\underset{\cdot}{I}_2$ lags $\underset{\cdot}{V}_B$ by arc cos $0{\cdot}71 = 44^\circ\ 46'$.

Therefore $\underset{\cdot}{I}_2$ lags $\underset{\cdot}{V}_A$ by $47^\circ\ 37'$

$\cos 47^\circ\ 37' = 0{\cdot}6741$ and $\sin 47^\circ\ 37' = 0{\cdot}7387$.

Hence,
$$\begin{aligned}\underset{\cdot}{I}_2 &= 887{\cdot}2\,(0{\cdot}6741 - j0{\cdot}7387)\\ &= 598{\cdot}1 - j655{\cdot}4\end{aligned}$$

From (2)
$$\begin{aligned}\underset{\cdot}{I}_3 &= 104{\cdot}9 - j\,(241 - 238{\cdot}1)\\ &= 104{\cdot}9 - j2{\cdot}9\\ \underset{\cdot}{I}_B &= \underset{\cdot}{I}_2 - \underset{\cdot}{I}_3\\ &= 598{\cdot}1 - j655{\cdot}4 - 104{\cdot}9 + j2{\cdot}9\\ &= 493{\cdot}2 - j652{\cdot}5\\ &= 818\,\underline{/\;-52^\circ\ 54'}\ \text{amperes.}\end{aligned}$$

Therefore $\underset{\cdot}{I}_B$ lags $\underset{\cdot}{V}_A$ by $52^\circ\ 54'$ and $\underset{\cdot}{I}_B$ lags $\underset{\cdot}{V}_B$ by $(52^\circ\ 54' - 2^\circ\ 51') = 50^\circ\ 3'$

Power factor of station B = cos $50^\circ\ 3'$ = **0·642 lagging.**

CHAPTER XII

TRANSMISSION LINES—GENERAL

(i) Voltage regulation and efficiency of lines.

138 *Show how the cross-sectional area of a feeder of given length supplying a given amount of power with a given percentage loss varies with the voltage of transmission.*

A load of 10 *kW is supplied from a distribution centre* 4 *miles away through a* 2*-core cable, each conductor having a resistance of* 0·118 *ohm per thousand yards. Find the voltage at the sending end if the loss of energy in the cable is* 10 *per cent of that supplied to the feeder.*

(*C. and G. Final, Pt. II,* 1940)

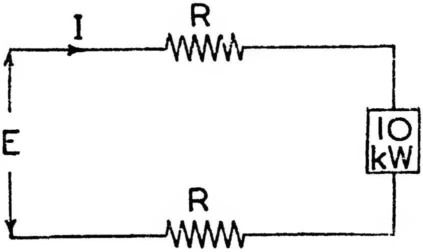


Fig. 98

$$\text{Resistance of both cores} = 2 \times 0{\cdot}118 \times 4 \times \frac{1760}{1000}$$
$$= 1{\cdot}662 \text{ ohms.}$$

The power loss is 10 per cent of that supplied to the feeder, therefore the load delivered is 90 per cent of the feeder input and = 10 kW. Hence

$$\text{Loss in cable} = \frac{10}{9} \text{ kW}$$
$$= 1{\cdot}111 \text{ kW.}$$
$$I^2 \times 1{\cdot}662 = 1111 \text{ watts}$$
$$I = 25{\cdot}86 \text{ amperes.}$$
$$\text{Power input} \quad EI = 11{\cdot}111 \text{ kW.}$$
$$E \times 25{\cdot}86 = 11111 \text{ watts.}$$
$$E = \mathbf{429{\cdot}6 \text{ volts}} = \textbf{sending end voltage.}$$

139. *Deduce an expression for calculating the approximate voltage regulation of a short transmission line.*

A single-phase line has an impedance of 5 ∠60° *ohms and supplies a load of* 120 *amperes,* 3300 *volts at* 0·8 *lagging power factor. Calculate the sending end voltage and draw a vector diagram approximately to scale.*

(*C. and G. Final, Pt. II*, 1940)

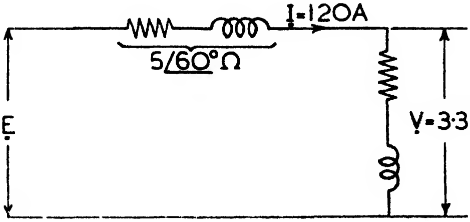


Fig. 99

Since the load power factor is 0·8 lagging the phase angle between V and I (Fig. 99) is arc tan $\frac{0\cdot6}{0\cdot8}$.

Therefore, if the current is represented by

$$I = 120\ (1 + j0) \text{ amperes}$$

the load voltage will be represented by

$$V = 3300\ (0\cdot8 + j0\cdot6)$$
$$= 2640 + j1980 \text{ volts.}$$

Line impedance (assumed for both conductors) $= 5\angle 60°$ ohms

$= 5\ (\cos 60° + j\sin 60°)$

$= 2\cdot5 + j4\cdot33$ ohms.

Line impedance drop $= 120\ (2\cdot5 + j4\cdot33)$

$= 300 + j520$ volts.

Sending end voltage = V + impedance drop in the line

$= (2640 + j1980) + (300 + j520)$

$= 2940 + j2500$

= **3859 volts.**

The vector diagram is drawn approximately to scale in Fig. 100

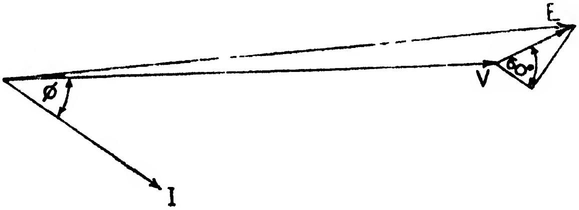


Fig. 100

140. *A balanced star-connected load of* 300 + *j*100 *ohms is supplied by a* 3*-phase line* 40 *miles long with an impedance of* 0·6 + *j*0·7 *ohm per mile* (*line to neutral*). *Find the voltage at the receiving end when the voltage at the sending end is* 33 *kV. What is the phase angle between these voltages?*

(*C. and G. Final, Pt. II,* 1943)

Line impedance per phase $= 40\,(0{\cdot}6 + j0{\cdot}7)$

$= 24 + j28$ ohms.

Sending end phase voltage $= 33 \div \sqrt{3}$ kV

$= 19050$ volts

$= 19050\,(1 + j0)$ volts.

Total impedance per phase $= 24 + j28 + 300 + j100$

$= 324 + j128$ ohms.

Line current $= \dfrac{19050}{324 + j128}$ amperes.

Phase p.d. at the load $= \dfrac{19050}{324 + j128} \times (300 + j100)$ volts

$= 19050 \times \dfrac{(300 + j100)\,(324 - j128)}{324^2 + 128^2}$

$= 19050\,(0{\cdot}906 - j0{\cdot}04943)$

Line voltage at the load $= \sqrt{3} \times 19050\,\sqrt{(0{\cdot}906^2 + 0{\cdot}04943^2)}$ volts

$= 33\,\sqrt{(0{\cdot}8208 + 0{\cdot}0024)}$ kV

$=$ **29·94 kV.**

Phase angle between sending and receiving end voltage $= \arctan \dfrac{0{\cdot}04943}{0{\cdot}906}$

$= \arctan 0{\cdot}05455$

$= 3°\ 7'$ with **the receiving end voltage lagging.**

141. *Deduce an approximate expression for the voltage drop in a short transmission line.*

A 3*-phase line,* 3 *miles long, delivers* 3000 *kW at power factor* 0·8 (*lagging*) *to a load. If the voltage at the supply end is* 11 *kV, determine the voltage at the load and the efficiency of transmission. The resistance per mile of each conductor is* 0·4 *ohm and the reactance* (*line to neutral*) *per mile of each onductor is* 0·3 *ohm.*

(*I.E.E., Pt. II, May* 1942)

Total resistance per phase $= 3 \times 0{\cdot}4$

$= 1{\cdot}2$ ohms.

Total reactance per phase $= 3 \times 0{\cdot}3$

$= 0{\cdot}9$ ohm.

The vector diagram per phase is given by Fig. 101 (b).

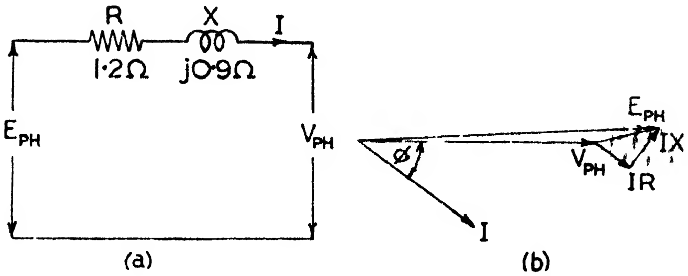


Fig. 101

The exact expression for E_{PH} in the above diagram is

$$E_{PH} = \sqrt{(V_{PH} + IR\cos\varnothing + IX\sin\varnothing)^2 + (IX\cos\varnothing - IR\sin\varnothing)^2}$$

Usually the term ($IX\cos\varnothing - IR\sin\varnothing$) is small enough to be neglected by comparison with the other term under the square root sign and if this is so (which is equivalent to assuming that E_{PH} and V_{PH} are in phase), then

$$E_{PH} = V_{PH} + IR\cos\varnothing + IX\sin\varnothing \text{ approximately.}$$

On the assumption that the power factor is the same at the input end as at the load, which follows from the above,

$$I = \frac{3000 \times 10^3}{\sqrt{3} \times 11 \times 10^3 \times 0{\cdot}8}$$
$$= 196{\cdot}8 \text{ amperes.}$$
$$IR = 236{\cdot}2 \text{ volts and } IX = 177{\cdot}1 \text{ volts.}$$

Hence,
$$\frac{11000}{\sqrt{3}} = V_{PH} + (236{\cdot}2 \times 0{\cdot}8 + 177{\cdot}1 \times 0{\cdot}6)$$
$$6352 = V_{PH} + 189 + 106$$
$$V_{PH} = 6057 \text{ volts.}$$

Line voltage at the load $= \sqrt{3} \times 6057 \times 10^{-3}$ kV
$=$ **10·5 kV.**

I^2R loss in transmission $= 3 \times 196{\cdot}8^2 \times 1{\cdot}2 \times 10^{-3}$ kW
$= 139{\cdot}5$ kW.

Input to the line $= 3139{\cdot}5$ kW.

Efficiency of transmission $= \frac{3000}{3139{\cdot}5} \times 100$ per cent
$=$ **95·6 per cent.**

142. *A 3-phase load of* 3000 *kVA*, 0·8 *power factor, is supplied at* 11 *kV from a step-down transformer having a ratio of* 3 : 1. *The primary side of the transformer is connected to a transmission line, the constants of which are: resistance per conductor*, 2 *ohms; reactance per conductor* 3 *ohms. The resistance and reactance per phase of the primary windings of the transformer (which*

are star-connected) are 5 *ohms and* 10 *ohms respectively, and the corresponding values for the secondary windings (which are delta-connected) are* 1·5 *ohms and* 3 *ohms respectively. Determine the voltage and power factor at the sending end of the transmission line.* (*I.E.E., Pt. II, November*, 1942)

The transformer is connected star-delta and has a **line** voltage ratio of 3 : 1. Therefore the **phase** voltage and turns ratios are $\sqrt{3} : 1$.

Let the resistance and reactance of both sides of the transformer be referred to the primary side.

$$\text{Equivalent resistance per phase} = 5 + (\sqrt{3})^2 \times 1{\cdot}5 = 9{\cdot}5 \text{ ohms} = R_T$$

$$\text{Equivalent reactance per phase} = 10 + (\sqrt{3})^2 \times 3 = 19 \text{ ohms} = X_T$$

Therefore, including the constants of the transmission line,

$$\text{total resistance per phase} = 2 + 9{\cdot}5 = 11{\cdot}5 \text{ ohms} = R_O$$

$$\text{Total reactance per phase} = 3 + 19 = 22 \text{ ohms} = X_O$$

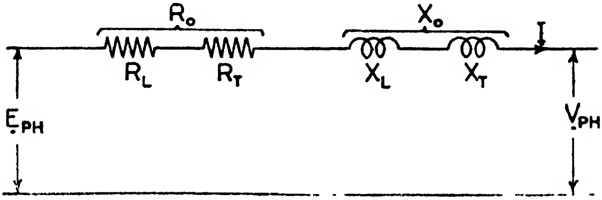


Fig. 102

$$\text{Load phase voltage referred to the primary} = 11\sqrt{3} \text{ kV} = 19{\cdot}05 \text{ kV} = V_{ph}$$

If the magnetizing current is small enough to be neglected,

$$\text{primary line current } I = \frac{3000 \times 10^3}{\sqrt{3} \times 33 \times 10^3} = 52{\cdot}49 \text{ amperes.}$$

Let the current $\underset{\cdot}{I}$ be the reference vector, i.e.

$$\underset{\cdot}{I} = 52{\cdot}49\,(1 + j0) \text{ amperes.}$$

Now, since the load power factor is 0·8 (assumed lagging), therefore $\cos\varnothing = 0{\cdot}8$ and $\sin\varnothing = 0{\cdot}6$, and the load phase voltage may be written

$$\underset{\cdot}{V}_{PH} = 19{\cdot}05\,(0{\cdot}8 + j0{\cdot}6) \text{ kV} = 15{\cdot}24 + j11{\cdot}43 \text{ kV.}$$

$$\text{Total impedance drop referred to the primary} = \underset{\cdot}{I}\underset{\cdot}{Z}_o = 52{\cdot}49\,(11{\cdot}5 + j22) \text{ volts} = 603{\cdot}9 + j1154 \text{ volts} = 0{\cdot}6039 + j1{\cdot}154 \text{ kV.}$$

Then,

$$\underset{\cdot}{E}_{PH} = \underset{\cdot}{V}_{PH} + \underset{\cdot}{I}\underset{\cdot}{Z}_o = 15{\cdot}24 + j11{\cdot}43 + 0{\cdot}6 + j1{\cdot}154 = 15{\cdot}84 + j12{\cdot}58 \text{ kV.}$$

$$E_{PH} = 20{\cdot}23 \text{ kV.}$$

Line voltage at sending end $= E_{PH}\sqrt{3} =$ **35 kV.**

Phase angle at sending end $= \text{arc tan}\,\frac{12{\cdot}58}{15{\cdot}84}$

$= \text{arc tan } 0{\cdot}7941$

$= 38^\circ\ 27'$

Power factor at sending end $= \cos 38^\circ\ 27'$

$=$ **0·783 lagging.**

143. *A balanced load, equivalent to three equal inductive impedances each of* 48 *ohms resistance and* 30 *ohms reactance, is mesh connected at the end o, a* 3-*phase transmission line which has a resistance of* 3 *ohms per conductor and a reactance (line to neutral) of* 1 *ohm. The voltage at the sending end is maintained at* 3·3 *kV. Find the voltage at the load and the efficiency of transmission.* (*C. and G. Final, Pt. II,* 1944)

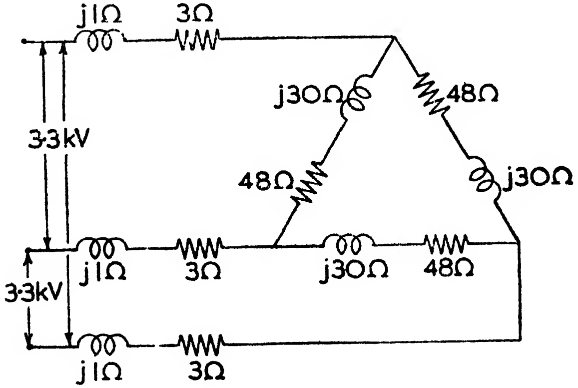


Fig. 101(a)

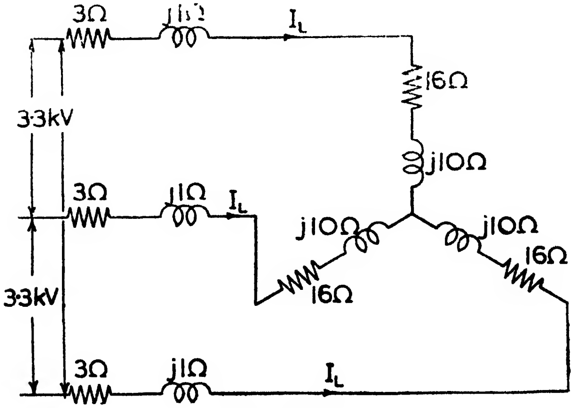


Fig. 103(b)

The load can be represented by the equivalent star-connected load (Fig. 103b) in which each of the impedances is one-third of the delta connected impedances.

Referring to the equivalent circuit diagram,

Total resistance per phase $= 3 + 16$
$= 19$ ohms.

Total reactance per phase $= 1 + 10$
$= 11$ ohms.

Total impedance per phase $= \sqrt{19^2 + 11^2}$
$= 21{\cdot}95$ ohms.

Line current $= \dfrac{3300}{\sqrt{3}} \times \dfrac{1}{21{\cdot}95}$
$= 86{\cdot}82$ amperes $= I_L$

Voltage drop in conductor resistance $= 3 \times 86{\cdot}82$ volts
$= 260{\cdot}46$ volts.

Voltage drop in conductor reactance $= 1 \times 86{\cdot}82$ volts
$= 86{\cdot}82$ volts.

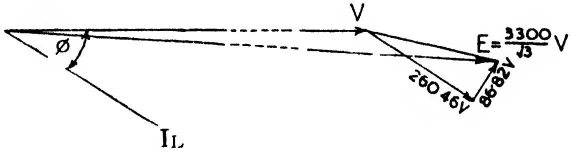


Fig. 104

From the vector diagram (Fig. 104) where V is the phase voltage across the equivalent star load,

$$(V + 260{\cdot}5\cos\varnothing + 86{\cdot}8\sin\varnothing)^2 + (86{\cdot}8\cos\varnothing - 260{\cdot}5\sin\varnothing)^2 = \left(\frac{3300}{\sqrt{3}}\right)^2$$

Now, $\tan\varnothing = \dfrac{\text{Load reactance}}{\text{Load resistance}}$
$= 0{\cdot}625$

Therefore the angle $\varnothing = 32°$, $\cos\varnothing = 0{\cdot}848$, $\sin\varnothing = 0{\cdot}53$.

$$(V + 260{\cdot}5 \times 0{\cdot}848 + 86{\cdot}8 \times 0{\cdot}53)^2 + (86{\cdot}8 \times 0{\cdot}848 - 260{\cdot}5 \times 0{\cdot}53)^2 = 3630000$$

$$(V + 221 + 46)^2 + (73{\cdot}6 - 138)^2 = 3630000$$

$$(V + 267)^2 = 3630000 \text{ very nearly.}$$

$$V = 1638 \text{ volts.}$$

Therefore, **line voltage across the load** $= 1638\sqrt{3}$
$=$ **2835 volts.**

Power delivered to load per phase $= I_L^2 \times 16$ watts
Power transmitted per phase $= I_L^2 \times 19$ watts

Transmission efficiency $= \dfrac{16}{19} \times 100$ per cent
$=$ **84·2 per cent.**

144. *A 3-phase, 50-c.p.s., overhead transmission line 60 miles long with 132 kV between the lines at the receiving end, has the following constants:*

Resistance per mile per phase = 0·25 *ohm*
Inductance per mile per phase = 2·0 *millihenrys*
Capacitance per mile per phase = 0·014 *microfarad.*

Determine, using an approximate method of allowing for the capacitance, the voltage, current and power factor at the sending end when the load at the receiving end is 70000 *kW at* 0·8 *power factor lagging.*

Draw a vector diagram for the circuit assumed.

(*London B.Sc. Eng., July,* 1945)

For a 60-mile length of line,

Resistance per phase = 60 × 0·25 = 15 ohms.
Inductance per phase = 60 × 2·0 = 120 millihenrys.
Capacitance per phase = 60 × 0·014 = 0·84 microfarad.

Using the nominal-T approximation the equivalent circuit is shown in Fig. 105 for one phase.

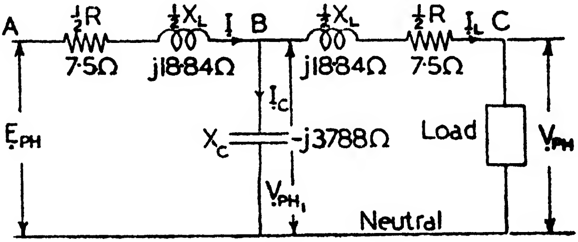


Fig. 105

Load current
$$I_L = \frac{70000}{\sqrt{3} \times 132 \times 0{\cdot}8}$$
$$= 382{\cdot}7 \text{ amperes.}$$

Taking this current as the reference vector,

$$\underset{\cdot}{I}_L = 382{\cdot}7\ (1 + j0)$$
$$V_{PH} = \frac{132}{\sqrt{3}} \text{ kV}$$
$$= 76{\cdot}23 \text{ kV.}$$

As the load power factor is 0·8 lagging,

$$\underset{\cdot}{V}_{PH} = 76{\cdot}23\ (0{\cdot}8 + j0{\cdot}6) \text{ kV}$$
$$= 61{\cdot}0 + j45{\cdot}74 \text{ kV.}$$
$$X_L = j\omega L$$
$$= j2\pi \times 50 \times 0{\cdot}12$$
$$= j37{\cdot}68 \text{ ohms.}$$
$$X_c = -j\,\frac{1}{\omega C}$$
$$= -j\,\frac{10^6}{2\pi \times 50 \times 0{\cdot}84}$$
$$= -j3788 \text{ ohms.}$$

Voltage drop between

B and C $= \underset{\cdot}{I}_L \underset{\cdot}{Z}_{BC}$

$= 382{\cdot}7\ (1 + j0)\ (7{\cdot}5 + j18{\cdot}84)$ volts,

$= 2{\cdot}87 + j7{\cdot}21$ kV.

Therefore, $\underset{\cdot}{V}_{PH_1} = \underset{\cdot}{V}_{PH} + \underset{\cdot}{I}_L \underset{\cdot}{Z}_{BC}$

$= (61{\cdot}0 + j45{\cdot}74) + (2{\cdot}87 + j7{\cdot}21)$ kV

$= 63{\cdot}87 + j52{\cdot}95$ kV.

The current in the capacitance is

$$\underset{\cdot}{I}_c = \frac{\underset{\cdot}{V}_{PH_1}}{X_c}$$

$$= \frac{(63{\cdot}87 + j52{\cdot}95) \times 10^3}{-j3788} \text{ amperes,}$$

$= j16{\cdot}86 - 13{\cdot}98$ amperes.

Input line current $= \underset{\cdot}{I}_L + \underset{\cdot}{I}_c$

i.e. $\underset{\cdot}{I} = 382{\cdot}7 + j16{\cdot}86 - 13{\cdot}98$

$= 368{\cdot}7 + j16{\cdot}86$ amperes.

Hence $I = \sqrt{368{\cdot}7^2 + 16{\cdot}86^2} =$ **369 amperes.**

Voltage drop between

A and B $= \underset{\cdot}{I}\ \underset{\cdot}{Z}_{AB}$

$= (368{\cdot}7 + j16{\cdot}86)\ (7{\cdot}5 + j18{\cdot}84) \times 10^{-3}$ kV

$= 2{\cdot}45 + j7{\cdot}07$ kV.

Therefore $\underset{\cdot}{E}_{PH} = \underset{\cdot}{V}_{PH_1} + \underset{\cdot}{I}\ \underset{\cdot}{Z}_{AB}$

$= 63{\cdot}87 + j52{\cdot}95 + 2{\cdot}45 + j7{\cdot}07$ kV

$= 66{\cdot}32 + j60{\cdot}02$ kV.

Input phase voltage $= \sqrt{66{\cdot}32^2 + 60{\cdot}02^2} = 89{\cdot}45$ kV.

Input line voltage $= 89{\cdot}45\ \sqrt{3} =$ **154·9 kV.**

Input phase angle = Phase difference between $\underset{\cdot}{E}_{PH}$ and $\underset{\cdot}{I}$

$$= \text{arc tan } \frac{60{\cdot}02}{66{\cdot}32} - \text{arc tan } \frac{16{\cdot}86}{368{\cdot}7}$$

$= 42° 9' - 2° 40'$

$= 39° 29'$

Input power factor $= \cos 39° 29. =$ **0·772 lagging.**

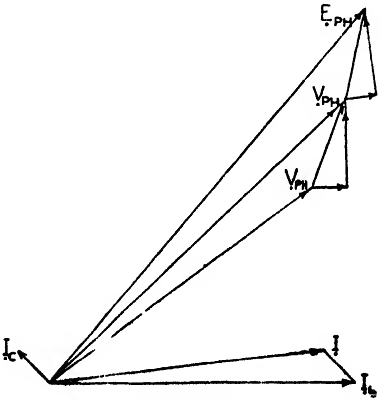


Fig. 106

145. *Draw and explain the vector diagram for a transmission line assuming that half the line capacitance is concentrated at each end of the line.*

A 50-*frequency*, 3-*phase line* 100 *km. long delivers a load of* 40000 *kVA at* 110 *kV and a lagging power factor of* 0·7. *The line constants* (*line to neutral values*) *are: resistance* 11 *ohms, inductive reactance* 38 *ohms, capacitive susceptance* 3 × 10-4 *mhos, leakage negligible. Find the sending-end voltage, current, power factor and power input.*

(*C. and G. Final, Pt. II*, 1937)

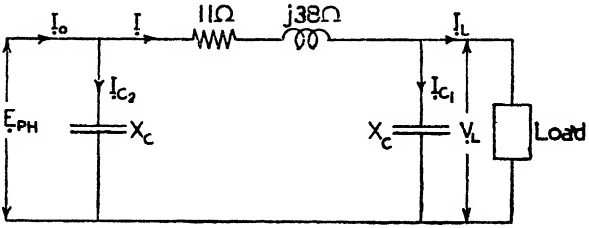


Fig. 107

Line current of the load $= \dfrac{40000}{\sqrt{3} \times 110}$

$= 210$ amperes $= I_L$ (Fig. 107)

Phase voltage at receiving end $= \dfrac{110000}{\sqrt{3}} = 63520$ volts

Let $\underset{\cdot}{I}_L = 210\,(1 + j0)$, then since the power factor of this current is 0·7, $\cos\varnothing_L = 0{\cdot}7$ and $\sin\varnothing_L = 0{\cdot}7141$.

Therefore, $\underset{\cdot}{V}_L = 63{\cdot}52\,(0{\cdot}7 + j0{\cdot}7141)$ kV.

$$\underset{\cdot}{I}_{C_1} = \frac{\underset{\cdot}{V}_L}{X_C}$$

$$= 63520\,(0{\cdot}7 + j0{\cdot}7141)\,j1{\cdot}5 \times 10^{-4}$$

$$= 6{\cdot}352\,(j1{\cdot}05 - 1{\cdot}071) \text{ amperes}$$

$$= j6{\cdot}67 - 6{\cdot}8 \text{ amperes.}$$

Now, $\underset{\cdot}{I} = \underset{\cdot}{I}_L + \underset{\cdot}{I}_{C_1}$

$$= 210 + j6{\cdot}67 - 6{\cdot}8$$

$$= 203{\cdot}2 + j6{\cdot}67 \text{ amperes.}$$

Impedance drop in each line $= \underset{\cdot}{I}\underset{\cdot}{Z}$

$$= (203{\cdot}2 + j6{\cdot}67)\,(11 + j38) \text{ volts}$$

$$= 1982 + j7794 \text{ volts}$$

$$= 1{\cdot}982 + j7{\cdot}794 \text{ kV.}$$

Sending end phase voltage $= V_L + IZ$

$= 63{\cdot}52\,(0{\cdot}7 + j0{\cdot}7141) + 1{\cdot}982 + j7{\cdot}794$ kV

$= 46{\cdot}44 + j53{\cdot}15$ kV $= E_{PH}$

$E_{PH} = 70{\cdot}6$ kV.

Sending end line voltage $= 70{\cdot}6\sqrt{3}$ kV $=$ **122·3 kV.**

Current in input susceptance $= \dfrac{E_{PH}}{X_C}$

$= (46{\cdot}44 + j53{\cdot}15)\,10^3 \times j1{\cdot}5 \times 10^{-4}$

$= j6{\cdot}966 - 7{\cdot}972$ amperes $= I_{C_2}$

Sending end current $I_0 = I + I_{C_2}$

$= 203{\cdot}2 + j6{\cdot}67 + j6{\cdot}966 - 7{\cdot}972$

$= 195{\cdot}2 + j13{\cdot}64$

$I_0 =$ **195·7 amperes.**

Phase angle between E_{PH} and $I_L = \text{arc tan}\ \dfrac{53{\cdot}15}{46{\cdot}44}$

$= 48^\circ\ 51'$, E_{PH} leading

Phase angle between I_0 and $I_L = \text{arc tan}\ \dfrac{13{\cdot}64}{195{\cdot}2}$

$= 4^\circ$, I_0 leading

Therefore, phase angle between E_{PH} and $I_0 = 48^\circ\ 51' - 4^\circ$

$= 44^\circ\ 51'$

Sending end power factor $= \cos 44^\circ\ 51'$

$=$ **0·709**

Input power $= \sqrt{3} \times 122{\cdot}3 \times 195{\cdot}7 \times 0{\cdot}709$ kW

$=$ **29390 kW.**

146. *Deduce an expression for the approximate voltage-drop in a short transmission line. Explain how the effect of capacitance is taken into account in calculations of the approximate voltage-drop in long lines, and draw a vector diagram for such a line.*

A 3-*phase line,* 100 *miles long, has constants* per mile per conductor *as follows: Resistance,* 0·25 *ohm: inductance,* 2 *millihenrys: capacitance, to neutral,* 0·015 *microfarad. Calculate the voltage required at the generating end in order that a load of* 10 *MVA at* 0·8 *power factor (lagging) may be supplied at* 120 *kV. The frequency is* 50 *cycles per sec.*

(*I.E.E., May,* 1943)

The equivalent circuit for a 100-mile length of line is shown in Fig. 108, using the nominal-T method. Fig. 109 is the corresponding vector diagram.

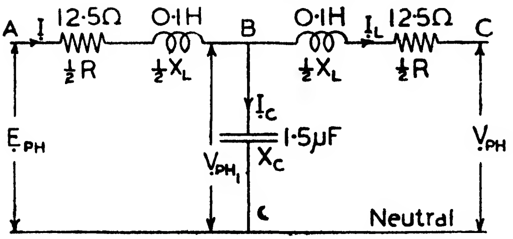


Fig. 108

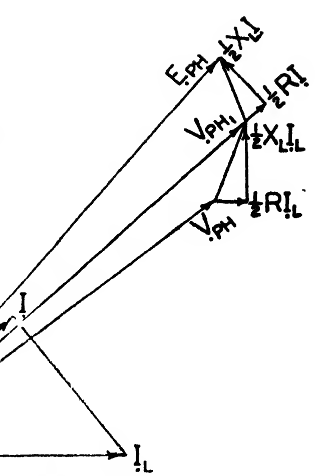


Fig. 109

$$X_L = j\omega L = j \times 2\pi \times 50 \times 0{\cdot}2 = j62{\cdot}8 \text{ ohms}$$

$$X_C = -j\frac{1}{\omega C} = -j\frac{10^6}{2\pi \times 50 \times 1{\cdot}5} = -j2120 \text{ ohms}$$

$$\text{Load current } I_L = \frac{10 \times 10^6}{\sqrt{3} \times 120 \times 10^3} = 48{\cdot}1 \text{ amperes.}$$

Taking this current as the reference vector,

$$I_L = 48{\cdot}1\ (1 + j0) \text{ amperes}$$

$$V_{PH} = \frac{120}{\sqrt{3}} \text{ kV}$$

$$69{\cdot}3 \text{ kV.}$$

Since the power factor is 0·8, therefore $\cos\phi = 0{\cdot}8$ and $\sin\phi = 0{\cdot}6$. Referred to the reference vector V_{PH} may be written

$$\begin{aligned} V_{PH} &= 69{\cdot}3\ (0{\cdot}8 + j0{\cdot}6) \\ &= 55{\cdot}44 + j41{\cdot}58 \text{ kV} \end{aligned}$$

$$\begin{aligned} \text{Voltage drop between B and C} &= I_L \cdot Z_{BC} \\ &= 48{\cdot}1\ (1 + j0)\ (12{\cdot}5 + j31{\cdot}4) \\ &= 601{\cdot}3 + j1510 \text{ volts} \\ &= 0{\cdot}6013 + j1{\cdot}510 \text{ kV.} \end{aligned}$$

Therefore,

$$\begin{aligned} V_{PH_1} &= V_{PH} + I_L Z_{BC} \\ &= 55{\cdot}44 + j41{\cdot}58 + 0{\cdot}6013 + j1{\cdot}510 \\ &= 56{\cdot}04 + j43{\cdot}09 \text{ kV.} \end{aligned}$$

The current in the capacitance is

$$\begin{aligned} I_C &= \frac{V_{PH_1}}{X_C} \\ &= \frac{(56{\cdot}04 + j43{\cdot}09) \times 10^3}{-j2120} \\ &= j26{\cdot}4 - 20{\cdot}3 \text{ amperes.} \end{aligned}$$

Input line current $\underset{\cdot}{I} = \underset{\cdot}{I}_L + \underset{\cdot}{I}_C$
$= 48{\cdot}1 + j26{\cdot}4 - 20{\cdot}3$
$= 27{\cdot}8 + j26{\cdot}4$ amperes.

Voltage drop between A and B $= \underset{\cdot}{I}{\cdot}\underset{\cdot}{Z}_{AB}$
$= (27{\cdot}8 + j26{\cdot}4)\ (12{\cdot}5 + j31{\cdot}4)$
$= -\ 481{\cdot}5 + j1203$ volts
$= -\ 0{\cdot}4815 + j1{\cdot}203$ kV.

Input phase voltage $\underset{\cdot}{E}_{PH} = \underset{\cdot}{V}_{PH_1} + \underset{\cdot}{I}{\cdot}\underset{\cdot}{Z}_{AB}$
$= 56{\cdot}04 + j43{\cdot}09 - 0{\cdot}4815 + j1{\cdot}203$
$= 55{\cdot}56 + j44{\cdot}29$ kV
$E_{PH} = 71{\cdot}05$ kV.

Input line voltage $= 71{\cdot}05\sqrt{3} =$ **123 kV.**

147. *Draw a circuit diagram for the "nominal T" approximation to a transmission line having resistance, inductance and capacitance. Thence draw a vector diagram for this equivalent circuit.*

A 3-*phase transmission line,* 60 *miles long, has the following constants: Resistance per line,* 12 *ohms; inductance per line,* 0·115 *henry; capacitance between each line and neutral,* 1·2 *microfarad. A load of* 30 *MW at* 0·8 *power factor (lagging) is connected to the distant end and is to be supplied at* 120 *kV,* 50 *c/s. Determine the voltage at the sending end of the line.*

(*I.E.E., May,* 1944)

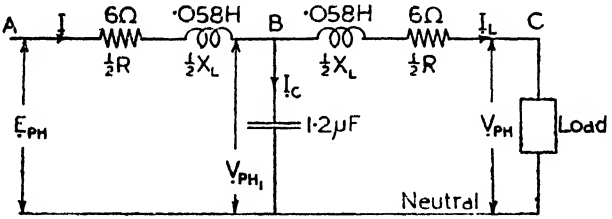


Fig. 110

Using the 'nominal-T' method the circuit diagram for the 60-mile length of line is given by Fig. 110.

$\tfrac{1}{2}X_L = 0{\cdot}5j\ (2\pi \times 50 \times 0{\cdot}115)$
$= j18{\cdot}06$ ohms.

$V_{PH_1} = \dfrac{120}{\sqrt{3}}$ kV
$= 69{\cdot}3$ kV.

Load current $I_L = \dfrac{30 \times 10^6}{\sqrt{3} \times 120 \times 10^3 \times 0{\cdot}8}$
$= 180{\cdot}4$ amperes.

Let $\underset{\cdot}{I}_L = 180{\cdot}4\ (1 + j0)$

Therefore $\underset{\cdot}{V}_{PH} = 69{\cdot}3\ (0{\cdot}8 + j0{\cdot}6)$
$= 55{\cdot}44 + j41{\cdot}58$ kV.

Voltage drop between B and C $= \underset{\cdot}{I}_L\ (6 + j18{\cdot}06)$
$= 180{\cdot}4\ (6 + j18{\cdot}06)$ volts
$= 1{\cdot}082 + j3{\cdot}258$ kV.

Voltage from B to neutral $= 55{\cdot}44 + j41{\cdot}58 + 1{\cdot}082 + j3{\cdot}258$

$= 56{\cdot}52 + j44{\cdot}84 \text{ kV} = \underset{\cdot}{V}_{PH_1}$

Current in the capacitance $= \underset{\cdot}{V}_{PH_1} \cdot j\omega C$

$= (56{\cdot}52 + j44{\cdot}84)\, 10^3 \times j \times 2\pi \times 50 \times 1{\cdot}2 \times 10^{-6}$

$= j21{\cdot}31 - 16{\cdot}9$ amperes.

Therefore, $\underset{\cdot}{I} = \underset{\cdot}{I}_L + \underset{\cdot}{I}_C$

$= 180{\cdot}4 + j21{\cdot}31 - 16{\cdot}9$

$= 163{\cdot}5 + j21{\cdot}31$ amperes.

Voltage drop between A and B $= \underset{\cdot}{I}\,(6 + j18{\cdot}06)$

$= (163{\cdot}5 + j21{\cdot}31)\,(6 + j18{\cdot}06)$

$= 596{\cdot}1 + j3080$ volts

$= 0{\cdot}596 + j3{\cdot}08 \text{ kV} = \underset{\cdot}{V}_{AB}$

Input phase voltage $\underset{\cdot}{E}_{PH} = \underset{\cdot}{V}_{PH_1} + \underset{\cdot}{V}_{AB}$

$= 56{\cdot}52 + j44{\cdot}84 + 0{\cdot}596 + j3{\cdot}08 \text{ kV}$

$= 57{\cdot}12 + j47{\cdot}92 \text{ kV}$

$E_{PH} = 74{\cdot}56 \text{ kV}.$

Input line voltage $= 74{\cdot}56\sqrt{3} = \mathbf{129{\cdot}1 \text{ kV}}.$

(ii) Parallel operation.

148. *Discuss the advantages and disadvantages of supplying important substations by either (a) two overhead lines operating in parallel, each line following a different route, or (b) a ring main.*

Two overhead lines are connected in parallel to supply a load of 10 *MW at* 0·8 *power factor (lagging) and* 30 *kV. The resistance and reactance of one line (A) are* 5·5 *ohms and* 13·5 *ohms, respectively; those of the other line (B) are* 6 *ohms and* 11 *ohms respectively. Calculate (a) the kVA supplied by each line, (b) the power supplied by each line.* (*I.E.E., Pt. II, May,* 1944)

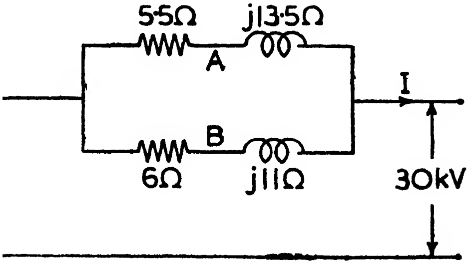


Fig. 111

Impedance of line A $= \underset{\cdot}{Z}_A = 5{\cdot}5 + j13{\cdot}5$ ohms

$= 14{\cdot}58\underline{/67^\circ\ 50'}$ ohms

Impedance of line B $= \underset{\cdot}{Z}_B = 6 + j11$ ohms

$= 12{\cdot}53\underline{/61^\circ\ 24'}$ ohms.

$\underset{\cdot}{Z}_A + \underset{\cdot}{Z}_B = 5{\cdot}5 + j13{\cdot}5 + 6 + j11$

$= 11{\cdot}5 + j24{\cdot}5$

$= 27{\cdot}06\underline{/64^\circ\ 51'}$ ohms.

If $\underset{\cdot}{P}$ is the total kVA supplied to the load,

$$\text{kVA supplied by line A} = \underset{\cdot}{P}\,\frac{\underset{\cdot}{Z}_B}{\underset{\cdot}{Z}_A + \underset{\cdot}{Z}_B}$$

$$\text{kVA supplied by line B} = \underset{\cdot}{P}\,\frac{\underset{\cdot}{Z}_A}{\underset{\cdot}{Z}_A + \underset{\cdot}{Z}_B}$$

these being **vector** expressions.

Now $\underset{\cdot}{P} = 12{\cdot}5$ MVA at power factor 0·8 lagging,

i.e. $\underset{\cdot}{P} = 12{\cdot}5 \angle \underline{-36^\circ\ 52'}$ MVA.

(a) **kVA supplied by A**

$$= 12500 \angle \underline{-36^\circ\ 52'} \times \frac{12{\cdot}53 \angle \underline{61^\circ\ 24'}}{27{\cdot}06 \angle \underline{64^\circ\ 51'}}$$

$$= \frac{12500 \times 12{\cdot}53}{27{\cdot}06} \angle \underline{-36^\circ\ 52' + 61^\circ\ 24' - 64^\circ\ 51'}$$

$$= 5790 \angle \underline{-40^\circ\ 19'} \text{ kVA}$$

= **5790 kVA lagging at power factor 0·763.**

kVA supplied by B

$$= 12500 \angle \underline{-36^\circ\ 52'} \times \frac{14{\cdot}58 \angle \underline{67^\circ\ 50'}}{27{\cdot}06 \angle \underline{64^\circ\ 51'}}$$

$$= \frac{12500 \times 14{\cdot}58}{27{\cdot}06} \angle \underline{-36^\circ\ 52' + 67^\circ\ 50' - 64^\circ\ 51'}$$

$$= 6730 \angle \underline{-33^\circ\ 53'} \text{ kVA}$$

= **6730 kVA lagging at power factor 0·83.**

(b) **Power supplied by A** = 5790 cos 40° 19′ kW
= 5790 × 0·763 kW
= **4415 kW.**

Power supplied by B = 6730 cos 33° 53′ kW
= 6730 × 0·83 kW
= **5585 kW.**

149. *Explain the object of duplicating 3-phase overhead transmission lines and of carrying two such lines over different routes. Draw a vector diagram for two 3-phase lines, of different impedances, operating in parallel, and deduce an expression for the current in each line in terms of the current supplied to the load.*

A 3-phase load of 5 MVA, at power factor 0·8 (lagging), is supplied at 33 kV from two lines connected in parallel. The resistance and reactance per conductor of one line are 6 ohms and 20 ohms respectively. The resistance and reactance per conductor for the other line are 4 ohms and 15 ohms. Calculate the current in each line. (*I.E.E., Pt. II, May,* 1943)

$$\text{Total line current} = \frac{5 \times 10^6}{\sqrt{3} \times 33 \times 10^3}$$

$$= 87{\cdot}5 \text{ amperes} = I$$

The circuit diagram per phase is as shown by Fig. 112.

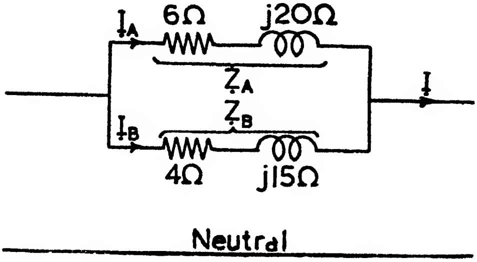


Fig. 112

Since the lines are in parallel, therefore

$$\left.\begin{aligned} I_A Z_A &= I_B Z_B \\ \text{and}\qquad I_A + I_B &= I \end{aligned}\right\}\text{these being \textbf{vector} expressions.}$$

Hence
$$I_A = I - I_B$$
$$= I - I_A \cdot \frac{Z_A}{Z_B}$$

Therefore,
$$I_A + I_A \cdot \frac{Z_A}{Z_B} = I$$
$$I_A \left\{1 + \frac{Z_A}{Z_B}\right\} = I$$
$$I_A = I \cdot \frac{Z_B}{Z_A + Z_B}$$

Similarly
$$I_B = I \cdot \frac{Z_A}{Z_A + Z_B}$$

Now,
$$\begin{aligned} Z_A &= 6 + j20 = 20{\cdot}88 \angle\underline{73^\circ\ 18'} \text{ ohms} \\ Z_B &= 4 + j15 = 15{\cdot}52 \angle\underline{75^\circ\ 4'} \text{ ohms} \\ Z_A + Z_B &= 10 + j35 = 36{\cdot}4 \angle\underline{74^\circ\ 3'} \text{ ohms.} \end{aligned}$$

Taking the voltage as the reference vector, the current lags by arc cos 0·8, hence

$$\begin{aligned} I &= 87{\cdot}5\ (0{\cdot}8 - j0{\cdot}6) \text{ amperes} \\ &= 87{\cdot}5 \angle\underline{-36^\circ\ 52'} \text{ amperes.} \end{aligned}$$

Therefore
$$\begin{aligned} I_A &= 87{\cdot}5 \angle\underline{-36^\circ\ 52'} \times \frac{15{\cdot}52 \angle\underline{75^\circ\ 4'}}{36{\cdot}4 \angle\underline{74^\circ\ 3'}} \\ &= 37{\cdot}31 \angle\underline{-35^\circ\ 51'} \text{ amperes} \\ &= \textbf{37·31 amperes at p.f. 0·811 lagging} \end{aligned}$$

$$\begin{aligned} I_B &= 87{\cdot}5 \angle\underline{-36^\circ\ 52'} \times \frac{20{\cdot}88 \angle\underline{73^\circ\ 18'}}{36{\cdot}4 \angle\underline{74^\circ\ 3'}} \\ &= 50{\cdot}2 \angle\underline{-37^\circ\ 37'} \text{ amperes} \\ &= \textbf{50·2 amperes at p.f. 0·792 lagging.} \end{aligned}$$

150. *A* 6600-*volt substation taking a total* 3-*phase load of* 5000 *kW at a lagging power factor of* 0·8 *is supplied through two feeder cables A and B*

in parallel which have impedances of $Z_A = 0{\cdot}5 + j0{\cdot}8$ *ohm and* $Z_B = 1{\cdot}0 + j0{\cdot}4$ *ohm per phase.*

Calculate the kW and kVA load carried by each cable.

(*C. and G. Final, Pt. II,* 1945)

Total kVA load $= \dfrac{5000}{0{\cdot}8} \angle \underline{\text{arc cos} 0{\cdot}8}$ kVA.

i.e. $P = 6250 \angle \underline{-36^\circ 52'}$ kVA.

$Z_A = 0{\cdot}5 + j0{\cdot}8 = 0{\cdot}943 \angle \underline{58^\circ}$ ohms.

$Z_B = 1{\cdot}0 + j0{\cdot}4 = 1{\cdot}077 \angle \underline{21^\circ 48'}$ ohms.

$Z_A + Z_B = 1{\cdot}5 + j1{\cdot}2 = 1{\cdot}921 \angle \underline{38^\circ 40'}$ ohms.

kVA carried by A $= P \cdot \dfrac{Z_B}{Z_A + Z_B}$

$= 6250 \angle \underline{-36^\circ 52'} \times \dfrac{1{\cdot}077 \angle \underline{21^\circ 48'}}{1{\cdot}921 \angle \underline{38^\circ 40'}}$

$= 3505 \angle \underline{-53^\circ 44'}$ kVA

= 3505 kVA at p.f. 0·592 lagging.

kW carried by A $= 3505 \cos 53^\circ 44'$ kW

$= 3505 \times 0{\cdot}592$ kW

= 2073 kW.

kVA carried by B $= 6250 \angle \underline{-36^\circ 52'} \times \dfrac{0{\cdot}943 \angle \underline{58^\circ}}{1{\cdot}921 \angle \underline{38^\circ 40'}}$

$= 3070 \angle \underline{-17^\circ 32'}$ kVA

= 3070 kVA at p.f. 0·954 lagging.

kW carried by B $= 3070 \cos 17^\circ 32'$ kW

$= 3070 \times 0{\cdot}954$ kW

= 2927 kW.

Note: $\text{kW}_A + \text{kW}_B = 2073 + 2927 = 5000$ kW, the total load specified.

151. *Two single-phase transmission lines are connected in parallel. Their impedances are* $1{\cdot}5 + j3$ *ohms and* $3 + j2$ *ohms respectively. The sending-end voltage is* 10000. *Deduce and draw a locus diagram for the receiving-end voltage for a total current of* 750 *amperes at various power factors. From the diagram determine the voltage regulation for a power factor, at the receiving end, of* 0·8, (*a*) *lagging, and* (*b*) *leading. Determine also the kW transmitted by each line under condition* (*a*). (*London B.Sc.Eng., July* 1944)

Let $Z_A = 1{\cdot}5 + j3 = 3{\cdot}354 \angle \underline{63^\circ 26'}$ ohms.

$Z_B = 3{\cdot}0 + j2 = 3{\cdot}605 \angle \underline{33^\circ 41'}$ ohms.

If Z_P is the combined impedance of both lines in parallel,

$$Z_P = \frac{Z_A \times Z_B}{Z_A + Z_B}$$

$$= \frac{3{\cdot}354 \angle \underline{63^\circ 26'} \times 3{\cdot}605 \angle \underline{33^\circ 41'}}{6{\cdot}727 \angle \underline{48^\circ 1'}}$$

Note: $Z_A + Z_B = 4{\cdot}5 + j5 = 6{\cdot}727 \angle \underline{48^\circ 1'}$

Therefore $Z_P = 1{\cdot}797 \angle \underline{49^\circ 6'}$

Let the total current $I = 750 \angle \underline{0^\circ}$ amperes.

$$\text{Voltage drop in the feeders} = \underset{\cdot}{I}\underset{\cdot}{Z}_P$$
$$= 750 \angle\underline{0^\circ} \times 1{\cdot}797 \angle\underline{49^\circ\, 6'} \text{ volts}$$
$$= 1348 \angle\underline{49^\circ\, 6'} \text{ volts.}$$

To obtain the locus of the sending-end voltage O′X′ is drawn horizontally and with centre O′ a circle is drawn to represent 10000 volts, the sending-end voltage. OO′ is drawn to represent the voltage drop in the impedance of the feeders, at an angle of 49° 6′ to the horizontal. Then OX drawn horizontally represents the total current vector. **With O as the origin the circle represents the locus of the receiving-end voltage, intercepts between O and the circle giving the receiving-end voltage in magnitude and phase referred to the current vector OX.**

(a) For a power factor of 0·8 lagging, $\varnothing_1 = 36^\circ\, 52'$

Then OV_1 (measured from the diagram) = 8660 volts.

$$\text{Percentage regulation} = \frac{10000 - 8660}{10000} \times 100$$
$$= \mathbf{13{\cdot}4 \text{ per cent.}}$$

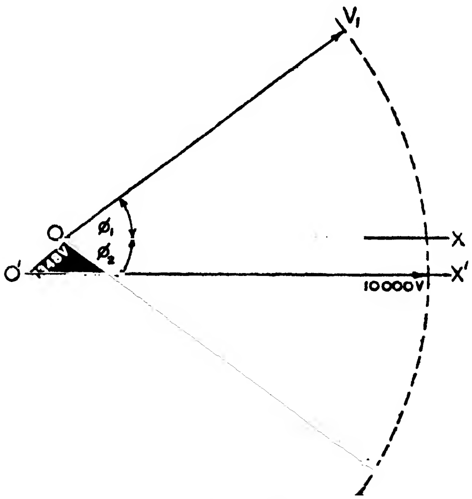


Fig. 113

(b) For a power factor of 0·8 leading, $\varnothing_2 = 36^\circ\, 52'$

Then OV_2 (measured from the diagram) = 9640 volts.

$$\textbf{Percentage regulation} = \frac{10000 - 9640}{10000} \times 100$$
$$= \mathbf{3{\cdot}6 \text{ per cent.}}$$

In (a) total kVA transmitted $= \dfrac{8660 \times 750}{1000} \angle \underline{-36^\circ 52'}$

$= 6500 \angle \underline{-36^\circ 52'}$ kVA $= \dot{P}$

kVA transmitted by line A $= \dfrac{6500 \angle \underline{-36^\circ 52'} \times 3{\cdot}605 \angle \underline{33^\circ 41'}}{6{\cdot}727 \angle \underline{48^\circ 1'}}$

$= 3482 \angle \underline{-51^\circ 12'}$ kVA.

kW transmitted by line A $= 3482 \cos 51^\circ 12'$

$=$ **2182 kW.**

kVA transmitted by line B $= \dfrac{6500 \angle \underline{-36^\circ 52'} \times 3{\cdot}354 \angle \underline{63^\circ 26'}}{6{\cdot}727 \angle \underline{48^\circ 1'}}$

$= 3240 \angle \underline{-21^\circ 27'}$ kVA.

kW transmitted by line B $= 3240 \cos 21^\circ 27'$

$=$ **3016 kW.**

152. *A* 3-*phase substation having a load of* 10000 *kW at* 0·8 *power factor, lagging, is supplied at* 66 *kV by two lines connected in parallel, the lines following different routes. The resistance and reactance per conductor of the lines are: For line A, R* = 6 *ohms,* X = 10 *ohms; for line B, R* = 8 *ohms,* X = 11 *ohms. Calculate* (*a*) *the voltage at the sending end,* (*b*) *the current in each line,* (*c*) *the phase difference between the currents in the lines.*

(*I.E.E., Pt. II, November,* 1942)

Total line current $= \dfrac{10000 \times 10^3}{\sqrt{3} \times 66 \times 10^3 \times 0{\cdot}8}$

$= 109{\cdot}3$ amperes.

Phase voltage at the load $= \dfrac{66}{\sqrt{3}}$

$= 38{\cdot}11$ kV.

Considering the phase voltage across the load as the reference vector, i.e.

$\dot{V}_{PH} = 38{\cdot}11\ (1 + j0)$ kV

the line current becomes $\dot{I} = 109{\cdot}3\ (0{\cdot}8 - j0{\cdot}6)$

$= 109{\cdot}3 \angle \underline{-36^\circ 52'}$ amperes.

The diagram for each phase is shown in Fig. 114.

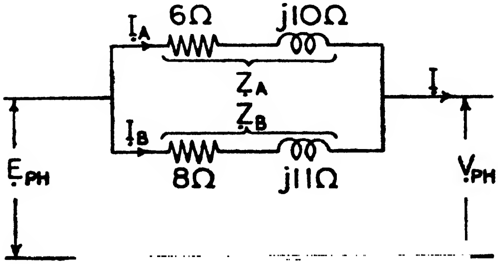


Fig. 114

$$\underset{\cdot}{Z}_A = 6 + j10 \qquad = 11{\cdot}66 \angle\underline{59^\circ\, 2'} \text{ ohms.}$$

$$\underset{\cdot}{Z}_B = 8 + j11 \qquad = 13{\cdot}6 \angle\underline{53^\circ\, 58'} \text{ ohms.}$$

$$\underset{\cdot}{Z}_A + \underset{\cdot}{Z}_B = 14 + j21 \qquad = 25{\cdot}24 \angle\underline{56^\circ\, 18'} \text{ ohms.}$$

Voltage drop in line impedance

$$= \underset{\cdot}{I}\cdot\frac{\underset{\cdot}{Z}_A\underset{\cdot}{Z}_B}{\underset{\cdot}{Z}_A + \underset{\cdot}{Z}_B}$$

$$= 109{\cdot}3 \angle\underline{-36^\circ\, 52'} \times \frac{11{\cdot}66 \angle\underline{59^\circ\, 2'} \times 13{\cdot}6 \angle\underline{53^\circ\, 58'}}{25{\cdot}24 \angle\underline{56^\circ\, 18'}}$$

$$= 686{\cdot}8 \angle\underline{19^\circ\, 50'} \text{ volts}$$

$$= 0{\cdot}6868\,(\cos 19^\circ 50' + j \sin 19^\circ 50')\,\text{kV}$$

$$= 0{\cdot}646 + j0{\cdot}233 \text{ kV.}$$

Sending-end phase voltage

$$\underset{\cdot}{E}_{PH} = 38{\cdot}11 + 0{\cdot}646 + j0{\cdot}233 \text{ kV}$$

$$= 38{\cdot}756 + j0{\cdot}233 \text{ kV}$$

$$E_{PH} = 38{\cdot}76 \text{ kV.}$$

Sending-end line voltage

$$= \sqrt{3} \times 38{\cdot}76 \text{ kV}$$

$$= \mathbf{67{\cdot}13 \text{ kV.}}$$

$$\underset{\cdot}{I}_A = \underset{\cdot}{I}\cdot\frac{\underset{\cdot}{Z}_B}{\underset{\cdot}{Z}_A + \underset{\cdot}{Z}_B}$$

$$= 109{\cdot}3 \angle\underline{-36^\circ\, 52'} \times \frac{13{\cdot}6 \angle\underline{53^\circ\, 58'}}{25{\cdot}24 \angle\underline{56^\circ\, 18'}}$$

$$= 58{\cdot}88 \angle\underline{-39^\circ\, 12'}$$

$$= \mathbf{58{\cdot}88 \text{ amperes at p.f. } 0{\cdot}775 \text{ lagging.}}$$

$$\underset{\cdot}{I}_B = \underset{\cdot}{I}\cdot\frac{\underset{\cdot}{Z}_A}{\underset{\cdot}{Z}_A + \underset{\cdot}{Z}_B}$$

$$= 109{\cdot}3 \angle\underline{-36^\circ\, 52'} \times \frac{11{\cdot}66 \angle\underline{59^\circ\, 2'}}{25{\cdot}24 \angle\underline{56^\circ\, 18'}}$$

$$= 50{\cdot}51 \angle\underline{-34^\circ\, 8'} \text{ amperes}$$

$$= \mathbf{50{\cdot}51 \text{ amperes at p.f. } 0{\cdot}828 \text{ lagging.}}$$

Phase difference between

$\underset{\cdot}{I}_A$ and $\underset{\cdot}{I}_B$ $= 39^\circ\, 12' - 34^\circ\, 8'$

$= \mathbf{5^\circ\, 4'}$ **with $\underset{\cdot}{I}_B$ leading.**

153. *A total load of* 10000 *kW at* 33 *kV and power factor* 0·8 *lagging is delivered to a substation by two* 3*-phase feeders connected in parallel. One of the cables has a resistance of* 1·5 *ohms for each conductor and a reactance* (*line to neutral*) *of* 1·4 *ohms, and delivers* 6000 *kW at a power factor of* 0·75 *lagging. Calculate the corresponding values of resistance and reactance of the second cable.*

Give a vector diagram in explanation.

(*C. and G. Final, Pt. II*, 1940)

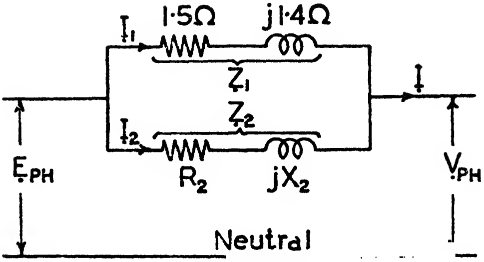


Fig. 115

Total line current $I = \dfrac{10000 \times 10^3}{\sqrt{3} \times 33 \times 10^3 \times 0{\cdot}8}$

$= 218{\cdot}7$ amperes.

Line current of first cable $I_1 = \dfrac{6000 \times 10^3}{\sqrt{3} \times 33 \times 10^3 \times 0{\cdot}75}$

$= 140$ amperes.

For I, $\cos \phi = 0{\cdot}8$, $\sin \phi = 0{\cdot}6$

For I_1, $\cos \phi_1 = 0{\cdot}75$, $\sin \phi_1 = 0{\cdot}6613$

Hence, taking the phase voltage as the reference vector

$$I = 218{\cdot}7\,(0{\cdot}8 - j0{\cdot}6)$$
$$= 218{\cdot}7 \angle -36^\circ\,52' \text{ amperes.}$$
$$I_1 = 140\,(0{\cdot}75 - j0{\cdot}6613)$$
$$= 140 \angle -41^\circ\,24' \text{ amperes.}$$

Now since the feeders are in parallel, the voltage drop in each must be the same and also the sum of the two feeder currents must equal the total current, i.e.

$$I_1Z_1 = I_2Z_2 \qquad (1)$$

and $$I_1 + I_2 = I \qquad (2)$$

These two equations are illustrated by the vector diagram, Fig. 116.

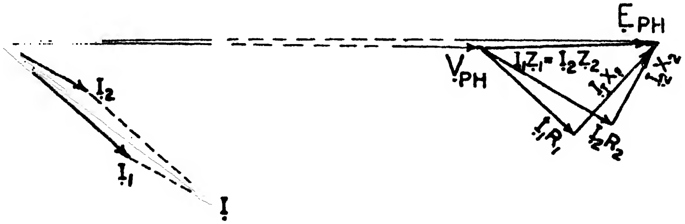


Fig. 116

From (2),
$$I_2 = I - I_1$$
$$= 218{\cdot}7\,(0{\cdot}8 - j0{\cdot}6) - 140\,(0{\cdot}75 - j0{\cdot}6613)$$
$$= 70 - j38{\cdot}64$$
$$= 79{\cdot}95 \angle -28^\circ\,54' \text{ amperes.}$$

Now,
$$Z_1 = 1{\cdot}5 + j1{\cdot}4$$
$$= 2{\cdot}052 \angle 43^\circ\,2' \text{ ohms.}$$

and from (1), $\underset{\cdot}{Z}_2 = \frac{I_1 Z_1}{\underset{\cdot}{I}_2}$

Hence,
$$\begin{aligned}\underset{\cdot}{Z}_2 &= \frac{140 \angle \underline{-41^\circ 24'} \times 2{\cdot}052 \angle \underline{43^\circ 2'}}{79{\cdot}95 \angle \underline{-28^\circ 54'}} \\ &= \frac{140 \times 2{\cdot}052}{79{\cdot}95} \angle \underline{-41^\circ 24' + 43^\circ 2' + 28^\circ 54'} \\ &= 3{\cdot}593 \angle \underline{30^\circ 32'} \text{ ohms} \\ &= 3{\cdot}593 (\cos 30^\circ 32' + j \sin 30^\circ 32') \\ &= 3{\cdot}095 + j1{\cdot}826 \text{ ohms.}\end{aligned}$$

Therefore **the second cable has a resistance per conductor of 3·095 ohms and a reactance (line to neutral) of 1·826 ohms.**

(iii) Economic size of cables—Kelvin's Law.

154. *Show that for a feeder cable if the load, the voltage and the power factor are given there is a most economical electrical efficiency of transmission which cannot be exceeded without involving an increase of the overall charges.*

Find the most economical cross-section of conductor and the efficiency of transmission to transmit 150 *amperes at* 230 *volts all the year round with a cost of* 0·35*d. per kWh for energy wasted. The* 2*-core cable costs (including installation costs)* £(0·80a + 0·1) *per yard where* a *is the cross-sectional area of each core in square inches. The length of the feeder is* 1 *mile and interest and depreciation charges total* 8 *per cent of the total capital cost. One mile of copper wire* 1 *sq. in. cross sectional area has a resistance of* 0·043 *ohm.*

(*C. and G. Final, Pt. II,*1941)

Let a = the cross-sectional area of each conductor required in sq. in.

Resistance of each core $= \frac{0{\cdot}043}{a}$ ohm.

$$\text{Energy wasted per annum} = 2 \times 150^2 \times \frac{0{\cdot}043}{a} \times \frac{1}{10^3} \times 8760 \text{ kWh}$$
$$= \frac{16950}{a} \text{ kWh.}$$

$$\text{Cost of energy wasted per annum} = \frac{£16950}{a} \times \frac{0{\cdot}35}{240}$$
$$= \frac{£24{\cdot}73}{a}$$

$$\text{Interest and depreciation charges per annum} = £\frac{8}{100} (0{\cdot}80a + 0{\cdot}1) \times 1760$$
$$= £(112{\cdot}6a + 14{\cdot}08)$$

For the most economical cross-sectional area of the cable the cost of the energy wasted per annum must be equal to the annual cost of the interest and depreciation on the **conductor** material of the cable. (This is Kelvin's Law.) Therefore, in this case

$$\frac{£24{\cdot}73}{a} = £112{\cdot}6a$$

$$a = \sqrt{\frac{24{\cdot}73}{112{\cdot}6}}$$

$$= \mathbf{0{\cdot}469\ sq.\ in.} = \textbf{cross-section required.}$$

Power loss in the cable $= \dfrac{2 \times 150^2 \times 0{\cdot}043}{0{\cdot}469 \times 1000}$ kW

$= 4{\cdot}13$ kW.

Power transmitted $= \dfrac{150 \times 230}{1000}$ kW

$= 34{\cdot}5$ kW.

Efficiency of transmission $= \dfrac{34{\cdot}5}{34{\cdot}5 + 4{\cdot}13} \times 100$ per cent

$=$ **89·3 per cent.**

155. *State the law of economy applied to an insulated cable. What practical considerations modify the size of conductor selected in any particular case?*

A load of 500 *kW at* 6600 *volts is to be transmitted for* 8 *hours a day for* 300 *days a year. The total cost of the cable is* £2·5 *per yard for each square inch cross section of copper plus* £0·33 *per yard independent of the size of the conductor. The cost of the energy wasted is* 0·25*d. per kWh, and interest and depreciation amount to* 8 *per cent per annum on the total capital cost. Find the most economical cross-sectional area of copper and criticize its use if the cable is* 10 *miles long. The resistance of* 1 *mile of copper wire* 1 *sq. in. in cross-section is* 0·046 *ohm.* (*C. and G. Final, Pt. II,* 1939)

Let a = the most economical cross-section of copper in sq. in.

Considering 1 mile of the cable,

resistance of each conductor $= \dfrac{0{\cdot}046}{a}$ ohm.

Assuming that the load is single-phase, unity power factor,

current flowing $= \dfrac{500 \times 10^3}{6600}$

$= 75{\cdot}76$ amperes.

Energy loss per annum in each conductor $= 75{\cdot}76^2 \times \dfrac{0{\cdot}046}{a} \times \dfrac{8 \times 300}{1000}$ kWh

$= \dfrac{633{\cdot}6}{a}$ kWh.

Annual cost of energy wasted $= \dfrac{£633{\cdot}6}{a} \times \dfrac{0{\cdot}25}{240}$

$= \dfrac{£0{\cdot}66}{a}$

Capital cost of the copper in each conductor per mile of cable

$$= £2{\cdot}5a \times 1760$$
$$= £4400a$$

Annual cost of interest and depreciation on the capital cost of the copper in one conductor per mile of cable

$$= \frac{£8}{100} \times 4400a$$
$$= £352a$$

By Kelvin's Law the most economical cross-section is when

$$£352a = \frac{£0{\cdot}66}{a}$$
$$a = \sqrt{\frac{0{\cdot}66}{352}}$$
$$= \mathbf{0{\cdot}0433\ sq.\ in.}$$

The resistance per mile of this conductor will be approximately 1 ohm. Hence on a 10-mile length of cable (2 conductors) the total resistance will be 20 ohms. The current flowing is 75·76 amperes and the voltage drop in the cable is 1515 volts. This voltage drop is practically 25 per cent of the voltage across the load and would be considered excessive. Thus a rather larger diameter conductor would be necessary in order to reduce this drop. It should also be noted that with the above cross-section the current density would be 1750 amperes per sq. in. which may be too high for the maximum permissible temperature rise. This is another reason for using a cross-section larger than the calculated value.

156. *Show how the variable losses in a transmission line supplying a variable load at constant voltage are taken into account in determining the most economical cross-section of conductor for the line.*

A 3-phase line is required to supply a factory at a constant voltage of 60 kV from a substation 30 miles distant. The load on the factory on each of the 300 working days per annum is as follows: 10 MW, 0·85 power factor, for 8 hours; 5 MW, 0·8 power factor for 4 hours; 2 MW, 0·9 power factor, for 6 hours; 0·5 MW, 0·95 power factor, for 6 hours. Calculate the most economical cross-section of conductor if the cost per mile of line completely erected is £(800 + 2600a), where a is the cross-section (in^2) of each conductor; the annual charges for interest and depreciation are equivalent to 8 per cent of the capital cost of the line; the cost of energy for supplying losses is 0·3d. per kWh; the resistance per mile of a single conductor 1 in^2 cross-section is 0·045 ohm. (*I.E.E., Pt. II, November,* 1944)

Let a = the most economical cross-section of the conductor in sq. in.

Annual charges for interest and depreciation

$$= £(2600a + 800) \times \frac{8}{100} \times 30$$
$$= £(6240a + 1920)$$

Resistance of each conductor $= \dfrac{0{\cdot}045}{a} \times 30$ ohms

$= \dfrac{1{\cdot}35}{a}$ ohms.

The currents flowing under the various load conditions will be:

(a) At 10 MW, 0·85 p.f. $I = \dfrac{10 \times 10^6}{\sqrt{3} \times 60 \times 10^3 \times 0{\cdot}85}$

$= 113{\cdot}2$ amperes.

(b) At 5 MW, 0·8 p.f. $I = \dfrac{5 \times 10^6}{\sqrt{3} \times 60 \times 10^3 \times 0{\cdot}8}$

$= 60$ amperes.

(c) At 2 MW, 0·9 p.f. $I = \dfrac{2 \times 10^6}{\sqrt{3} \times 60 \times 10^3 \times 0{\cdot}9}$

$= 21{\cdot}4$ amperes.

(d) At 0·5 MW, 0·95 p.f. $I = \dfrac{0{\cdot}5 \times 10^6}{\sqrt{3} \times 60 \times 10^3 \times 0{\cdot}95}$

$= 5{\cdot}07$ amperes.

The corresponding daily copper losses in the 3 lines will be

(a) $\text{Loss} = 3 \times 113{\cdot}2^2 \times \dfrac{1{\cdot}35}{a} \times \dfrac{8}{10^3}$

$= \dfrac{416}{a}$ kWh.

(b) $\text{Loss} = 3 \times 60^2 \times \dfrac{1{\cdot}35}{a} \times \dfrac{4}{10^3}$

$= \dfrac{58{\cdot}32}{a}$ kWh.

(c) $\text{Loss} = 3 \times 21{\cdot}4^2 \times \dfrac{1{\cdot}35}{a} \times \dfrac{6}{10^3}$

$= \dfrac{11{\cdot}2}{a}$ kWh.

(d) $\text{Loss} = 3 \times 5{\cdot}07^2 \times \dfrac{1{\cdot}35}{a} \times \dfrac{6}{10^3}$

$= \dfrac{0{\cdot}625}{a}$ kWh.

Total daily loss $= \dfrac{486{\cdot}1}{a}$ kWh.

Annual cost of energy wasted $= \dfrac{£486{\cdot}1 \times 0{\cdot}3 \times 300}{a \times 240}$

$= \dfrac{£182{\cdot}0}{a}$

For the most economical cross-section of the cable,

$$\frac{£182{\cdot}0}{a} = £6240a$$

$$a = \sqrt{\frac{182{\cdot}0}{6240}}$$

$= $ **0·171 sq. in.**

157. *A factory is to be supplied by a 3-phase cable from a substation 3 miles distant. The voltage at the factory is 11 kV, and the daily load for six days per week throughout the year is as follows: 600 kW at 0·9 power factor for 6 hours, 400 kW at 0·8 power factor for 2 hours, 60 kW at unity power factor for 16 hours. Determine the most economical cross-section of conductor for the cable if the cost of the complete cable, including laying, etc., is £(2,500a + 900) per mile and the cost of energy at the factory is £4 10s. per annum per kVA maximum demand plus 0·5d. per kWh, a being the cross-section (in square inches) of each conductor. Assume that the resistance per mile of conductor is 0·043 ohm. Allow 12½ per cent for interest and depreciation. Calculate also (a) the annual cost of the cable, (b) the average cost per kWh (taken over a year).* (*I.E.E., Pt. II, November,* 1942)

Capital cost of the cable $= 3 \times £(2500a + 900)$
$= £(7500a + 2700)$

Annual cost of interest and depreciation on this amount $= 0{\cdot}125 \times £(7500a + 2700)$
$= £(937{\cdot}5a + 337{\cdot}5)$

Resistance of each conductor $= \dfrac{0{\cdot}043}{a} \times 3$
$= \dfrac{0{\cdot}129}{a}$ ohms.

Note.—There appears to be an omission in the question here and it is believed that the resistance of the conductor should be taken as 0·043 ohm per mile **if the cross-section is 1 sq. in.**

The line currents flowing under each load condition are:

(i) At 600 kW, 0·9 p.f. $I = \dfrac{600 \times 10^3}{\sqrt{3} \times 11 \times 10^3 \times 0{\cdot}9}$
$= 35{\cdot}0$ amperes.

(ii) At 400 kW, 0·8 p.f. $I = \dfrac{400 \times 10^3}{\sqrt{3} \times 11 \times 10^3 \times 0{\cdot}8}$
$= 26{\cdot}3$ amperes.

(iii) At 60 kW, unity p.f. $I = \dfrac{60 \times 10^3}{\sqrt{3} \times 11 \times 10^3 \times 1{\cdot}0}$
$= 3{\cdot}15$ amperes.

The corresponding energy losses per week in the cable on these loads

(i) $$\text{Loss} = 35^2 \times \frac{0{\cdot}129}{a} \times \frac{6 \times 6}{1000} \times 3 \text{ kWh}$$
$$= \frac{17{\cdot}04}{a} \text{ kWh.}$$

(ii) $$\text{Loss} = 26{\cdot}3^2 \times \frac{0{\cdot}129}{a} \times \frac{2 \times 6}{1000} \times 3 \text{ kWh}$$
$$= \frac{3{\cdot}21}{} \text{ kWh.}$$

(iii) $$\text{Loss} = 3{\cdot}15^2 \times \frac{0{\cdot}129}{a} \times \frac{16 \times 6}{1000} \times 3 \text{ kWh}$$

$$\frac{0{\cdot}37}{a} \text{ kWh.}$$

Total weekly loss $$= \frac{20{\cdot}62}{a} \text{ kWh.}$$

Annual cost of copper loss $$= \frac{\pounds 20{\cdot}62}{a} \times \frac{52 \times 0{\cdot}5}{240}$$

$$= \frac{\pounds 2{\cdot}235}{a}$$

The maximum voltage drop in each conductor $$= 35 \times \frac{0{\cdot}129}{a}$$

$$= \frac{4{\cdot}52}{a} \text{ volts.}$$

Hence the maximum kVA demand is increased by

$$3 \times \frac{4{\cdot}52}{a} \times \frac{35}{1000} = \frac{0{\cdot}475}{a} \text{ kVA.}$$

Additional charge per annum due to line kVA $$= \frac{\pounds 0{\cdot}475}{a} \times 4{\cdot}5$$

$$= \frac{\pounds 2{\cdot}135}{a}$$

Total annual charge due to cable losses $$= \frac{\pounds 2{\cdot}235}{a} + \frac{\pounds 2{\cdot}135}{a}$$

$$= \frac{\pounds 4{\cdot}37}{a}$$

For the most economical size of cable,

$$\frac{\pounds 4{\cdot}37}{a} = \pounds 937{\cdot}5a$$

$$a = \sqrt{\frac{4{\cdot}37}{937{\cdot}5}}$$

$$= \mathbf{0{\cdot}068 \text{ sq. in.}}$$

(a) **Annual cost of cable** $$= \pounds\left(\frac{4{\cdot}37}{a} + 937{\cdot}5a\right)$$

$$= \pounds\left(\frac{4{\cdot}37}{0{\cdot}068} + 937{\cdot}5 \times 0{\cdot}068\right)$$

$$= \mathbf{\pounds 128}$$

(b) Total yearly kWh $$= (600 \times 6 + 400 \times 2 + 60 \times 16) \times 6 \times 52$$

$$= 1672320 \text{ kWh.}$$

Maximum demand kVA during the year $$= \frac{600 \text{ kW}}{0{\cdot}9} = \frac{2000}{3} \text{ kVA.}$$

Annual maximum demand charges $= \frac{2000}{3} \times £45$

$= £3000$

These charges are spread over 1672320 kWh taken by the load.

Average cost per kWh at the factory $= 0{\cdot}5\text{d.} + \frac{£3000}{1672320}$

$= 0{\cdot}5\text{d.} + 0{\cdot}43\text{d.}$

$= \mathbf{0{\cdot}93d.}$

158. *State and explain a law to determine the most economical size of feeders.*

Calculate the most economical size of feeder for a d.c. system if the distance between consumer and substation is 1500 *yd., the feeder being assumed to carry a constant current of* 300 *amperes for* 10 *hours a day. Interest and depreciation charges are* 8 *per cent and the cost of* 1 *kilowatt hour is* 0·75 *pence. The capital cost of the feeder which varies with the area is* 1*s. per lb. The resistance of* 1000 *yd. of conductor* 1 *sq. in. cross-section is* 0·0243 *ohm and the weight of* 1 *cu. in. is* 0·32 *lb.*

Explain under what conditions this cross-section would have to be modified.

(*London, B.Sc. Eng., July,* 1944)

Let a = the most economical cross-section of feeder in sq. in.

Weight of 1500 yd. of feeder (2 conductors) $= 2 \times 1500 \times 36 \times a \times 0{\cdot}32$ lb.

$= 34560$ lb. $\times$ a

Capital cost of copper $= 34560\text{s.} \times a$

$= £1728a$

Annual cost due to interest and depreciation $= \frac{8}{100} \times £1728a$

$= £138{\cdot}24a$

Total resistance of feeder $= 2 \times \frac{0{\cdot}0243}{a} \times \frac{1500}{1000}$

$= \frac{0{\cdot}0729}{a}$ ohms.

Power loss in feeder $= 300^2 \times \frac{0{\cdot}0729}{a} \times \frac{1}{1000}$ kW

$= \frac{6{\cdot}561}{a}$ kW.

Energy lost per annum $= \frac{6{\cdot}561}{a} \times 10 \times 365$ kWh

$= \frac{23947}{a}$ kWh.

$$\text{Annual cost of energy lost} = \frac{£23947}{a} \times \frac{0{\cdot}75}{240}$$

$$= \frac{£74{\cdot}82}{a}$$

For the most economical feeder cross-section,

$$\frac{74{\cdot}82}{a} = 138{\cdot}24a$$

$$a = \sqrt{\frac{74{\cdot}82}{138{\cdot}24}}$$

$$= \mathbf{0{\cdot}735\ sq.\ in.}$$

This cross-section would have to be modified if the current density consequent upon using it proved to be too high for the maximum permissible temperature rise. In this event it would be necessary to increase the cross-section. Also if the feeder were so long that the voltage drop incurred were too large to be compensated for at the substation then it would again be necessary to increase the cross-section to decrease the drop.

CHAPTER XIII

OVERHEAD TRANSMISSION LINES

(i) Inductance of lines.

159. *Develop an expression for the inductance in millihenrys per mile of a pair of parallel conductors of a diameter which is small compared with the distance between their centres.*

Calculate the impedance of a 50-*mile length of transposed single-circuit "Grid" line. The conductors have a diameter of* 0·77 *in., a resistance of* 0·255 *ohm per mile and the equivalent equilateral spacing is* 15 *ft.*

(*C. and G. Final, Pt. II*, 1943)

For a line in which the conductor diameter is small by comparison with the spacing between centres an approximate expression for the inductance per mile is given by:

$$L = 0{\cdot}74 \log_{10} \frac{d}{r} \text{ millihenrys,}$$

where d is the equivalent equilateral spacing between the conductors, r is the radius of each conductor, and L is the inductance of each conductor.

For a length of 50 miles with d = 15 ft. and r = 0·385 in.

Total inductance of each conductor $= \left(0{\cdot}74 \log_{10} \frac{180}{0{\cdot}385}\right) \times 50$

$= 98{\cdot}8$ millihenrys.

Reactance between each conductor and neutral $= 2\pi \times 50 \times 98{\cdot}8 \times 10^{-3}$ ohms

$= 31{\cdot}04$ ohms.

Resistance of each conductor $= 0{\cdot}255 \times 50$ ohms

$= 12{\cdot}75$ ohms.

Hence, **impedance per conductor (line to neutral)** $= \sqrt{12{\cdot}75^2 + 31{\cdot}04^2}$

$= \sqrt{162{\cdot}6 + 963{\cdot}4}$

$=$ **33·56 ohms.**

160. *Derive from first principles an expression for the inductance per phase per mile for a* 3-*phase transmission line with conductors of diameter d arranged in the same horizontal plane at successive distances D apart. The conductors are regularly transposed.*

Calculate the inductance per mile per phase for such a line with d = 0·75 *in. and D* = 3 *ft.* (*C. and G. Final, Pt. II*, 1945)

If three conductors A, B and C are unsymmetrically spaced by distances D_1 D_2 and D_3 respectively and the conductors are regularly transposed, the inductive voltage drop is the same as in a line of the same conductor size and length and having an equilateral spacing of $\sqrt[3]{D_1D_2D_3}$. Therefore the inductance per phase of such a line may be calculated by assuming equilaterally spaced conductors at an equivalent spacing of $\sqrt[3]{D_1D_2D_3}$

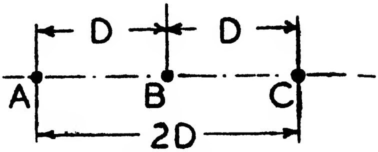


Fig. 117

When the conductors are in the same horizontal plane as shown with successive spacings of D,

$D_1 = D,\ D_2 = D,\ D_3 = 2D.$

Hence the equivalent equilateral spacing is $\sqrt[3]{D \times D \times 2D} = 1{\cdot}26D$.

Therefore, **inductance per mile**

$$\textbf{per phase} = 0{\cdot}74 \log_{10} \frac{1{\cdot}26D}{\frac{d}{2}} \text{ millihenrys}$$
$$= 0{\cdot}74 \log_{10} \frac{2{\cdot}52D}{d} \text{ millihenrys}$$
$$= 0{\cdot}74 \log_{10} \frac{2{\cdot}52 \times 36}{0{\cdot}75} \quad ,,$$
$$= 0{\cdot}74 \times 2{\cdot}0826 \text{ millihenrys}$$
$$= \mathbf{1{\cdot}541\ millihenrys.}$$

(ii) Capacitance of lines.

161. *Deduce a formula for the capacitance to neutral in microfarads per mile in terms of the conductor size and spacing of a symmetrical* 3-*phase overhead line.*

A 3-*phase,* 50-*frequency overhead line has regularly transposed conductors equilaterally spaced* 12 *feet apart. The conductor diameter is* 0·88 *inch. Find the charging current per mile per volt.*

The capacitance per mile to neutral of a 3-phase overhead line with symmetrical conductor spacing is given by the expression:

$$C = \frac{0{\cdot}0388}{\log_{10} \frac{d}{r}} \text{ microfarads.}$$

If $d = 12$ ft. (144 in.) and $r = 0{\cdot}44$ in.

$$C = \frac{0{\cdot}0388}{\log \frac{144}{0{\cdot}44}} \text{ microfarad per mile}$$
$$= 0{\cdot}01537 \text{ microfarad per mile.}$$

Charging current per mile per volt (i.e. per $\frac{1}{\sqrt{3}}$volt per phase)

$$= \frac{1}{\sqrt{3}} \times \omega C$$
$$= \frac{2\pi \times 50 \times 0{\cdot}01537}{\sqrt{3}} \text{ microamperes}$$
$$= \mathbf{2{\cdot}8\ microamperes.}$$

162. *Recalculate Problem* 161 *assuming that the conductors are in the same horizontal plane with successive spacings of* 12 *ft. and are regularly transposed.*

As with the calculation of the inductance of an irregularly spaced line, the equivalent capacitance of each of the conductors of this line to neutral

is the same as that of one with equilaterally spaced conductors whose spacing is $\sqrt[3]{D_1D_2D_3}$. When, as in this problem, $D_1 = d$, $D_2 = d$ and $D_3 = 2d$ then the equivalent spacing is 1·26d.

$$\text{Therefore, capacitance per mile to neutral} = \frac{0{\cdot}0388}{\log_{10} \frac{1{\cdot}26d}{r}} \text{ microfarads}$$

$$= \frac{0{\cdot}0388}{\log_{10} \frac{1{\cdot}26 \times 144}{0{\cdot}44}} \text{ microfarads}$$

$$= \frac{0{\cdot}0388}{2{\cdot}6153} \text{ microfarad}$$

$$= 0{\cdot}01484 \text{ microfarad.}$$

$$\textbf{Charging current per mile per volt} = \frac{2\pi \times 50 \times 0{\cdot}01484}{\sqrt{3}} \text{ microamperes}$$

$$= \mathbf{2{\cdot}69 \text{ microamperes.}}$$

(iii) **Corona.**

163. *Give a short account of corona in high-voltage transmission lines and derive a formula for the disruptive critical voltage between two smooth circular wires, assuming the breakdown strength of air to be* 30 *kV per cm.*

In a 3-*phase overhead line the conductors have each a diameter of* 1·25 *in. and are arranged in delta formation. Assuming a critical voltage of* 230 *kV the air density factor* 0·95 *and the irregularity factor* 0·95, *find the minimum spacing distance between conductors, assuming fair weather conditions.*

(*C. and G. Final, Pt. II,* 1937)

When the spacing between the conductors is d inches and their radius is r inches, the critical voltage to neutral at which corona begins is given by

$$E_n = 2{\cdot}303\ m_o\ g_o\ \delta\ r \log_{10} \frac{d}{r}$$

where m_o = the irregularity factor due to the state of the surface and shape of the cross-section of the wire.

g_o = the breakdown strength for air in volts R.M.S. per inch

δ = the air density factor.

In this question,

$$E_n = \frac{230000}{\sqrt{3}} \text{ volts,}$$

$$m_o = 0{\cdot}95$$

$$g_o = \frac{30000 \times 2{\cdot}54}{\sqrt{2}}$$

$$= 53500 \text{ volts (R.M.S.) per inch.}$$

$$\delta = 0{\cdot}95$$

Hence, $$\frac{230000}{\sqrt{3}} = 2{\cdot}303 \times 0{\cdot}95 \times 53{,}500 \times 0{\cdot}95 \times 0{\cdot}625 \log_{10} \frac{d}{r}$$

and $$\log_{10} \frac{d}{r} = 1{\cdot}9$$

$$\frac{d}{r} = \text{antilog}_{10}\ 1{\cdot}9$$

$$= 79{\cdot}43$$

Since $$r = 0{\cdot}625 \text{ in.}$$

$$d = 79{\cdot}43 \times 0{\cdot}625 \text{ in.}$$

$$= 4{\cdot}14 \text{ ft.}$$

Therefore, **minimum spacing between conductors = 4·14 ft.**

(iv) Suspension insulators.

164. *Define string efficiency with reference to a suspension insulator assembly. Explain how this efficiency can be raised by the introduction of arcing horns or rings.*

If the voltage across the units in a 2-unit suspension insulator is 60 per cent and 40 per cent respectively of the line voltage, find the ratio of the capacitance of the insulator to that of its capacitance to earth.

(*C. and G. Final, Pt. II,* 1944)

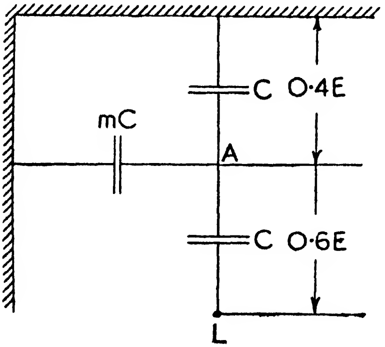


Fig. 118

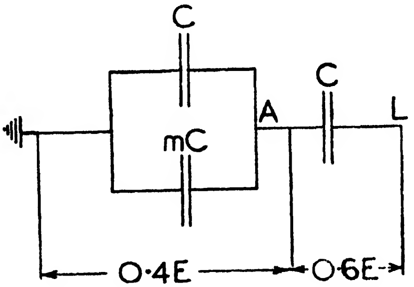


Fig. 119

Let the capacitance to earth be m times the self-capacitance of the insulator.

The total capacitance from A to earth (Fig. 118) is C + mC, and the diagram may be redrawn as in Fig. 119.

Then since the p.d. across a condenser is inversely proportional to its capacitance,

$$\frac{C + mC}{C} = \frac{0{\cdot}6E}{0{\cdot}4E} = \frac{3}{2}$$

i.e. $$1 + m = 1{\cdot}5$$

$$m = 0{\cdot}5$$

Therefore, **the self-capacitance of the insulator is twice that of its capacitance to earth.**

165. *Explain why suspension-type insulators have superseded pin-type insulators for high-voltage overhead lines. Sketch a sectional view of one unit of a suspension type insulator and describe the construction. Show how a string of insulators may be protected against damage when a flash-over occurs.*

Each line of a 3-phase system is suspended by a string of three similar insulators. If the voltage across the "line" unit is 10 *kV, determine the line voltage of the system. Assume that the shunt capacitance between each insulator and the earthed metalwork of the tower is one-tenth of the capacitance of the insulator itself.* (*I.E.E., Pt. II, May,* 1944)

Let the charges on the various capacitances due to the p.d.s across them at any instant be as shown in Fig. 120.

Then $Q_1 = CE_1$
$Q_2 = CE_2$
$Q_3 = CE_3 = 10C$ kilo-coulombs
$Q_A = 0{\cdot}1CE_1$
$Q_B = 0{\cdot}1C\ (E_1 + E_2)$

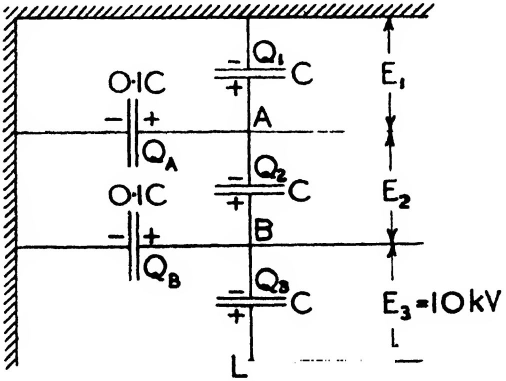


Fig. 120

The relation between the charges at the junction A is

$$\begin{aligned} Q_2 &= Q_1 + Q_A \\ &= CE_1 + 0{\cdot}1CE_1 \\ &= 1{\cdot}1CE_1 \end{aligned}$$

Hence,

$$CE_2 = 1{\cdot}1CE_1$$

$$E_2 = 1{\cdot}1E_1 \qquad (1)$$

The relation between the charges at the junction B is

$$\begin{aligned} Q_3 &= Q_2 + Q_B \\ &= CE_2 + 0{\cdot}1C\ (E_1 + E_2) \\ &= 1{\cdot}1CE_1 + 0{\cdot}1C\ (E_1 + 1{\cdot}1E_1) \\ &= 1{\cdot}1CE_1 + 0{\cdot}21CE_1 \\ &= 1{\cdot}31CE_1 \end{aligned}$$

Hence, $CE_3 = 1{\cdot}31CE_1$

$$E_3 = 1{\cdot}31E_1 \qquad (2)$$

Since $E_3 = 10$ kV,

from (2) $E_1 = \dfrac{10}{1{\cdot}31}$

$= 7{\cdot}634$ kV.

From (1) $E_2 = 1{\cdot}1E_1$

$= 1{\cdot}1 \times 7{\cdot}634$

$= 8{\cdot}397$ kV.

Line p.d. to earth $= E_1 + E_2 + E_3 = 26{\cdot}03$ kV.

Assuming that the neutral of the system is at earth potential

line voltage $= \sqrt{3} \times 26{\cdot}03 =$ **45·1 kV.**

166. *Each of three insulators forming a string has a self-capacitance of C farads. The shunting capacitance of the connecting metalwork of each insulator is 0·2C to earth and 0·1C to the line. A guard ring increases the capacitance to the line of the metalwork of the lowest insulator to 0·3C. Calculate the string efficiency of this arrangement (a) with the guard ring, (b) without the guard ring.*

(a) **With the guard ring**

Let the charges on the various condensers due to the p.d.s across them be as shown in Fig. 121.

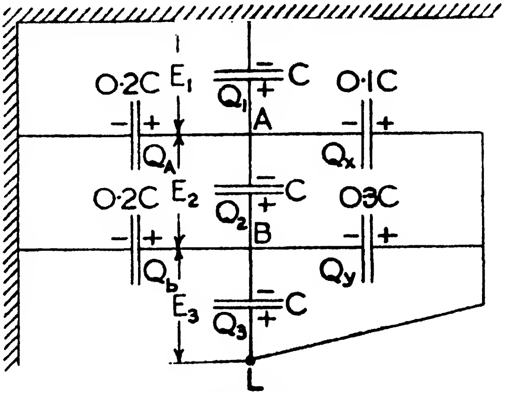


Fig. 121

Then the relations between the charges and the p.d.s are

$$Q_1 = CE_1 \quad (1)$$
$$Q_a = 0{\cdot}2CE_1 \quad (2)$$
$$Q_x = 0{\cdot}1C(E_2 + E_3) \quad (3)$$
$$Q_2 = CE_2 \quad (4)$$
$$Q_b = 0{\cdot}2C(E_1 + E_2) \quad (5)$$
$$Q_y = 0{\cdot}3CE_3 \quad (6)$$
$$Q_3 = CE_3 \quad (7)$$
$$Q_2 + Q_x = Q_1 + Q_a \quad (8)$$
$$Q_3 + Q_y = Q_2 + Q_b \quad (9)$$
$$E = E_1 + E_2 + E_3 \quad (10)$$

From equations (1), (2), (3), (4) and (8)

$$CE_2 + 0{\cdot}1C\ (E_2 + E_3) = CE_1 + 0{\cdot}2CE_1$$

i.e. $$12E_1 - 11E_2 - E_3 = 0 \quad (11)$$

From equations (4), (5), (6), (7) and (9)

$$CE_3 + 0{\cdot}3CE_3 = CE_2 + 0{\cdot}2C\ (E_1 + E_2)$$

i.e. $$2E_1 + 12E_2 - 13E_3 = 0 \quad (12)$$

Multiply (11) by 13,

$$156E_1 - 143E_2 - 13E_3 = 0 \quad (13)$$

Subtract (12) from (13),

$$154E_1 - 155E_2 = 0$$
$$E_2 = \frac{154}{155} E_1$$

Also multiply (11) by 12,

$$144E_1 - 132E_2 - 12E_3 = 0 \quad (14)$$

and multiply (12) by 11,

$$22E_1 + 132E_2 - 143E_3 = 0 \quad (15)$$

Add (14) and (15),

$$166E_1 - 155E_3 = 0$$
$$E_3 = \frac{166}{155} E_1$$

Therefore
$$\begin{aligned} E &= E_1 + E_2 + E_3 \\ &= E_1\left(1 + \frac{154}{155} + \frac{166}{155}\right) \\ &= 3{\cdot}0645E_1 \end{aligned}$$

Whence $E_1 = 0{\cdot}327E$, $E_2 = 0{\cdot}325E$, and $E_3 = 0{\cdot}35E$.

$$\text{String efficiency} = \frac{\text{p.d. from line to earth}}{\text{N} \times \text{p.d. across line insulator}}$$

where N = number of insulators

$$\begin{aligned} &= \frac{E}{3 \times 0{\cdot}35E} \times 100 \text{ per cent} \\ &= \mathbf{95{\cdot}24 \text{ per cent.}} \end{aligned}$$

(b) **Without the guard ring**

Equations (1) to (10) remain the same with the exception of (6) which becomes

$$Q_v = 0{\cdot}1CE_3 \qquad (6a)$$

Equation (12) then becomes

$$CE_3 + 0{\cdot}1CE_3 = CE_2 + 0{\cdot}2C\ (E_1 + E_2)$$

i.e.
$$2E_1 + 12E_2 - 11E_3 = 0 \qquad (12a)$$

Solving equations (10), (11) and (12a) in a similar manner to that given for part (a) gives

$$E_1 = 0{\cdot}31E,\ E_2 = 0{\cdot}303E, \text{ and } E_3 = 0{\cdot}387E$$

$$\begin{aligned} \text{String efficiency} &= \frac{E}{3 \times 0{\cdot}387E} \times 100 \text{ per cent} \\ &= \mathbf{86{\cdot}1 \text{ per cent.}} \end{aligned}$$

(v) Calculation of sag.

167. *Deduce an approximate expression for calculating the sag in an overhead line.*

A span of 450 *ft. between level supports is expected to have a sag of* 9*ft. when the wind pressure is* 8 *lb. per sq. ft. of projected area. The circular copper conductor has a diameter of* 0·5 *in., weighs* 0·76 *lb. per ft. run and has a breaking stress of* 60000 *lb. per sq. in. Calculate the factor of safety under these conditions.*

How would the sag and tension of the conductor be affected by an extreme fall of temperature? (*C. and G. Final, Pt. II,* 1942)

Assuming that the wind pressure per ft. run is w_1 lb. acting horizontally, and that the weight of the conductor is w_o lb. per ft. run acting vertically, then

$$\begin{aligned} w_1 &= 1 \times \frac{d}{12} \times 8 \text{ lb.} \\ &= 1 \times \frac{0{\cdot}5}{12} \times 8 \text{ lb.} \\ &= 0{\cdot}33 \text{ lb.} \\ w_o &= 0{\cdot}76 \text{ lb.} \end{aligned}$$

Therefore the effective weight per ft. run on which the sag must be calculated is

$$\begin{aligned} w &= \sqrt{w_0^2 + w_1^2} \\ &= \sqrt{0{\cdot}76^2 + 0{\cdot}33^2} \\ &= 0{\cdot}83 \text{ lb.} \end{aligned}$$

The approximate formula for the sag is

$$D = \frac{wl^2}{2T}$$

where w = the effective weight per ft. run,
l = half the span in ft.,
T = the line tension in lb.

In this question $D = 9$ ft.
$w = 0{\cdot}83$ lb.
$l = 225$ ft.

Hence,

$$9 = \frac{0{\cdot}83 \times 225^2}{2 \times T}$$

$$T = 2334 \text{ lb.}$$

The breaking tension is

$$\begin{aligned} T_{\max} &= 60000 \times \frac{\pi \times 0{\cdot}5^2}{4} \\ &= 11790 \text{ lb.} \end{aligned}$$

Factor of safety $= \dfrac{11790}{2334} = \mathbf{5{\cdot}05.}$

An extreme fall of temperature would cause the sag in the conductor to decrease and the tension to increase owing to the contraction in the length of the line.

168. *Obtain an expression for the vertical sag in an overhead line assuming a parabolic configuration.*

A transmission line has a span of 600 *ft. between level supports. The conductor has a cross-sectional area of* 0·2 *sq. in., weighs* 2360 *lb. per* 1000 *yd., and has a breaking stress of* 60000 *lb. per sq. in. Calculate the sag for a factor of safety of* 5, *allowing a maximum wind pressure of* 25 *lb. per sq. ft. of projected surface.*

Show how to allow for the presence of ice on the wire.

(*C. and G. Final, Pt. II,* 1936)

Weight of conductor per ft. run $= \dfrac{2360}{3000}$

$= 0{\cdot}787$ lb.

Diameter of conductor $= \sqrt{\dfrac{4 \times 0{\cdot}2}{\pi}}$

$= 0{\cdot}5046$ in.

Projected area per ft. run $= 1 \times \dfrac{0{\cdot}5046}{12}$ sq. ft.

$= 0{\cdot}042$ sq. ft.

Wind pressure per ft. run $= 0{\cdot}042 \times 25$

$= 1{\cdot}05$ lb.

Effective weight of conductor per ft. run $= \sqrt{0{\cdot}787^2 + 1{\cdot}05^2}$
$= 1{\cdot}312$ lb.

Breaking tension $= 60000 \times 0{\cdot}2$
$= 12000$ lb.

Hence, allowing for a factor of safety of 5,
tension in the wire $= 12000 \div 5$
$= 2400$ lb.

Sag in conductor $= \dfrac{wl^2}{2T}$

$$= \frac{1{\cdot}312 \times 300^2}{2 \times 2400}$$

$= \mathbf{24{\cdot}6\ ft.}$

To allow for the presence of ice on the wire assume that the radial thickness of the coating is r. The density of ice is approximately 57 lb. per cu. ft., therefore the weight of ice per ft. run is

$$w_2 = \pi\left[\left(\frac{d}{2} + r\right)^2 - \left(\frac{d}{2}\right)^2\right] \div 144 \times 57$$
$$= 1{\cdot}25r\,(d + r)\ \text{lb.}$$

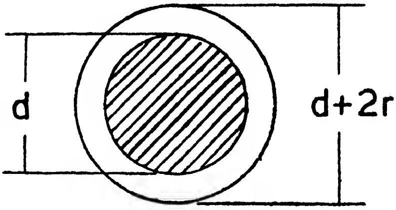


Fig 122

This must be added to the vertical weight of the conductor w_o per ft. run.

In addition the ice causes the projected area per ft. run to be increased and the wind pressure per ft. run is now

$$w_1 = 1 \times \frac{d + 2r}{12} \times 25\ \text{lb.}$$
$$= \frac{25}{12}\,(d + 2r)\ \text{lb.}$$

Then the effective weight per ft. run, w, needs to be recalculated allowing for the increased vertical and horizontal components due to the ice, and the new value of w must be used in the sag formula.

169. *In the transmission lines of the National Grid Scheme the line conductors each have an effective diameter of* 0·77 *in., weight* 571*lb. per* 1000 *ft. and an ultimate strength of* 17722 *lb. Calculate the height above ground at which a conductor with a span of* 900 *ft. should be supported in order that the total tension shall not exceed half the ultimate strength with a* ½*-in. radial coating of ice and a horizontal wind pressure of* 8 *lb. per sq. ft. of projected area. The ground clearance required is* 22 *ft. Weight of* 1 *cu. ft. of ice* = 57 *lb.*

(*London B.Sc.Eng.*, 1932)

Total vertical weight of conductor
per ft. run $= w_o + w_2$ (*See* Problem 168)
$= w_o + 1{\cdot}25r\ (d + r)$
$= 0{\cdot}571 + 1{\cdot}25 \times 0{\cdot}5\ (0{\cdot}77 + 0{\cdot}5)$
$= 0{\cdot}571 + 0{\cdot}795$ lb.
$= 1{\cdot}366$ lb. $= w_o$

Horizontal pressure due to wind

$$\text{per ft. run} = \frac{d + 2r}{12} \times 8 \text{ lb.}$$
$$= \frac{0{\cdot}77 + 1{\cdot}0}{12} \times 8 \text{ lb.}$$
$$= 1{\cdot}18 \text{ lb.} = w_1$$

Hence, effective weight of conductor

$$\text{per ft. run} = \sqrt{w_1^2 + w_3^2}$$
$$= \sqrt{1{\cdot}18^2 + 1{\cdot}366^2}$$
$$= 1{\cdot}805 \text{ lb.} = w$$

The maximum permissible tension $= 17722 \times 0{\cdot}5$ lb.
$= 8861$ lb.

Half span $= 450$ ft.

Therefore, using the approximate formula for the sag,

$$\text{Sag} = \frac{wl^2}{2T}$$
$$= \frac{1{\cdot}805 \times 450^2}{2 \times 8861}$$
$$= 20{\cdot}6 \text{ ft. or } 20 \text{ ft. } 7 \text{ in.}$$

As the clearance at the ground must be 22 ft.

Height of supports $= 22$ ft. $+ 20$ ft. 7 in.
$=$ **42 ft. 7 in.**

(vi) Line surges.

170. *Explain what is meant by the surge impedance of a line and show upon what factors it depends.*

An overhead transmission line 186 *miles long, having a surge impedance of* 500 *ohms is short circuited at one end and a steady voltage of* 3000 *volts is suddenly applied at the other end.*

Neglecting the resistance, explain, with diagrams, how the current and voltage change at different parts of the line and calculate the current at the sending end of the line 0·004 *second after the voltage is applied.*

(*London B.Sc.Eng.*, *July*, 1945)

The speed at which the wave travels along the line is approximately 186000 miles per second so that, as the line is 186 miles long, the wave travels the length of the line once in 0·001 second and is then reflected.

During the first traversal of the line the voltage builds up to 3000 volts all the way along the line and when t = 0·001 second the voltage is 3000 volts everywhere. Similarly the current flowing at each point in the line is

$$I = \frac{\text{Line p.d.}}{\text{Surge impedance}} = \frac{3000 \text{ volts}}{500 \text{ ohms}} = 6 \text{ amperes.}$$

At t = 0·001 second the first reflection takes place, the voltage being reflected with change of sign and the current without any change in sign. Between t = 0·001 and 0·002 second the combination of incident and reflected waves along the line results in the line voltage dropping to zero, commencing at the far end (short circuit) and spreading back to the sending end where the voltage is zero at t = 0·002 second. The current reflection causes the

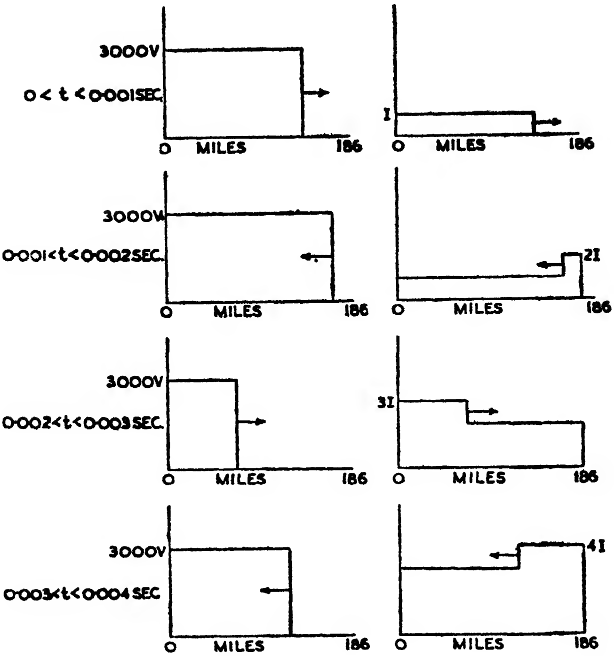


Fig. 123

current to become 2I at the far end when t = 0·001 second and the wave travels back to make the sending end current 2I (12 amperes) when t = 0·002 second.

The second reflection occurs at t = 0·002 second at the sending end and between t = 0·002 and 0·003 second a voltage wave travels from the sending end to the short circuit. A current wave of 3I also travels between these two points in the same time.

Thus it will be seen that each reflection at the sending end causes a voltage wave of 3000 volts to travel out along the line; while each reflection at the short circuited end causes a zero voltage wave to travel back along the line. At each reflection at either end the current is increased by an increment I, in this case of 6 amperes.

When t = 0·004 second the current has reached the sending end after 3 reflections and is about to experience a fourth.

Therefore, sending end current = 18 amperes and is about to increase to 24 amperes.

171. *State three ways in which surges may be caused in an overhead transmission line. Describe one method of protecting the terminal apparatus from damage from overvoltage.*

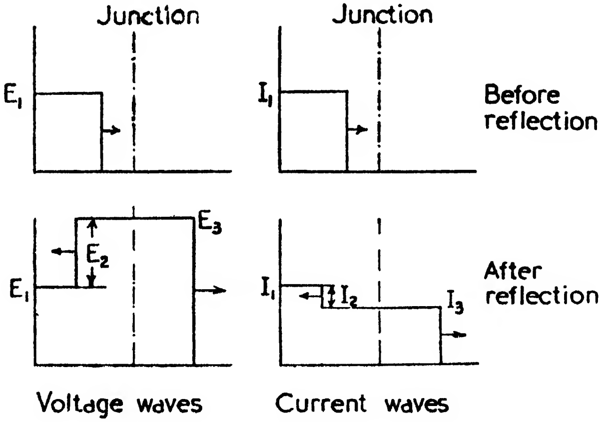
A surge of 20 kV magnitude travels along a cable towards its junction

with an overhead line. The inductance and capacitance of the cable are 0·8 *millihenrys and* 1·0 *microfarad, and of the overhead line,* 6 *millihenrys and* 0·05 *microfarad respectively. Calculate the voltage rise at the junction due to the surge.* (*C. and G. Final, Pt. II,* 1945)

The surge impedances are:

for the cable,
$$Z_1 = \sqrt{\frac{L_1}{C_1}} = \sqrt{\frac{0{\cdot}8 \times 10^{-3}}{1{\cdot}0 \times 10^{-6}}} = 28{\cdot}28 \text{ ohms.}$$

For the overhead line,
$$Z_2 = \sqrt{\frac{L_2}{C_2}} = \sqrt{\frac{0{\cdot}6 \times 10^{-3}}{0{\cdot}05 \times 10^{-6}}} = 109{\cdot}5 \text{ ohms.}$$



Voltage waves Current waves

Fig. 124

Referring to Fig. 124, let E_1 and I_1 be the voltage and current arriving at the junction where $E_1 = 20$ kV.

Let E_2 and I_2 be the reflected voltage and current, and E_3 and I_3 be the voltage and current which are transmitted into the overhead line.

Then
$$\frac{E_1}{I_1} = Z_1$$

Since the wave is travelling from a low impedance cable into a higher impedance line, at the junction the voltage is reflected without change of sign and the current with change of sign. Hence

$$\frac{E_2}{I_2} = -Z_1$$

Also the voltage reflected back into the cable is the algebraic difference between the incident voltage and the voltage transmitted into the overhead line, i.e.

$$E_2 = E_3 - E_1 \qquad (1)$$

The current reflected back into the cable is the algebraic difference between the incident and transmitted currents, i.e.

$$I_2 = I_1 - I_3 \qquad (2)$$

Now each of the voltages referred to above equals the product of the corresponding current and the surge impedance into which it is flowing.

Therefore, $I_2Z_1 = I_3Z_2 - I_1Z_1$

and $$I_2 = I_3 \cdot \frac{Z_2}{Z_1} - I_1 \qquad (3)$$

Equating (2) and (3),

$$I_1 - I_3 = I_3 . \frac{Z_2}{Z_1} - I_1$$

$$I_3 \left(1 + \frac{Z_2}{Z_1}\right) = 2I_1$$

$$I_3 = \frac{2Z_1}{Z_1 + Z_2} \cdot I_1$$

Now $E_3 = I_3Z_2$ and $I_1 = \dfrac{E_1}{Z_1}$

Hence, $E_3 = \dfrac{2Z_2}{Z_1 + Z_2} \cdot E_1$ = the voltage at the junction due to the surge.

In this example, $E_3 = \dfrac{2 \times 109{\cdot}5}{28{\cdot}28 + 109{\cdot}5} \times 20$ kV

= 31·78 kV or a rise of 11·78 kV.

(vii) **Line protection.**

172. *Explain, with connection and vector diagrams, the action of the Peterson coil on a transmission line.*

Deduce an expression for the inductance of such a coil in terms of the capacitance of the line and the frequency of the system. Calculate this inductance for a 50-*frequency line in which the capacitance (line to neutral) is* 2 *microfarads.*

(*C. and G. Final, Pt. II,* 1943)

The Peterson coil is an iron-cored reactance connected between the neutral point of a 3-phase system and earth for the purpose of extinguishing arcs which might otherwise develop in the event of an earth fault on one line.

Fig. 125 shows the arrangement, L being the Peterson coil and c the faulty line. The two sound lines have capacitances C to earth. Fig. 126 shows the voltage and current vector diagrams. V_{an} V_{bn} and V_{cn} are the voltages of the respective lines to the neutral and when c is earthed V_{ac} and V_{bc} are the voltages of the sound lines to earth.

The currents flowing are:

$I_a = V_{ac} \cdot \omega C$ from a to earth leading V_{ac} by 90°,

$I_b = V_{bc} \cdot \omega C$ from b to earth leading V_{bc} by 90°,

$I_L = \dfrac{V_{nc}}{\omega L}$ from n to earth lagging V_{nc} by 90°.

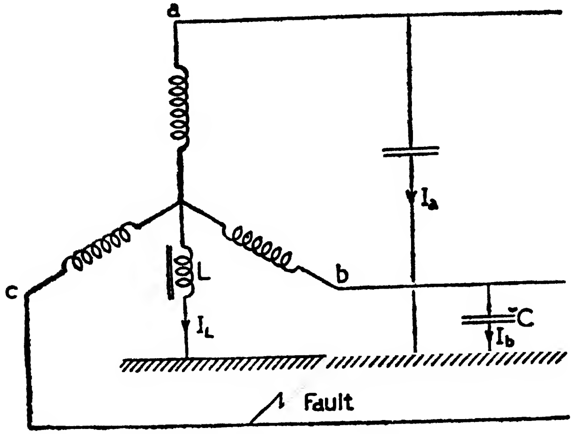


Fig. 125

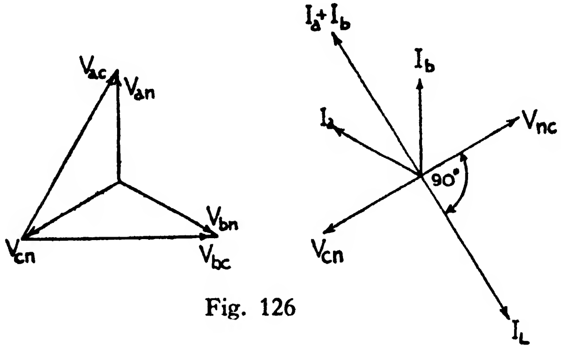


Fig. 126

The resultant of I_a and I_b is $I_a + I_b = \sqrt{3}\cdot V_{ac}\cdot\omega C$ and is in phase opposition to I_L.

Hence if $I_L = I_a + I_b$ there is no current through the earth fault and there will be no arc. For this to be the case,

$$\frac{V_{nc}}{\omega L} = \sqrt{3}\cdot V_{ac}\cdot\omega C$$

But $$V_{ac} = \sqrt{3}\ V_{nc},$$

Hence $$\frac{V_{nc}}{\omega L} = \sqrt{3}\cdot\sqrt{3}\cdot V_{nc}\cdot\omega C$$

i.e. $$L = \frac{1}{3\omega C}$$

In this problem, $$L = \frac{1}{3 \times 2\pi \times 50 \times 2 \times 10^{-6}}$$

$$= \mathbf{533\ henrys.}$$

173. *What is the object of a Peterson coil? Describe, with the aid of sketches, how it functions.*

Calculate the reactance of a coil suitable for a 33-*kV* 3-*phase transmission system of which the capacitance to earth of each conductor is* 4·5 *microfarads.*

(*I.E.E., Pt. II, May,* 1940)

From the proof given in the previous example,

$$L = \frac{1}{3\omega C}$$

Therefore the reactance necessary is

$$\omega L = \frac{1}{3\omega C} \times \omega$$

$$= \frac{1}{3C}$$

$$= \frac{1}{3 \times 4{\cdot}5 \times 10^{-6}}$$

$$= \mathbf{74100\ ohms.}$$

CHAPTER XIV

UNDERGROUND CABLES

(i) Insulation.

174. *Describe the construction of an underground cable for a* 66-*kV* 3-*phase circuit, and sketch the cross-section. In what respects does the cable differ from one for a* 6·6-*kV circuit? Give reasons for the differences.*

A single-core cable, for a working voltage of 6·5 *kV* (*between the core and sheath*), *has a conductor of* 0·4 *in. overall diameter, which is insulated with impregnated paper to a radial thickness of* 0·3 *in. and lead-covered. Calculate the maximum electric stress* (*R.M.S. volts per cm.*) *on the insulation, assuming the dielectric constant to be* 3·2. (*I.E.E., May, Pt. II,* 1942)

Maximum stress on the insulation, $$g_m = \frac{E}{r \log_e \frac{r_1}{r}}$$

where E = R.M.S. voltage between conductor and sheath,
r = the radius of the conductor,
r_1 = the external radius over the insulation.

In this question,
$E = 6500$ volts,
$r = 0{\cdot}2 \times 2{\cdot}54$ cm.,
$r_1 = 0{\cdot}5 \times 2{\cdot}54$ cm.

Hence,
$$g_m = \frac{6500}{0{\cdot}2 \times 2{\cdot}54 \log_e \frac{0{\cdot}5}{0{\cdot}2}}$$
$$= \frac{6500}{0{\cdot}2 \times 2{\cdot}54 \times 0{\cdot}9163}$$
$$= \mathbf{13960\ volts/cm.\ (R.M.S.)}$$

Note.—The dielectric constant is not needed in this calculation.

175. *Explain the principle of capacitance grading of the insulation of high-voltage cables.*

The inner conductor of a concentric cable has a diameter of 3 *cm., the diameter over the insulation being* 8·5 *cm. The cable is insulated with two materials having permittivities of* 5 *and* 3 *respectively with corresponding safe working stresses of* 38 *kV per cm. and* 26 *kV per cm. Calculate the radial thickness of each insulating layer and the safe working voltage of the cable.*
(*C. and G. Final, Pt. II,* 1943)

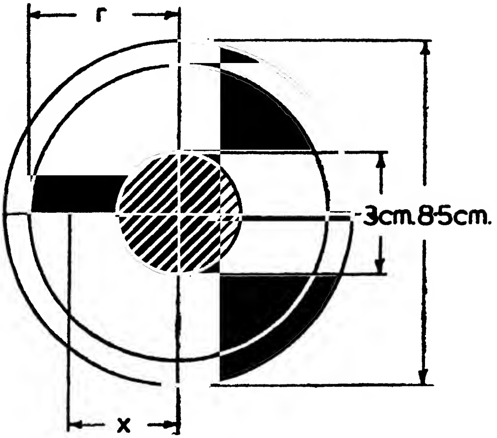


Fig. 127

Let r (Fig. 127) be the outer radius of the inner insulating material, and the charge on the conductor be q per cm. length. Then if k_1 is the permittivity of the inner dielectric and K_2 the permittivity of the outer dielectric and since the electric stress at any radius x is

$$g = \frac{2q}{kx},$$

therefore the maximum stress in the inner dielectric (i.e. at the surface of the conductor) is

$$g_1 = \frac{2q}{5 \times 1 \cdot 5} = \frac{2q}{7 \cdot 5}$$

The maximum stress in the outer dielectric (at radius **r**) is

$$g_2 = \frac{2q}{3 \times r} = \frac{2q}{3r}$$

Therefore, $\frac{g_1}{g_2} = \frac{3r}{7 \cdot 5}$

But $g_1 = 38$ kV/cm. and $g_2 = 26$ kV/cm.

Hence $r = \frac{38}{26} \times \frac{7 \cdot 5}{3}$

$= 3 \cdot 65$ cm.

Therefore the **radial thicknesses of the dielectric** are:

inner = 2·15 cm., outer = 0·6 cm.

The p.d. across a dielectric between radii r_1 and r_2 is given by the expression

$$E = g_m r_1 \log_e \frac{r_2}{r_1}$$

where g_m = the maximum stress in the insulation.

For the inner dielectric, $g_m = 38$ kV/cm., $r_1 = 1 \cdot 5$ cm., $r_2 = 3 \cdot 65$ cm.

Therefore E_1 (peak) $= 38 \times 1 \cdot 5 \times \log_e \frac{3 \cdot 65}{1 \cdot 5}$

$= 2 \cdot 303 \times 38 \times 1 \cdot 5 \times \log_{10} \frac{3 \cdot 65}{1 \cdot 5}$

$= 2 \cdot 303 \times 38 \times 1 \cdot 5 \times 0 \cdot 3861$

$= 50 \cdot 69$ kV.

For the outer dielectric, $g_m = 26$ kV/cm., $r_1 = 3 \cdot 65$ cm., $r_2 = 4 \cdot 25$ cm.

Therefore E_2 (peak) $= 2{\cdot}303 \times 26 \times 3{\cdot}65 \times \log_{10} \dfrac{4{\cdot}25}{3{\cdot}65}$

$= 2{\cdot}303 \times 26 \times 3{\cdot}65 \times 0{\cdot}0661$

$= 14{\cdot}45$ kV.

Peak voltage of cable $= E_1 + E_2$

$= 65{\cdot}14$ kV.

Safe working voltage (R.M.S.) $= \dfrac{65{\cdot}14}{\sqrt{2}}$

$=$ **46·08 kV.**

Note: If the safe working stresses given are in terms of R.M.S. values, the safe working voltage is 65·14 kV.

176. *Deduce an expression for the dielectric stress in a single-core cable and indicate how the potential gradient is controlled by using a metallic intersheath.*

In a 66-*kV lead-sheathed, paper-insulated cable the material has a permissible potential gradient of* 40 *kV per cm. Calculate the minimum overall diameter of the cable and the voltage at which the intersheath must be maintained. What is the economic conductor diameter and overall diameter of a similar cable with no intersheath?* (*C. and G. Final, Pt. II,* 1939)

The purpose of the intersheath is to divide up the insulation into two layers, in each of which the maximum stress is the same. This is done by maintaining the intersheath at a definite potential between that of the conductor and the sheath.

Let r be the conductor radius (Fig. 128), r_1 the outer radius of the inner insulating layer and r_2 the outer radius of the outer insulating layer. Let E_1 be the voltage between the conductor and the intersheath and E_2 the voltage between the intersheath and the sheath.

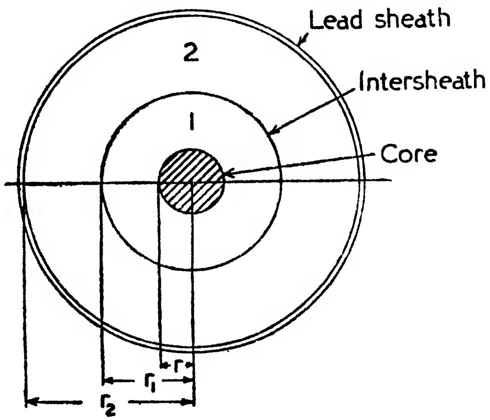


Fig. 128

Then the maximum stress in each layer is given by

$$g_m = \frac{E_1}{r \log_e \dfrac{r_1}{r}} = \frac{E_2}{r_1 \log_e \dfrac{r_2}{r_1}} \qquad (1)$$

It may be shown that for a given cable voltage E and a given maximum stress g_m, the minimum overall diameter r_2 is obtained when

$$E_1 = \frac{E}{e}, \; r = \frac{E}{e.g_m}, \text{ and } r_1 = \frac{E}{g_m} \qquad (2)$$

If the relations given in (2) are substituted in equation (1) it will be found that the minimum overall diameter is

$$r_2 = r_1 \cdot e^{\frac{E_2}{E}}$$

In the question, $g_m = 40$ kV (assumed peak), and $E = 66\sqrt{2}$ kV (peak)

Hence from (2),
$$40 = \frac{66\sqrt{2}}{2{\cdot}718\ r}$$
$$r = 0{\cdot}86 \text{ cm.}$$

Therefore
$$r_1 = 0{\cdot}86 \times 2{\cdot}718$$
$$= 2{\cdot}34 \text{ cm.}$$

$$r_2 = 2{\cdot}34 \times 2{\cdot}718^{\frac{E_2}{E}}$$

Now
$$E_2 = E - E_1$$
$$= 66 - \frac{66}{2{\cdot}718} \text{ kV (R.M.S.)}$$
$$= 41{\cdot}7 \text{ kV.}$$

Hence,
$$r_2 = 2{\cdot}34 \times 2{\cdot}718^{\frac{41{\cdot}7}{66}}$$
$$= 4{\cdot}39 \text{ cm.}$$

i.e. **Minimum overall diameter of the intersheathed cable = 8·78 cm.**

For a similar cable with no intersheath, let r = the conductor radius and R = the overall radius. Then the most economical conductor is when

$$\frac{R}{r} = e = 2{\cdot}718$$

Therefore
$$E = g_m r \log_e \frac{R}{r}$$
$$= g_m \cdot r$$
$$r = \frac{E}{g_m}$$
$$= \frac{66\sqrt{2}}{40}$$
$$= 2{\cdot}33 \text{ cm.}$$
$$R = 2{\cdot}33 \times 2{\cdot}718$$
$$= 6{\cdot}33 \text{ cm.}$$

Therefore **the most economic dimensions for the conductor with no intersheath are:**

Conductor diameter = 4·66 cm., overall diameter = 12·66 cm.

177. *Deduce an expression for the voltage gradient in the dielectric of a single-core cable in terms of the voltage V between the conductor and the sheath, R the radius of the sheath and r the radius of the conductor.*

Determine the value of r which gives the minimum voltage gradient for fixed values of V and R and calculate the best value of r if V is 93 *kV and the maximum permissible gradient is* 50 *kV per cm.*

(*London, B.Sc. Eng., July*, 1944)

The highest value of the voltage gradient occurs at the surface of the conductor and is given by

$$g_m = \frac{V}{r \log_e \frac{R}{r}} \qquad (1)$$

If V and R are fixed and r is variable, the value of r which gives the minimum value of g_m is that which makes $\frac{dg_m}{dr}$ equal to zero.

$$\frac{dg_m}{dr} = \frac{-V\left(\log_e \frac{R}{r} + r \cdot \frac{r}{R}\left(-\frac{R}{r^2}\right)\right)}{\left(r \log_e \frac{R}{r}\right)^2}$$

For g_m to be a minimum,

$$\log_e \frac{R}{r} + r \cdot \frac{r}{R}\left(-\frac{R}{r^2}\right) = 0$$

$$\log_e \frac{R}{r} - 1 = 0$$

$$\log_e \frac{R}{r} = 1$$

$$\frac{R}{r} = e$$

$$r = \frac{R}{e}$$

If this value of r is substituted in (1) **the minimum value of g_m becomes**

$$g_m = \frac{V}{r}$$

The significance of these results is that if the overall and core radii are in the ratio of e, then for a given core radius the voltage gradient at the surface of the conductor, which is always where the insulation is most heavily stressed, is a minimum. Under these circumstances the voltage gradient at the conductor surface is inversely proportional to the core radius.

If V = 93 kV and the voltage gradient at the conductor surface must not exceed 50 kV per cm., then the best value of r is given by

$$50 = \frac{93\sqrt{2}}{r}$$

$$\mathbf{r = 2{\cdot}63\ cm.}$$

Note.—It is assumed here that the value given for V is R.M.S. and that for the voltage gradient is peak value.

178. *A single-core cable* 5 *miles long has an insulation resistance of* 0·4 *megohm. The core diameter is* 20 *mm. and the diameter of the cable over the insulation is* 5 *cm. Calculate the resistivity of the insulating material. Prove the formula used.* (*C. and G. Final, Pt. I,* 1943)

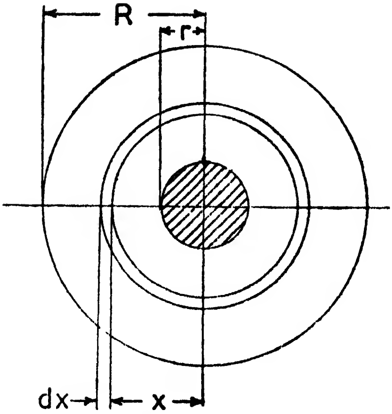


Fig. 129

Consider an element of the insulating material at a radius x from the centre of the conductor, and having a thickness dx in a length l of the cable, all dimensions being in cm.

Let ρ = the resistivity of the material in megohms per cm. cube.

Insulation resistance through the element $= \dfrac{\rho\,dx}{2\pi x l}$

Total insulation resistance

$$= \int_r^R \frac{\rho\,dx}{2\pi x l}$$

$$= \frac{\rho}{2\pi l}\int_r^R \frac{dx}{x}$$

$$= \frac{\rho}{2\pi l}\log_e \frac{R}{r}$$

$$= \frac{2{\cdot}303\,\rho}{2\pi\, l}\log_{10}\frac{D}{d} \text{ megohms}$$

$$= \frac{\rho}{2{\cdot}728\, l}\log_{10}\frac{D}{d} \text{ megohms.}$$

In the problem given, the insulation resistance = 0·4 megohm, l = 5 × 5280 × 12 × 2·54 cm. = 0·8046 × 10^6 cm.

$$\frac{D}{d} = \frac{5}{2} = 2{\cdot}5$$

$$\log_{10}\frac{D}{d} = 0{\cdot}3979$$

Therefore,

$$\rho = \frac{0{\cdot}4 \times 2{\cdot}728 \times 0{\cdot}8046 \times 10^6}{0{\cdot}3979}$$

$$= \mathbf{2{\cdot}207 \times 10^6} \textbf{ megohms per cm. cube.}$$

179. *Deduce an expression for the insulation resistance of a mile of single-core concentric cable which has a diameter over the insulation D cm., a conductor diameter d cm. and a resistivity of ρ megohms per cm. cube for the insulating material.*

Find the resistivity of the insulating material of a high-voltage concentric cable which has inner and outer diameters of 0·15 *in. and* 1 *in. respectively, and an insulation resistance of* 5000 *megohms for* 1 *mile.*

(*C. and G. Final, Pt. I*, 1937)

The insulation resistance in megohms is given by

$$R = \frac{\rho}{2{\cdot}728\ l} \log_{10} \frac{D}{d}$$

where
ρ = the resistivity in megohms per inch cube
l = the length of cable in inches,
D = the outer diameter in inches,
d = the inner diameter in inches.

In this problem,

R = 5000 megohms,
$l = 5280 \times 12$ in.,
D = 1 in.
d = 0·15 in.

Hence,

$$5000 = \frac{\rho}{2{\cdot}728 \times 5280 \times 12} \log_{10} \frac{1}{0{\cdot}15}$$

$$= \frac{0{\cdot}8239\ \rho}{2{\cdot}728 \times 5280 \times 12}$$

$$\rho = \frac{5000 \times 2{\cdot}728 \times 5280 \times 12}{0{\cdot}8239}$$

$$= \mathbf{1049 \times 10^6}$$ **megohms per inch cube.**

(ii) **Capacitance.**

180. *Deduce an expression for the capacitance per mile of a single-core, metal-sheathed cable in terms of the core diameter, the diameter inside the sheath and the permittivity of the dielectric. Explain how the dimensions of a high-voltage cable may be reduced by permittivity grading.*

An oil-filled cable has a core diameter of 2 *cm. and an inside sheath diameter of* 6 *cm. Calculate the charging current by* 5 *miles of this cable when used as one phase of a* 3-*phase,* 66-*kV*, 50-*c/s system. The permittivity is* 2·8.

(*C. and G. Final, Pt. II*, 1944)

The capacitance per mile of a single-core metal-sheathed cable is

$$C = \frac{0{\cdot}0894\ k}{\log_e \frac{R}{r}} \text{ microfarads}$$

where
k = the permittivity of the dielectric,
R = the inside radius of the sheath,
r = the core radius.

For a 5-mile length of the cable given,

$$C = \frac{0{\cdot}0894 \times 2{\cdot}8}{\log_e \frac{6}{2}} \times 5 \text{ microfarads}$$

$$= \frac{0{\cdot}0894 \times 2{\cdot}8 \times 5}{1{\cdot}0986} \text{ microfarads}$$

$$= 1{\cdot}139 \text{ microfarads}$$

$$\text{Phase voltage} = \frac{66}{\sqrt{3}}\ \text{kV.}$$

$$\text{Hence, \textbf{charging current}} = \frac{66}{\sqrt{3}} \times 10^3 \times 2\pi \times 50 \times 1\cdot139 \times 10^{-6}\ \text{amperes}$$

$$= \mathbf{13\cdot63}\ \text{amperes.}$$

181. *Show how the capacitance of a* 3*-phase,* 3*-core, lead-sheathed cable can be represented by a combination of star-connected and delta-connected condensers. Derive an expression for the charging current of such a cable in terms of the capacitance measured between any two of the cores, with the third core and the sheath insulated.*

Calculate the kVA taken by 10 *miles of such a cable which has a capacitance of* 0·3 *microfarad per mile measured between two of the cores, when it is connected to* 10000*-volt,* 50*-frequency bus-bars.*

(*C. and G. Final, Pt. II,* 1940)

Fig. 130 (a) shows the arrangement of condensers which is equivalent to the capacitances in a 3-phase, lead-sheathed cable, C_1 representing the intercore capacitances and C_2 the capacitance of each core to the sheath.

This diagram may be replaced by the equivalent diagram Fig. 130(b) in which the star capacitances C_3 take the place of the delta capacitances C_1. For the two circuits to be equivalent $C_3 = 3C_1$. Now it is obvious that the star point N and the sheath will be equipotential hence the cable may also be represented by the system of condensers shown in Fig. 131.

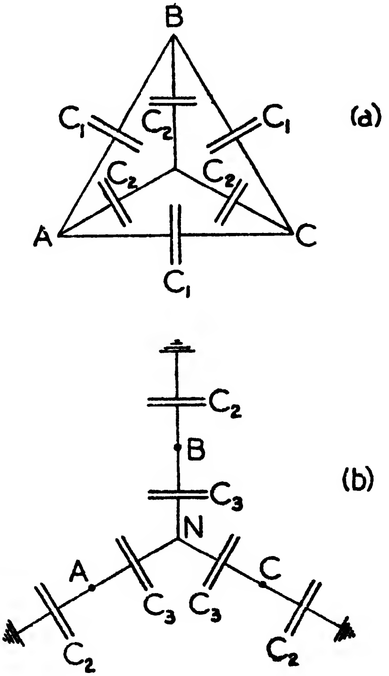


Fig. 130

Line charging current for the star-connected system of Fig. 131.

$$= V_{ph} \cdot \omega \cdot (C_2 + C_3)$$

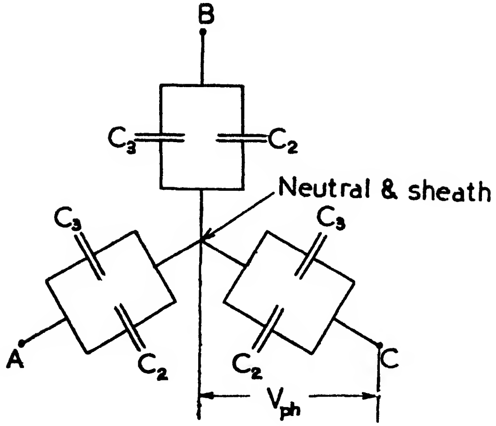


Fig. 131

If C_4 is the capacitance measured between A and B with C insulated, then

$$C_4 = \tfrac{1}{2}(C_2 + C_3), \text{ therefore}$$

$$C_2 + C_3 = 2C_4$$

Line charging current $= V_{ph} \cdot \omega \cdot 2C_4$

If V is the line voltage,

Line charging current $= \dfrac{2V\,\omega\,C_4}{\sqrt{3}}$ amperes

$= 7{\cdot}25fVC_4$ where f is the supply frequency.

In the question, $C_4 = 0{\cdot}3$ microfarad/mile × 10 miles

$= 3{\cdot}0$ microfarads.

Line charging current $= \dfrac{2 \times 10000 \times 2\pi \times 50 \times 3{\cdot}0 \times 10^{-6}}{\sqrt{3}}$ amperes

$= I_c$

Charging kVA $= \sqrt{3}VI_c \times 10^{-3}$

$$= \frac{\sqrt{3} \times 10000 \times 2 \times 10000 \times 2\pi \times 50 \times 3}{\sqrt{3} \times 10^9}$$

$= 188{\cdot}4$ kVA_r.

182. *Explain how the capacitance of a* 3-*core,* 3-*phase lead-sheathed cable can be represented by a system of condensers. Deduce an expression for the charging current from a single measurement of the capacitance between any two of the cores.*

In such a cable the capacitance between the three cores bunched together and the lead sheath is 0·55 *microfarad per mile and that between two of the cores connected together and the third core is* 0·523 *microfarad per mile. Calculate* (*a*) *the capacitance per mile measured between any two of the cores,* (*b*) *the kVA required to keep* 10 *miles of this cable charged when connected to* 33-*kV,* 3-*phase,* 50-*frequency bus-bars.*

(*C. and G. Final, Pt. II,* 1937)

The proof for the first part of this question is the same as that given in the preceding problem. From the data given in the second part and using the notation of Figs. 130 and 131.

$$3C_2 = 0{\cdot}55 \text{ microfarad,}$$
$$\therefore C_2 = 0{\cdot}183 \text{ microfarad.}$$

If B and C are joined together the capacitance between A and B consists of two capacitances of 2 $(C_2 + C_3)$ and $(C_2 + C_3)$ in series.

Therefore, capacitance between A and B

$$= \tfrac{2}{3}(C_2 + C_3)$$
$$= 0{\cdot}523 \text{ microfarad.}$$

Since $C_2 = 0{\cdot}183$ microfarad, therefore

$$\tfrac{2}{3}(0{\cdot}183 + C_3) = 0{\cdot}523$$
$$0{\cdot}183 + C_3 = 0{\cdot}785$$
$$C_3 = 0{\cdot}602 \text{ microfarad per mile.}$$

Hence $$C_4 = \tfrac{1}{2}(C_2 + C_3)$$
$$= \mathbf{0{\cdot}393 \text{ microfarad per mile}}$$
$$= \textbf{capacitance per mile between any 2 cores.}$$

(b) Capacitance of 10 miles between 2 cores = 3·93 microfarads.

Charging kVA of 10 miles of cable

$$= \sqrt{3}VI_c \times 10^{-3}$$
$$= \sqrt{3}\,V \times \frac{2V\,\omega\,C_4}{\sqrt{3}}$$
$$= \frac{2 \times (33 \times 10^3)^2 \times 2\pi \times 50 \times 3{\cdot}93}{10^9}$$
$$= 2690 \text{ kVA}_r$$

Therefore to keep 10 miles of the cable charged requires 2690 kVA$_r$

183. *A* 9-*mile length of single-phase concentric cable takes a charging current of* 6 *amperes when connected to* 11000-*volt,* 50-*cycle bus-bars. The inner conductor has a diameter of* 0·4 *in. and the insulation has a radial thickness of* 0·6 *in. Calculate the permittivity of the dielectric.*

Prove any formula used. (*C. and G. Final, Pt. I,* 1940)

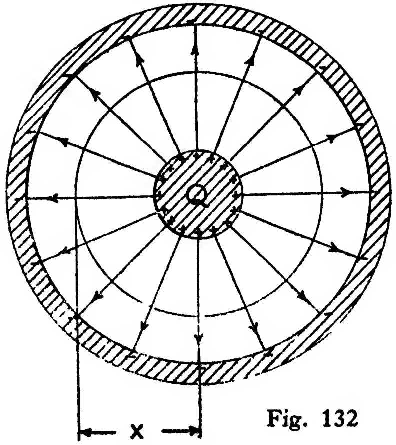


Fig. 132

Consider 1 cm. length of the cable with a charge of Q e.s.u. on the centre conductor.

Total electric flux passing through a cylinder radius x enclosing the centre conductor $\Big\}$ = $4\pi Q$ lines

Electric flux density at P $= \dfrac{4\pi Q}{2\pi x}$

$$= \frac{2Q}{x} \text{ lines/cm.}^2$$
$$= D_x$$

But $D_x = k\,\epsilon_x$

where $\epsilon_x =$ the electric force at radius x

Hence, $\epsilon_x = \frac{2Q}{kx}$

The work done in transferring unit charge a distance dx in the direction of the field is ϵ_x dx.

Work done in transferring unit charge from the centre conductor to the sheath is the potential difference V between them.

Therefore
$$V = \int_r^R \epsilon_x\,dx$$
$$= \int_r^R \frac{2Q}{kx}\,dx$$
$$= \frac{2Q}{k}\log_e \frac{R}{r} \text{ e.s.u.}$$

Capacitance $C = \frac{Q}{V} = \frac{k}{2\log_e \frac{R}{r}}$ e.s.u. per cm. length.

In this problem,

reactance of 9-mile length $= \frac{11000}{6}$ ohms.

Capacitance $= \frac{6}{2\pi \times 50 \times 11000}$ farads.

Capacitance per cm. length
$$= \frac{6 \times 9 \times 10^{11}}{2\pi \times 50 \times 11000 \times 9 \times 5280 \times 30{\cdot}48}$$
$$= 1{\cdot}079 \text{ e.s.u.}$$

Outer radius $R = (0{\cdot}2 + 0{\cdot}6)$ in.
$= 0{\cdot}8$ in.

Inner radius $r = 0{\cdot}2$ in.

Therefore $\frac{R}{r} = 4{\cdot}0$

$$\log_{10}\frac{R}{r} = 0{\cdot}6021$$
$$\log_e \frac{R}{r} = 0{\cdot}6021 \times 2{\cdot}303$$
$$= 1\,3863$$

Permittivity $= 2C\log_e \frac{R}{r}$
$$= 2 \times 1{\cdot}079 \times 1{\cdot}3863$$
$$= 3{\cdot}0$$

CHAPTER XV

ECONOMICS OF GENERATION AND DISTRIBUTION

(i) **Generation costs.**

184. *The annual working costs of a coal-fired turbine-driven electric generating station can be represented by the formula £ $(a + b \times kW + c \times kWh)$, where a, b, c, are constants for a particular station, kW is the total power installed and kWh is the energy produced per annum. Explain the significance of the constants a, b, c, and the factors upon which their numerical values depend. Determine their values for a* 60-*MW station operating with an annual load factor of* 50 *per cent for which* (1) *the capital cost of the buildings and equipment is £*500000; (2) *the annual cost of fuel, oil, taxation, wages and salaries of the operating staff is £*90000; (3) *the interest and depreciation charges on the buildings and equipment are at the rate of* 10 *per cent per annum; and* (4) *the annual costs of organization, interest on cost of site, etc., are £*50000. (*I.E.E., Pt. II, November,* 1942)

In the expression for an electric generating station

Total annual working costs = £ (a + b × kW + c × kWh),

£a = a fixed charge, independent of the maximum power output or the total energy output of the station. It is due to the annual cost of the central organization, interest on the capital cost of land for the site, etc.

£b = a constant which when multiplied by the maximum kW demand on the station gives the semi-fixed annual charge, which is independent of the total energy output per annum. It arises from the interest and depreciation on the capital cost of buildings and equipment.

£c = a constant, which, multiplied by the total kWh output per annum, gives the annual cost of fuel, oil, taxation, wages and salaries, etc., of the operating staff.

In this problem, **a = £50000**

Also, b × maximum kW = 10 per cent of £500000

Hence, b × 60000 = £50000

b = 0·833

At a load factor of 50 per cent and a maximum demand of 60000 kW,

annual energy output = 60000 × 8760 × 0·5 kWh

= 262800000 kWh.

Hence, £c × 262800000 = £90000

c = 0·0003425

185. *Explain the term "load factor" as applied to electric generating stations. Discuss the effect of load factor on (a) the choice of plant for a generating station with steam turbines, (b) the running costs of such a station.*

The output of a generating station is 525×10^6 *kWh per annum, and the average load factor is* 60 *per cent. If the annual fixed charges are* £1·25 *per kW of installed plant and the annual running charges are* 0·15*d. per kWh, what is the cost per kWh of energy at the bus-bars?*

(*I.E.E., Pt. II, May,* 1943)

$$\text{Average kW supplied per annum} = \frac{525 \times 10^6}{8760} = 59940 \text{ kW}$$

$$\text{Load factor} = \frac{\text{Average load in kW}}{\text{Maximum demand in kW}}$$

$$\text{Therefore,} \quad 0{\cdot}6 = \frac{59940}{\text{Maximum demand in kW}}$$

$$\text{Maximum kW demand during the year} = \frac{59940}{0{\cdot}6} = 99900 \text{ kW.}$$

Since the plant installed must be based on the maximum demand, the fixed annual charges at the rate of £1·25 per kW of plant will be

$$= £(99900 \times 1{\cdot}25) = £124875$$

This cost must be spread over 525×10^6 kWh and added to the running charges to obtain the total price per kWh at the bus-bars.

$$\textbf{Cost per kWh at the bus-bars} = 0{\cdot}15\text{d.} + \frac{124875 \times 240}{525 \times 10^6}\text{d.} = 0{\cdot}15\text{d.} + 0{\cdot}057\text{d.} = \mathbf{0{\cdot}207d.}$$

186. *State the chief items of expenditure for an electric supply undertaking and classify them into fixed and running charges.*

An electric supply undertaking having a maximum load of 100 *MW generates* 375 *million kWh per annum and supplies consumers having an aggregate maximum demand of* 165 *MW. The annual expenses, including capital charges, are: fuel,* £200000; *fixed expenses connected with generation,* £300000; *transmission and distribution expenses,* £350000. *Assuming that* 90 *per cent of the fuel cost is assigned to the running charges, and that the losses in transmission and distribution are* 15 *per cent of the kWh generated, deduce a two-part tariff to represent the actual cost of supply to consumers.*

(*I.E.E., May,* 1942)

Total fixed charges = Fixed charges for generation + transmission and distribution expenses + 10 per cent of fuel cost

= £300000 + £350000 + £20000

= £670000 per annum.

This total fixed cost must be spread over the aggregate maximum demand of all consumers and charged to each of them at the corresponding rate per kW of maximum demand.

$$\text{Rate per kW of maximum demand} = \frac{£670000}{165000}$$
$$= £4{\cdot}06, \text{ say } £4 \text{ 1s. 3d.}$$

The running charges are 90 per cent of the fuel cost

$$= £180000 \text{ per annum.}$$

This cost must be spread over the actual number of kWh delivered to the consumers, which, owing to losses in transmission and distribution total 85 per cent of the kWh generated per annum.

$$\text{Running cost per kWh} = \frac{180000 \times 240}{375 \times 10^6 \times 0{\cdot}85} \text{ d.}$$
$$= 0{\cdot}1355\text{d., say } 0{\cdot}14\text{d.}$$

Therefore, a suitable **tariff** would be:

£4 1s. 3d. per kW of maximum demand + 0·14d. per kWh consumed.

187. *Define the terms "diversity factor" and "load factor": give a short account of their influence upon central station design and the cost of supply.*

A generating station has a maximum demand of 80000 *kW and a yearly load factor of* 40 *per cent. Generating costs, inclusive of station capital costs, are* £2 *per annum per kW demand, plus* 0·2*d. per kWh transmitted. The capital charge for the transmission system is* £45000 *per annum and for the distribution system* £60000, *the respective diversity factors being* 1·1 *and* 1·3. *The efficiency of the transmission system is* 90 *per cent and that of the distribution system, including substation losses,* 80 *per cent. Find the yearly cost per kW demand and the cost per kWh supplied* (*a*) *at the substations,* (*b*) *on the consumers' premises.* (*C. and G. Final, Pt. II,* 1942)

Capital costs of station and fixed costs of generation = £2 per kW × 80000 kW

$$= £160000 \text{ per annum.}$$

Capital charges on the transmission system = £45000 per annum.

Therefore total fixed charges associated with the supply of energy to the substations = £205000 per annum.

Aggregate of all maximum demands by the substations = Maximum demand on station × diversity factor

$$= 80000 \times 1{\cdot}1 \text{ kW}$$
$$= 88000 \text{ kW.}$$

Therefore the fixed charges are spread over 88000 kW at the substations.

(a) **Yearly cost per kW demand at the substations**

$$= \frac{£205000}{88000}$$
$$= £2{\cdot}33$$
$$= £2 \text{ 7s.}$$

Since the transmission efficiency is 90 per cent, for every kWh transmitted only 0·9 kWh reaches the substation.

Therefore at the substation 0·9 kWh incurs a running charge of 0·2d.

i.e. **Cost per kWh supplied** = **0·222d.**

(b) The diversity factor for the distribution system is 1·3, therefore Aggregate of maximum demands of

all consumers = Maximum demand on substation × diversity factor
$\doteq$ 88000 × 1·3 kW
= 114400 kW.

Capital charges on distribution system = £60000

Therefore total fixed charges associated with the supply of energy to the consumer = £205000 + £60000
= £265000 per annum.

This cost must be spread over 114400 kW.

Therefore, **yearly cost per kW demand on consumers' premises** $= \frac{£265000}{114400}$
= £2·32
= **£2 6s. 5d.**

For each kWh supplied to a substation only 0·8 kWh reaches the consumer and this costs 0·222d.

Cost per kWh supplied = 0·278d.

(ii) **Tariffs.**

188. *Explain the economic basis of a typical two-part tariff.*

An industrial load can be supplied on the following alternative tariffs: (a) H.V. supply at £3 per kVA per annum plus 0·25d. per kWh, or (b) L.V. supply at £3 5s. per kVA per annum plus 0·3d. per kWh. Transformers and switchgear suitable for the high-voltage supply cost £2 8s. per kVA, the full-load transformation losses being 2 per cent. The fixed charges are 25 per cent per annum on the capital cost of the high-voltage plant and the installation works at full-load. Find the number of working hours per week above which the H.V. supply is the cheaper. There are 50 working weeks in a year.

(*C. and G. Final, Pt. II*, 1943)

Suppose the load is 100 kW and let x = the number of working hours per week above which the high-voltage supply is cheaper, i.e. at which the two tariffs give equal annual costs.

Since the transformation losses are 2 per cent, the rating of the transformers and switchgear would be $\frac{100 \times 100}{98}$ kW.

$$\text{Cost of transformers and switchgear} = \frac{10^4}{98} \times \pounds 2{\cdot}4 = \pounds 244{\cdot}9$$

$$\text{Fixed charges per annum on high-voltage plant} = \pounds 244{\cdot}9 \times 25 \text{ per cent} = \pounds 61{\cdot}225$$

$$\text{Total kWh used per annum by the load} = 100 \times x \times 50 = 5000x$$

On the high-voltage supply the total kWh which would have to be paid for would be $5000x \times \frac{100}{98}$

(a) On high-voltage supply,

$$\begin{aligned}\text{annual cost} &= \pounds 3 \times \text{kVA} + \frac{\pounds 0{\cdot}25 \times \text{kWh}}{240} + \text{charges on H.V. plant,} \\ &= \pounds 3 \times \frac{100}{0{\cdot}98} + \frac{\pounds 0{\cdot}25 \times 5000x \times 100}{240 \times 98} + \pounds 61{\cdot}225 \\ &= \pounds(367{\cdot}525 + 5{\cdot}315x)\end{aligned}$$

(b) On the L.V. supply,

$$\begin{aligned}\text{annual cost} &= \pounds 3{\cdot}25 \times \text{kVA} + \frac{\pounds 0{\cdot}3 \times \text{kWh}}{240} \\ &= \pounds 3{\cdot}25 \times 100 + \frac{\pounds 0{\cdot}3 \times 5000x}{240} \\ &= \pounds(325 + 6{\cdot}25x)\end{aligned}$$

For these two annual costs to be equal,

$$\begin{aligned}325 + 6{\cdot}25x &= 367{\cdot}525 + 5{\cdot}315x \\ 0{\cdot}935x &= 42{\cdot}525 \\ x &= 45{\cdot}5\end{aligned}$$

Therefore, **above 45·5 hours per week the high-voltage supply is cheaper.**

189. *A 25-h.p. induction motor is supplied with energy on a two-part tariff of £4 10s. per kVA of maximum demand per annum, plus 0·5d. per kWh. Motor (A) has an efficiency of 89 per cent and a power factor of 0·83. Motor (B), with an efficiency of 90 per cent and power factor 0·91 costs £10 more. With motor A the power factor would be raised to 0·91 (lagging), by installing condensers at a cost of £4 per kVA.*

If the service required from the motor is equivalent to 2280 hours per annum at full-load, compare the annual charges in the two cases. Assume interest and depreciation charges to be 12½ per cent per annum for the motor and 8 per cent per annum for the condensers. (*I.E.E., Pt. II, November,* 1944)

Motor A. Input to motor on full-load

$$= \frac{25 \times 746 \times 100}{1000 \times 89} \text{ kW} = 20{\cdot}95 \text{ kW} = 25{\cdot}24 \text{ kVA at } 0{\cdot}83 \text{ p.f.}$$

When the power factor is raised to 0·91,

$$\text{kVA demand of the motor} = \frac{20{\cdot}95}{0{\cdot}91} = 23{\cdot}0 \text{ kVA.}$$

Cost of energy supplied to motor

$$\begin{aligned}\text{per annum} &= £23 \times 4{\cdot}5 + \frac{£20{\cdot}95 \times 2280 \times 0{\cdot}5}{240}\\ &= £103{\cdot}5 + £99{\cdot}51\\ &= £203\end{aligned}$$

By the use of condensers the phase angle of the load is improved from $\varnothing_1$ ($\cos \varnothing_1 = 0{\cdot}83$) to $\varnothing$ ($\cos \varnothing = 0{\cdot}91$).

Reactive kVA necessary for this

$$\begin{aligned}\text{improvement} &= \text{Load kW } (\tan \varnothing_1 - \tan \varnothing)\\ &= 20{\cdot}95\ (0{\cdot}672 - 0{\cdot}4557)\\ &= 4{\cdot}532 \text{ kVA}_r\end{aligned}$$

$$\begin{aligned}\text{Annual charge on the condensers} &= £4 \text{ per kVA} \times 4{\cdot}532 \text{ kVA} \times 8 \text{ per cent},\\ &= £1{\cdot}45\end{aligned}$$

Total annual charges, exclusive of interest and depreciation on motor = £204·45

Motor B. Input to motor on full-

$$\begin{aligned}\text{load} &= \frac{25 \times 746 \times 100}{1000 \times 90}\\ &= 20{\cdot}72 \text{ kW}\\ &= 22{\cdot}77 \text{ kVA at } 0{\cdot}91 \text{ p.f.}\end{aligned}$$

Cost of energy supplied to

$$\begin{aligned}\text{motor per annum} &= £22{\cdot}77 \times 4{\cdot}5 + \frac{£20{\cdot}72 \times 2280 \times 0{\cdot}5}{240}\\ &= £102{\cdot}465 + £98{\cdot}42\\ &= £200{\cdot}885\end{aligned}$$

Hence, exclusive of interest and depreciation charges on the motor itself, B is cheaper than A by £(204·45 − 200·885), i.e. by £3·565 per annum. However, the price of B exceeds that of A by £10, hence there will be an additional annual charge on B of 12½ per cent of £10 due to increased interest and depreciation.

Therefore, **B is cheaper than A** by £3·565 − 12½ per cent of £10, i.e. **by £2 6s. (approx.) per annum.**

(iii) Power factor improvement.

190. *In a small generating station the first cost of that part of the generating plant affected by power factor is £6 per kVA and the cost of power-factor improvement plant is £1 5s. per reactive kVA. Assuming that the rates of interest, depreciation, operating costs and losses are the same for both, find the value of the power factor to which it will be economical for the plant to be operated.*

(*C. and G. Final, Pt. II,* 1940)

Let £X = the cost per kVA of that part of the generating plant affected by power factor,

£Y = the cost per reactive kVA of power factor improvement plant,

r = the annual rate of interest and depreciation on each.

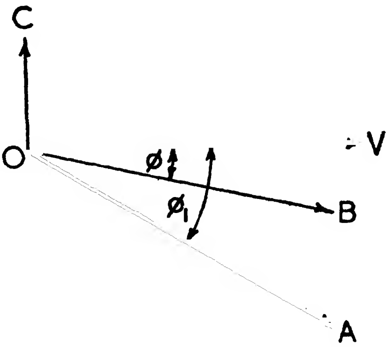


Fig. 133

Let OA (Fig. 133) represent the kVA of the load without the power factor improvement and OB the combined load with it. Then OC is the reactive kVA of the power factor improvement plant.

$$\text{Now OA} \cos\phi_1 = \text{OB} \cos\phi$$

$$\text{OB} = \text{OA}\cdot\frac{\cos\phi_1}{\cos\phi}$$

$$\text{Saving in plant kVA} = \text{OA} - \text{OB}$$

$$= \text{OA}\left(1 - \frac{\cos\phi_1}{\cos\phi}\right)$$

Annual saving in

$$\text{plant cost} = £\text{rX}\cdot\text{OA}\left(1 - \frac{\cos\phi_1}{\cos\phi}\right)$$

$$= £\text{rX}\cdot\text{OA}\cdot\cos\phi_1\left(\frac{1}{\cos\phi_1} - \frac{1}{\cos\phi}\right)$$

$$\text{Reactive kVA required} = \text{OC or AB}$$

$$= \text{OA} \sin\phi_1 - \text{OB} \sin\phi$$

$$= \text{OA} \cos\phi_1 (\tan\phi_1 - \tan\phi)$$

Annual cost of power factor improvement plant

$$= £\text{rY}\cdot\text{OA} \cos\phi_1 (\tan\phi_1 - \tan\phi)$$

$$\text{Nett annual saving} = £\left[\text{rX}\cdot\text{OA} \cos\phi_1\left(\frac{1}{\cos\phi_1} - \frac{1}{\cos\phi}\right) - \text{rY}\cdot\text{OA} \cos\phi_1 (\tan\phi_1 - \tan\phi)\right]$$

$$= £\text{P}$$

To find the condition for maximum saving, differentiate P with respect to ϕ and equate to zero, i.e.

$$\frac{d\text{P}}{d\phi} = \text{rX}\cdot\text{OA} \cos\phi_1\left(-\frac{\sin\phi}{\cos^2\phi}\right) - \text{rY}\cdot\text{OA} \cos\phi_1 \times\left(-\frac{1}{\cos^2\phi}\right)$$

$$= 0$$

Hence, $$\text{X}\frac{\sin\phi}{\cos^2\phi} = \text{Y}\frac{1}{\cos^2\phi}$$

i.e. $$\sin\phi = \frac{\text{Y}}{\text{X}}$$

This gives the most economical phase angle $\varnothing$ at which the plant may be operated.

In the question, $Y = £1{\cdot}25$ per reactive kVA.
$X = £6$ per kVA.

Therefore, $\sin\varnothing = \frac{1{\cdot}25}{6} = 0{\cdot}2083$

$\varnothing = 12°$ and

the most economic power factor $= \cos\varnothing = \mathbf{0{\cdot}978}$

191. *Explain the object of an industrial tariff of the form £a + b (pence), where £a is the charge per annum per kVA of maximum demand and b is the price per kWh.*

A factory takes a steady 3-*phase load of* 100 *kVA, at* 0·7 *power factor (lagging), from* 400-*volt* 50-*c/s. supply mains, and is charged on a tariff of the above form, £a being £*4 10*s. and b being* 0·5*d. It is desired to improve this power factor, by the installation of condensers, to a value which will reduce to a minimum the annual charges for electricity and the condenser installation. Calculate the kVA rating of the condensers and draw a diagram of connections. Assume the cost of condensers, including installation to be £*4 *per kVA, and interest and depreciation to be at the rate of* 12 *per cent per annum.*

(*I.E.E., Pt. II, November,* 1943)

In Problem 190 it was shown that the most economical phase angle, $\varnothing$, for a plant was given by

$$\sin\varnothing = \frac{Y}{X}$$

where X = the cost per kVA of the plant,
Y = the cost per reactive kVA of the power factor improvement plant.

This expression can be adapted to suit the case of the purchase of electrical energy by a consumer on the basis of his maximum demand and a flat rate per kWh.

Let £X = the annual charge per kVA maximum demand,
£Y = the annual charge per kVA for interest and depreciation on condensers for power factor improvement.

Then it can be shown as before that the most economical phase angle is given by

$$\sin\varnothing = \frac{Y}{X}$$

In this question,

$$X = £4{\cdot}5 \text{ per kVA of maximum demand,}$$

$$Y = £4 \text{ per kVA} \times 12 \text{ per cent} = £0{\cdot}48$$

Hence,

$$\sin\varnothing = \frac{0{\cdot}48}{4{\cdot}5} = 0{\cdot}1067$$

$$\varnothing = 6^\circ\, 7'$$

$$\cos\varnothing = 0{\cdot}9943, \text{ which is the most economical power factor.}$$

Reactive kVA of load $= 100 \sin\varnothing_1$
$= 100 \sin(\text{arc}\cos 0{\cdot}7)$
$= 71{\cdot}41 \text{ kVA}_r$

Reactive kVA of load and condensers $= \text{OB} \sin\varnothing$ (Fig. 134a)
$= \text{OA}\,\dfrac{\cos\varnothing_1}{\cos\varnothing}\sin\varnothing$
$= \text{OA}\cos\varnothing_1{\cdot}\tan\varnothing$
$= 100 \times 0{\cdot}7 \times 0{\cdot}1072$
$= 7{\cdot}50 \text{ kVA}_r$

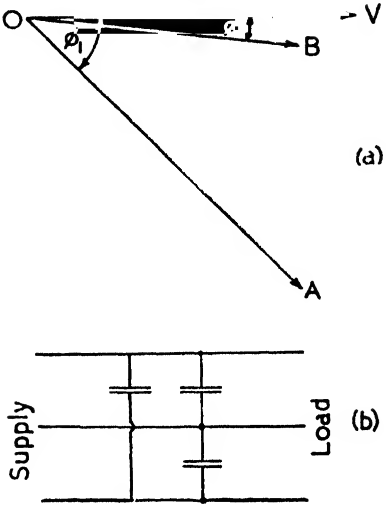


Fig. 134

Therefore, **reactive kVA of condensers** $= 71{\cdot}41 - 7{\cdot}50$
$= \mathbf{63{\cdot}91 \text{ kVA}_r}$

192. *Give an account of the influences of load factor, diversity factor and power factor upon the cost of supply. An industrial load takes* 800000 *units a year, the power factor being* 0·707 *lagging. The recorded maximum demand is* 500 *kVA. The tariff is* £5 *per annum per kVA maximum demand plus* 0·4*d. per unit. Calculate the yearly cost of supply and find the annual saving in cost by installing phase-advancing plant costing* £3 *per kVA, which raises the plant power factor from* 0·707 *to* 0·9 *lagging. Allow* 10 *per cent per annum on the cost of the phase advancing plant to cover all additional costs.*

(*C. and G. Final, Pt. II,* 1945)

Yearly cost of supply = £5 × maximum kVA demand + total kWh × 0·4d.

$$= £5 \times 500 + \frac{£800000 \times 0{\cdot}4}{240}$$

$$= \mathbf{£3833\ 6s.\ 8d.}$$

✓ When the power factor is increased from 0·707 to 0·9 the maximum demand is reduced to $\frac{500 \times 0{\cdot}707}{0{\cdot}9} = 392{\cdot}8$ kVA. The total number of units remains the same at 800000 per annum.

New yearly cost of supply = £392·8 × 5 + £1333 6s. 8d.
= £3297 6s. 8d.

Reactive kVA required to improve power factor from 0·707 to 0·9
= kW demand $(\tan\varnothing_1 - \tan\varnothing)$
where $\varnothing_1 = \text{arc cos } 0{\cdot}707$
$\varnothing = \text{arc cos } 0{\cdot}9$
= 500 × 0·707 (1 − 0·484)
= 182 kVA_r

Annual cost of phase advancing plant = $£182 \times 3 \times \frac{10}{100}$
= £54·6

The annual saving arises from the reduction in the charges for maximum demand, since the charges for the units consumed remains the same throughout.

Therefore, **annual saving** = Initial charges for kVA − (new charges for kVA + annual cost of phase advancers)
= £2500 − £(1964 + 54·6)
= **£481 8s.**

193. *A load of* 500 *kW at a lagging power factor of* 0·75 *is taken by an industrial consumer. The tariff is* £2 *a year per kVA demand plus a flat rate per kWh. Phase advancing condensers cost* £3 *per kVA and all charges associated with the condensers total* 12 *per cent per annum of the capital cost. Calculate* (*a*) *the most economical power factor at which the installation should be operated,* (*b*) *the annual saving in cost effected by improving the power factor to this value.* (*C. and G. Final, Pt. II,* 1937)

(a) Annual charges per kVA demand = £2 = X

Annual charges on condensers per kVA = £3 × 12 per cent
= £0·36 = Y

The most economical power factor is $\cos\varnothing$ where

$$\sin\varnothing = \frac{Y}{X} = \frac{0{\cdot}36}{2} = 0{\cdot}18$$

$$\varnothing = 10^\circ\ 22'$$

Most economical power factor = $\cos\varnothing$ = **0·984**

(b) Without the power factor improvement,

total kVA demanded $= \dfrac{500}{0{\cdot}75}$

$= 666{\cdot}67$ kVA.

Charges for this kVA demand $= £2 \times 666{\cdot}67$

$= £1333{\cdot}3$

When the power factor is improved to 0·984,

new kVA demanded $= \dfrac{500}{0{\cdot}984}$

$= 508$ kVA.

New charges on this kVA demand $= £2 \times 508$

$= £1016$

Saving on kVA demanded $= £1333{\cdot}3 - £1016$

$= £317{\cdot}3$

This saving is partly offset by the annual cost of the condensers used to improve the power factor.

Initial phase angle $= \text{arc cos } 0{\cdot}75$

$= 41^\circ\, 24'$

New phase angle $= \text{arc cos } 0{\cdot}984$

$= 10^\circ\, 22'$

Therefore, as in Problem 185,

reactive kVA to be taken by the condensers $=$ kW demand $(\tan 41^\circ\, 24' - \tan 10^\circ\, 22')$

$= 500\,(0{\cdot}8816 - 0{\cdot}1829)$

$= 349{\cdot}4\ \text{kVA}_r$

Hence, annual cost of the condensers $= 349{\cdot}4 \times £3 \times \dfrac{12}{100}$

$= £125{\cdot}8$

Therefore, **nett saving in cost** $= £317{\cdot}3 - £125{\cdot}8$

$=$ **£191 10s. per annum**

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(*Numbering refers to Problem Numbers*)